

SECTION A (29 Marks)

Examination Number: _____ Maths Teacher: _____

(Govender, Owen, Botha, Willows, Ungere)

QUESTION 1

a) Solve for x if $2x^2 + 9x = 5$ (3)

$$2x^2 + 9x - 5 = 0$$

$$(2x-1)(x+5) = 0 \quad \checkmark \text{ factors}$$

$$x = \frac{1}{2} \quad x = -5 \quad \checkmark a$$

b) Given: $(2x-1)(3-x) \geq 0$ (3)

(1) Solve for x :

$$CV : \frac{1}{2} ; 3$$

$$\frac{1}{2} \leq x \leq 3 \quad \checkmark a$$

a - correct inequality signs

(2) Hence, find the sum of all the integers that satisfy the inequality. (2)

$$\text{Sum} = 1 + 2 + 3 \quad \checkmark ca$$

$$= 6 \quad \checkmark a \text{ (must be 6)}$$

(c) Simplify the expression (5)

$$\sqrt{\frac{3^{2x+1} - 3^{x-1}}{3^{x-2}} + 1}$$

$$= \sqrt{\frac{3^{2x} \sqrt{3} (3 - 3^{-1})}{3^{2x} (3^{-2})} + 1} \quad \checkmark \text{ "cancel"}$$

$$= \sqrt{\frac{8}{3} \sqrt{\frac{1}{9}} + 1} \quad \checkmark \text{ simplify fractions.}$$

$$= \sqrt{25} \quad \checkmark a$$

$$= 5 \quad \checkmark a$$

(d) Solve for y if $3^{2y+1} - 8 \cdot 3^y - 3 = 0$ (5)

Let $k = 3^y \quad \checkmark$
 $\therefore 3k^2 - 8k - 3 = 0 \quad \checkmark a \text{ sub.}$
 $(3k+1)(k-3) = 0 \quad \checkmark \text{ factors.}$

$$k = -\frac{1}{3} \quad k = 3$$

$$3^y \neq -\frac{1}{3} \quad \checkmark \text{ reject } 3^y = 3$$

$$\text{no solution } \underline{y = 1} \quad \checkmark a$$

e) If $g(x) = x - 3$ is a factor of $f(x) = x^3 + (mx)^2 - 11x - 15m$, determine the value(s) of m . (5)

$$f(3) = 0 \checkmark = 0$$

$$3^3 + (3m)^2 - 11(3) - 15m = 0 \checkmark$$

$$9m^2 - 15m - 6 = 0$$

$$3m^2 - 5m - 2 = 0 \checkmark$$

quadratic eq = 0

$$(3m+1)(m-2) = 0 \checkmark$$

$$m = -\frac{1}{3} \checkmark \quad m = 2 \checkmark$$

f) The solution to a quadratic equation is $x = 4 + \sqrt{1-3p}$. Complete the table below by finding a value for p in each case where $p \in \mathbb{R}$. (3)

p	x is:
0	Rational
-2	Irrational
1	Non-real

my table answer.

g) Without solving the equation, describe the roots of the equation $k^2x^2 - kx + \frac{1}{4} = 0$. (3)

$$b^2 - 4ac \checkmark$$

$$= (-k)^2 - 4(k^2)(\frac{1}{4}) \checkmark$$

$$= 0$$

sub in for Δ

Real, rational, equal

QUESTION 2

(a) Given: $f(x) = -\frac{4}{x}$
 Determine $f'(x)$ from first principles. (4)

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-\frac{4}{x+h} - (-\frac{4}{x})}{h} \checkmark$$

sub in correct formula.

