Memo

KEARSNEY COLLEGE
Founded in 1921

TRIALS EXAMINATION

TUESDAY 9th SEPTEMBER 2014

TIME: 3 HOURS **MARKS: 150**

MATHEMATICS
PAPER II

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY:

1. This question paper consists of **22 pages** with **5 sections**. Please check that your paper is complete. An information sheet has been printed on the inside of the cover sheet.
2. Write down your Examination number and Maths teacher's name on each section in the space provided.
3. Read the questions carefully.
4. Answer all the questions.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated. Ensure that your calculator is in **DEGREE** mode.
6. Round off your answers to **one decimal digit** where necessary.
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.

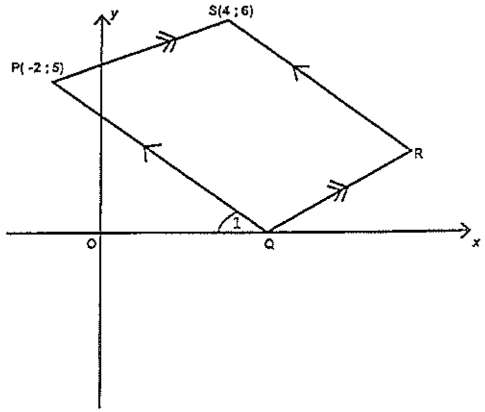
ANSWER ONLY – NO MARKS

SECTION A (29 Marks)

Examination Number: ______________ **Maths Teacher:** ______________
(Govender, Owen, Botha, Willows, Ungerer)

QUESTION 1

In the diagram, PQRS is a parallelogram.
P is the point $(-2\,;5)$, S is the point $(4\,;6)$ and Q is on the x-axis.
The equation of the line RS is given by $2y = -x + 6$



(a) Determine the size of $\hat{Q}_1$ correct to one decimal digit. (3)

$y = -\frac{1}{2}x + 3$ ✓A re-arranging

$m_{PQ} = -\frac{1}{2}$ ✓A $\tan\theta = -\frac{1}{2}$

KA: $26{,}6°$

$P\hat{Q}X = 180° - 26{,}6° = 153{,}4°$

$\therefore \hat{Q}_1 = 26{,}6°$ ✓A

(b) Determine the equation of PQ in the form $y = mx + c$ (3)

$y = -\frac{1}{2}x + c$ ✓A

sub $(-2;5)$ $5 = -\frac{1}{2}(-2) + c$ ✓correct substitution

$5 = 1 + c$

$4 = c$ $\therefore y = -\frac{1}{2}x + 4$ ✓A

(c) Determine the co-ordinates of Q. (2)

Let $y = 0$ $\left.\begin{array}{l} -\frac{1}{2}x + 4 = 0 \\ -\frac{1}{2}x = -4 \end{array}\right\}$ ✓A

$x = 8$

$Q(8;0)$ ✓A must give co-ordinates

[8]

QUESTION 2

(a) Determine the centre and radius of the circle with equation

$(x+2)^2 - 12 = 4y - y^2$ (4)

$(x+2)^2 + y^2 - 4y = 12$ ✓A re-arranging

$(x+2)^2 + y^2 - 4y + 4 = 12 + 4$

$(x+2)^2 + (y-2)^2 = 16$ ✓A

Centre: $(-2;2)$ ✓A   Radius: 4 ✓A

(b) Determine the equation of the tangent to the circle at the point $(2\,;2)$ (2)

$m_{rad}: \frac{2-2}{2+2} = 0$ ✓

$x = 2$ ✓

[6]

QUESTION 3

A large company employs 9 people.
The commission that each person earned (in rands) in August is reflected below.

3 900	5 700	7 300	10 600	13 000	13 600	15 100	15 800	17 100

(a) Calculate the mean of the above data. (2)

$\bar{x} = R\,11\,344{,}44$ ✓✓   Must be 2dp – money to get both marks

(b) Calculate the standard deviation for the data. (2)

$\sigma = R\,4460{,}97$ ✓✓

(c) The company rates the staff according to the amount of commission earned.
A person whose commission is more than one standard deviation above the mean receives rating of "good." How many people will receive a rating of "good" for August? (2)

$(11\,344{,}44\,;15\,805{,}41)$ ✓

$\therefore$ 1 person ✓   Answer only - 2 marks

[6]

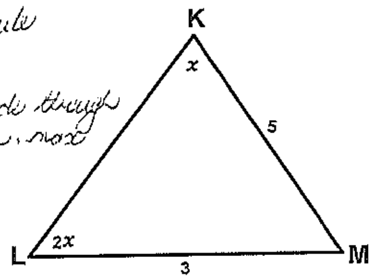
QUESTION 4

(a) If $\sin^2 25^\circ = m$, calculate, **without using a calculator**, the value of $\cos 20^\circ . \cos 70^\circ + \sin 20^\circ . \sin 70^\circ$ in terms of m

$\cos(70^\circ - 20^\circ)$ ✓A
$\cos 50^\circ$
$\cos 2(25^\circ)$ ✓A
$1 - 2\sin^2 25^\circ$ ✓A
$1 - 2m$ ✓A

(4)

(b) In Δ KLM, $KM = 5 units$; $LM = 3 units$; $\hat{K} = x$ and $\hat{L} = 2x$ and $0^\circ < x < 45^\circ$.
Calculate the value of x, giving your answer correct to one decimal digit.



$\frac{\sin 2x}{5} = \frac{\sin x}{3}$ ✓A using sin-rule

*if divide through by sin x, max 3/5

$3\sin 2x = 5\sin x$
$3(2\sin x \cos x) = 5\sin x$
$6\sin x \cos x - 5\sin x = 0$ ✓A
$\sin x(6\cos x - 5) = 0$
$\sin x = 0$ OR $\cos x = \frac{5}{6}$ ✓A
no values in $x \in (0^\circ; 45^\circ)$ ✓A
$x = 33,6^\circ$ ✓A

(5)

[9]

TOTAL : 29 MARKS

SECTION B (31 Marks)

Examination Number: ______________ **Maths Teacher:** ______________
(Govender, Owen, Botha, Willows, Ungerer)

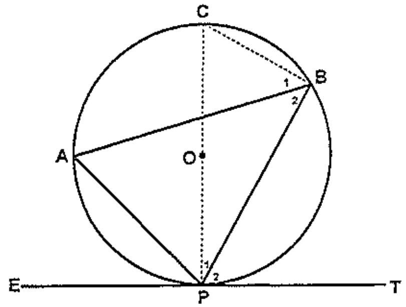
QUESTION 5

Using the given diagram, prove, by filling in the spaces, the theorem that states:

" ***The angle between a tangent to a circle and a chord drawn from the point of contact is equal to the angle in the alternate segment.***"

Given: Circle with centre O ; EPT is a tangent at P
PB is a chord and point A is on the major segment.

Required to prove: $\hat{P}_2 = \hat{A}$



Construction: Draw diameter PC and draw CB.

Proof: $\hat{P}_1 + \hat{P}_2 = 90^\circ$ ✓A — tangent ⊥ radius ✓A
$(\hat{B}_{1+2})$ $C\hat{B}P = 90^\circ$ ✓A — angle in a semi-circle
$\therefore \hat{P}_1 + \hat{C} = 90^\circ$ ✓A — angles of a Δ
$\therefore \hat{P}_2 = \hat{C}$
But $\hat{C} = \hat{A}$ ✓A — angles in the same segment ✓A
$\therefore \hat{P}_2 = \hat{A}$