$$= \lim_{h \rightarrow 0} \frac{-4x + 4x + 4h}{x(x+h)} \checkmark$$

✓ LCD

$$= \lim_{h \rightarrow 0} \frac{4h}{xh(x+h)} \checkmark$$

"cancel" h

$$= \lim_{h \rightarrow 0} \frac{4}{x(x+h)} \checkmark$$

P-1 for notation

$$= \frac{4}{x^2} \checkmark$$

$\lim_{h \rightarrow 0} (f(x+h) - f(x))^{1/4}$

(b) Evaluate $\frac{dy}{dx}$ if $y = \frac{x^3 - 8\sqrt{x^3}}{4x}$. Leave answers with positive exponents. (5)

$$y = x^2 \sqrt{x} - 2x \checkmark$$

derivative

$$\frac{dy}{dx} = 2x \sqrt{x} - 2 \checkmark$$

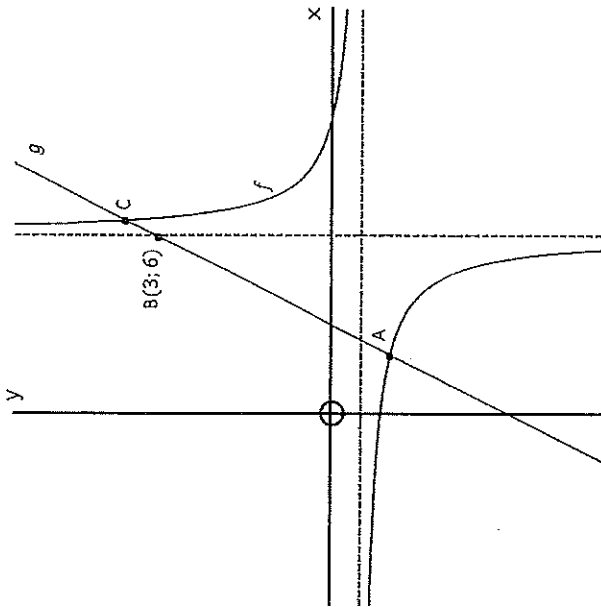
derivative

$$= 2x + \frac{1}{x^{1/2}} \checkmark$$

positive exponent.

QUESTION 3

The sketch below represents the graphs of $f(x) = \frac{2}{x-3} - 1$ and $g(x) = 4x + e$. Point $B(3; 6)$ lies on the graph of g and the two graphs intersect at points A and C.



a) Write down the equations of the asymptotes of f .

$x = 3$ ✓ $y = 1$ ✓ $y = -1$ ✓ $x = 3$ ✓ $y = 1$ ✓ $y = -1$ ✓

b) Show that the equation of $g(x)$ is $y = 4x - 6$, if the graph of g has an angle of inclination of 76° with the x -axis.

$d = \tan 76^\circ = 4\sqrt{a}$ using \tan .

$6 = 4(3) + e$ ✓ sub

$e = -6$ ✓ a

(c) Determine the coordinates of A and C.

(5) equating functions.

$\frac{2}{x-3} - 1 = 4x - 6$ ✓ equating functions.

$\frac{2}{x-3} = 4x - 5$ ✓ simplify

$4x^2 - 17x + 13 = 0$ ✓

$(4x - 13)(x - 1) = 0$

$x = \frac{13}{4}$ $x = 1$ } both x -values.

$y = 7$ $y = -2$

$B(\frac{13}{4}; 7)$ $A(1; -2)$ ✓ must have y -values

(d) For what values of x is $g(x) \geq f(x)$

(3)

$x \in [-1; 3) \cup [\frac{13}{4}; \infty)$ ✓ P-1 for incorrect brackets or inequality signs

OR. $1 \leq x < 3$ ✓ $x \geq \frac{13}{4}$ ✓ \sqrt{a} ✓ \sqrt{a} ✓

(e) Determine the equations of the axes of symmetry of f .

(3)

$y = (x-3) - 1 \rightarrow y = -(x-3) - 1$ ✓ \leftarrow $\frac{a}{a}$ method

$y = x - 4$ ✓ $y = -x + 2$ ✓ \sqrt{a} ✓ \sqrt{a} ✓

Find the equation of the tangent to the curve of $f(x) = \sqrt{x}$ at $x = 9$.

(5)

$f'(x) = \frac{1}{2} x^{-\frac{1}{2}}$ ✓ derivative

ca sub to find gradient

$m = \frac{1}{2} (9)^{-\frac{1}{2}} = \frac{1}{6}$ ✓

$y = \sqrt{9} = 3$ ✓ y-value

$B = \frac{1}{6} (9) + c$ ✓ sub in $y = mx + c$

$\frac{3}{2} = c$ ✓ ca

$y = \frac{1}{6} x + \frac{3}{2}$ ✓ ca

[14]

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(Govender, Owen, Botlia, Willows, Ungerefer)

QUESTION 5

(a) A geometric sequence has $T_3 = 20$ and $T_4 = 40$. Determine the general term of the sequence. (4)

$r = 2$ ✓
 $ar^2 = 20$
 $a = 5$ ✓
 $T_n = 5 \cdot 2^{n-1}$ ✓

(b) Given the sequence: $\frac{2}{1}; -\frac{1}{5}; -\frac{4}{25} \dots$

The terms in the numerator form an arithmetic sequence while the terms in the denominator form a geometric sequence.

(1) Write down the fourth term. (1)

$T_4 = \frac{-7}{125}$ ✓ a

(2) Determine the general term of the sequence. (3)

$T_n = \frac{-3n+5}{5^{n-1}}$ ✓

(3) Which term will be the first in the sequence to have a NUMERATOR smaller than -59 ? (2)

$-3n+5 < -59$ ✓
 $-3n < -64$
 $n > 21,3$ ✓ a
 $\therefore n = 22$ ✓

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QUESTION 6

One hundred and seventy-five movie critics were invited to preview a new movie. After seeing the movie, a survey was conducted and the results were recorded in a two-way contingency table.

	Age < 40	Age ≥ 40	Totals
Liked the movie	65	37	102
Did not like the movie	A	31	B
Totals	C	D	175

(a) Find the values of A, B, C and D.

$A = 42$ ✓ $B = 73$ ✓ $C = 107$ ✓ $D = 68$ ✓
 $\checkmark a$ $\checkmark a$ $\checkmark a$ $\checkmark a$

(b) A critic was chosen at random. What is the probability that the critic was less than 40 years old and did not like the movie.

$P(x) = \frac{42}{175}$ ✓ $\checkmark a$

(c) Given that the critic liked the movie, what is the probability that he/she was ≥ 40 years old?

$P(x) = \frac{37}{102}$ ✓ $\checkmark a$
 sample space.

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QUESTION 7

Given: $P(W) = 0,4$; $P(T) = 0,35$ and $P(T \cap W) = 0,14$

(a) Are the events W and T mutually exclusive? Give a reason for your answer. (2)

No. ✓ answer
 $P(T \cap W) \neq 0$ ✓ reason.

(b) Are the events W and T independent? Give a reason for your answer. (3)

$P(T) \times P(W)$
 $= 0,4 \times 0,35$ ✓
 $= 0,14$ ✓
 \therefore Independent events. ✓

[5]

QUESTION 8

The arithmetic sequence 4; 10; 16; ... is the sequence of first differences of a quadratic sequence with a first term equal to 3. Determine the 50th term of the sequence.

$3 \quad \checkmark \quad 7 \quad \checkmark \quad 17 \quad \checkmark \quad 33$
 $4; 10; 16$
 \checkmark 2nd level difference.
 $2a = 6 \quad 3a + b = 4 \quad a + b + c = 3$
 $a = 3 \checkmark a \quad b = -5 \checkmark a \quad c = 5 \checkmark a$

$T_n = 3n^2 - 5n + 5$

$T_{50} = 7255 \checkmark ca$

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QUESTION 9

In a geometric series, the sum of the first n terms is given by $S_n = p \left(1 - \left(\frac{1}{2} \right)^n \right)$ and

$$i_{\infty} = 10 \text{ and } r = \frac{1}{2}$$

a) Calculate the value of p .

(3)

$$a = p \left(1 - \frac{1}{2} \right) \quad \checkmark \text{ finding } a$$

$$= \frac{p}{2}$$

$$\frac{9}{1-r} = 10 \quad \checkmark \text{ So formula.}$$

$$\frac{p}{2} = 10$$

$$p = 10 \quad \checkmark a$$

b) Determine the second term of the series.

(3)

$$T_2 = S_2 - S_1 \quad \checkmark \text{ sub}$$

$$= 10 \left(1 - \left(\frac{1}{2} \right)^2 \right) - 10 \left(1 - \frac{1}{2} \right) \quad \checkmark \text{ ca}$$

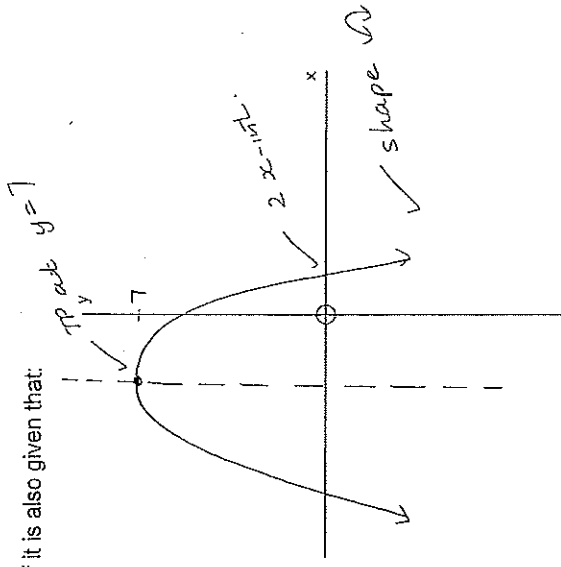
$$= \frac{5}{2} \quad \checkmark a.$$

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QUESTION 10

Sketch the graph of $h(x) = ax^2 + bx + c$ if it is also given that:

- $y \in (-\infty; 7]$
- $a \neq 0$
- $b < 0$
- One root of h is negative and the other is positive.

**QUESTION 11**

[3]

(a) Given $f(x) = 3x^2$

(1) Write down the domain and range of $f^{-1}(x)$.

(2)

$$x \in [0; \infty) \quad \checkmark a$$

$$y \in \mathbb{R} \quad \checkmark a$$

(2) Simplify the following expression: $f(x) + f\left(\frac{1}{x}\right) - \frac{1}{f'(x)}$ to a single term.

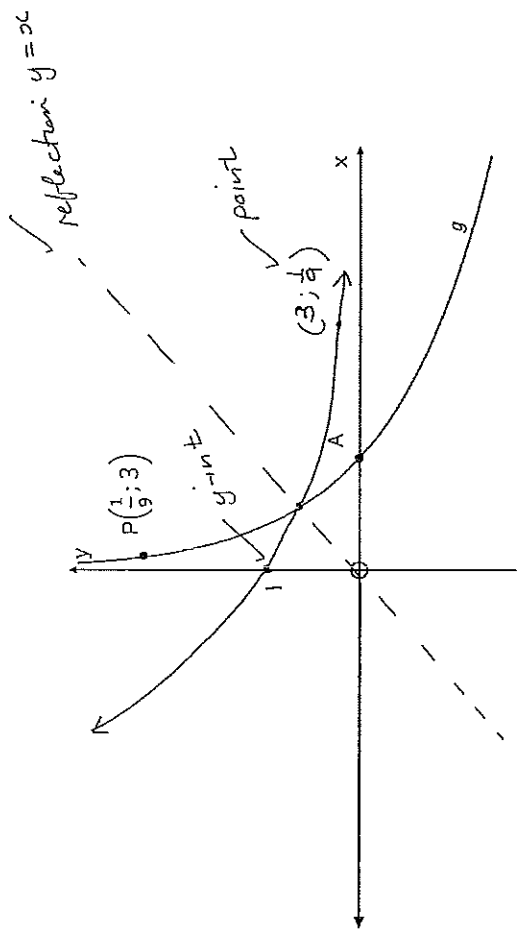
(4)

$$= 3x^2 + 3 \left(\frac{1}{x} \right)^2 - \frac{1}{6x} \quad \checkmark \text{ derivative}$$

$$= 3x^2 + \frac{3}{x^2} - \frac{1}{6x} \quad \checkmark \text{ simplify}$$

$$= \frac{18x^4 + 18 - x}{6x^2} \quad \checkmark a$$

(b) Given the graph of $g(x) = \log_{\frac{1}{9}} x$. A is the x-intercept of g and $P(\frac{1}{9}; 3)$ lies on the graph.



(1) Write down the coordinates of A.
 $A(1, 0)$ ✓ a

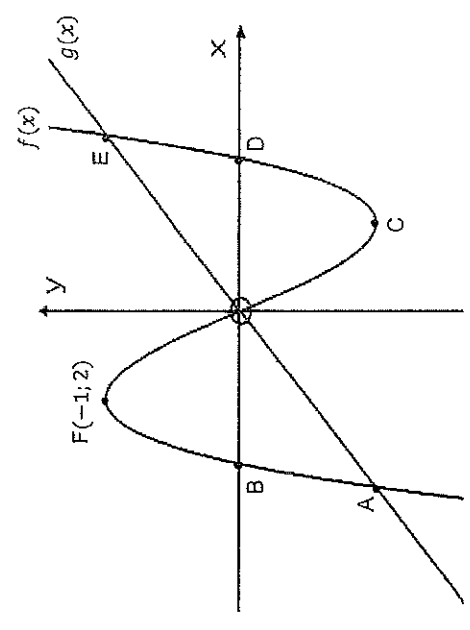
(2) Sketch the graph of $g^{-1}(x)$ on the axes above, indicating any intercepts with the axes and one other point.

(3) Write down the domain of $g^{-1}(x)$.
 $x \in \mathbb{R}$ ✓ a

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QUESTION 12

The graph below represents the functions f and g with $f(x) = ax^3 + bx$ and $g(x) = x$. $F(-1; 2)$ and C are the turning points of f . The graphs of f and g intersect at A , O and E . B , O and D are the x-intercepts of f .



(a) Show that $a = 1$ and $b = -3$. All working details must be shown. (5)

$$3ax^2 + b = 0 \quad \checkmark \text{ derivative} \quad \checkmark \text{ sub } x = -1$$

$$3a(-1)^2 + b = 0 \quad \checkmark \text{ sub } x = -1 \quad \checkmark \text{ solve for } a \text{ and } b.$$

$$3a + b = 0 \quad \dots (1) \quad a + b = -2$$

$$a = 1 \quad b = -3$$

No marks for $a=1$ $b=-3$ } given.

b). Find the coordinates of C. *derivative = 0*

$$3x^2 - 3 = 0$$

$$x^2 - 1 = 0$$

$$(x+1)(x-1) = 0$$

C: $x = 1$ ✓ a
 $y = -2$ ✓ a

c) Show that AC is parallel to the x-axis

$$x^3 - 3x = x \quad \checkmark \text{ equating functions}$$

$$x(x+2)(x-2) = 0$$

$$x = 0 \quad x = \pm 2 \quad \checkmark \text{ solve for } x$$

A : $x = -2$ ✓ a choosing correct x-value.

$\therefore y = -2$ ✓ showing y-values the same.

\therefore AC || x-axis.

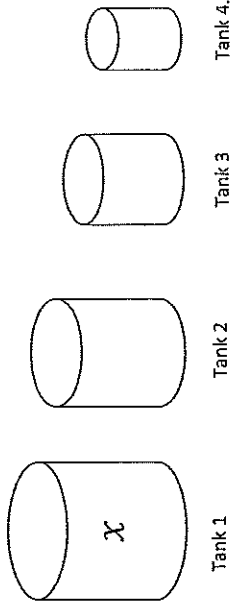
[12]

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QUESTION 13

Twenty water tanks are decreasing in size in such a way that the volume of each tank is half the volume of the previous tank. The first tank is empty, but the other 19 tanks are full of water. Let the volume of Tank 1 be x .



Determine whether it would be possible for the first water tank to hold all the water from the other 19 tanks. **Show all working details.**

$$a = \frac{x}{2} \quad \checkmark a \quad r = \frac{1}{2} \quad \checkmark a$$

$$S_{19} = \frac{\frac{x}{2} (1 - (\frac{1}{2})^{19})}{1 - \frac{1}{2}} \quad \checkmark \text{ sub for } 19 \text{ terms}$$

$$= x \quad \checkmark a$$

\therefore Yes it will hold all the water. [5]

QUESTION 14

(a) The estates department decide to purchase a new machine to mark the fields for different sporting events. The machine costs R220 000. The value of the machine will depreciate by 9% per annum according to a reducing balance interest rate.

(1) Determine the scrap value of the vehicle at the end of 5 years. (2)

$$\begin{aligned} \text{Scrap value} &= 220000 (1 - 0,09)^5 \\ &= \underline{R 137\ 287,07} \end{aligned}$$

(2) After 5 years the machine will need to be replaced. During this time the inflation rate will remain at 7,5% per annum. Determine the cost of a new machine at the end of 5 years. (2)

$$\begin{aligned} \text{New machine} &= 220000 (1 + 0,075)^5 \\ &= \underline{R 315\ 838,45} \end{aligned}$$

(3) The estates department estimates that they will need R165 000 by the end of 5 years. A sinking fund is set up with an interest rate of 8,5% p.a., compounded monthly. The first payment is made immediately and the last payment will be made at the end of the 5 year period. Calculate the monthly payment into the sinking fund. (5)

$$165000 = (x) \left[-1 + \frac{(1 + \frac{0,085}{12})^6 - 1}{\frac{0,085}{12}} \right]$$

✓ correct formula

$$x = R 2171,93$$

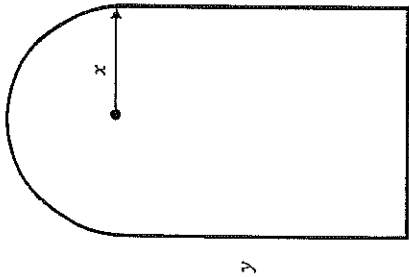
(b) Anthony takes out a loan of R900 000 to realise his dream to travel around the world. He plans to repay the loan on his return by making equal payments of R18 000 per month at an interest rate of 10,5% per annum compounded monthly. His first instalment is paid at the end of the first month back. How long will it take him to pay off the loan? (6)

$$900000 = \frac{18000 \left(1 - \left(1 + \frac{0,105}{12} \right)^{-h} \right)}{\frac{0,105}{12}}$$

✓ correct formula

$$h = 66 \text{ months.}$$

The sketch represents the door frame inside the Kearsney College Chapel.



(a) Determine an expression for the area of the door in terms of x , y and π .

$$A = 2xy + \frac{1}{2} \pi x^2$$

(b) The total area enclosed by the door frame is 14m^2 . Show that the expression for the perimeter of the frame in terms of x is: $P = \frac{14}{x} + 2x + \frac{\pi x}{2}$

$$P = 2y + 2x + \pi x \quad \text{a perimeter} \quad (4)$$

$$2y = \frac{14 - \frac{\pi x^2}{2}}{x}$$

$$= \frac{14 - \frac{\pi x^2}{2}}{x} + 2x + \pi x$$

$$= \frac{14}{x} - \frac{\pi x}{2} + 2x + \pi x \quad \text{simplify fraction}$$

$$= \frac{14}{x} + 2x + \frac{\pi x}{2}$$

(2)

c) Hence show that the perimeter will be a minimum if $x = \sqrt{\frac{28}{4+\pi}}$ (4)

$$\frac{dP}{dx} = -\frac{14}{x^2} + 2 + \frac{\pi}{2} = 0 \quad \text{derivative} \quad \checkmark$$

$$-14 + 2x^2 + \frac{\pi}{2}x^2 = 0$$

$$-28 + 4x^2 + \pi x^2 = 0 \quad \checkmark \text{ factorise}$$

$$x^2(4 + \pi) = 28$$

$$x^2 = \frac{28}{4 + \pi} \quad \checkmark$$

$$x = \sqrt{\frac{28}{4 + \pi}} \quad [10]$$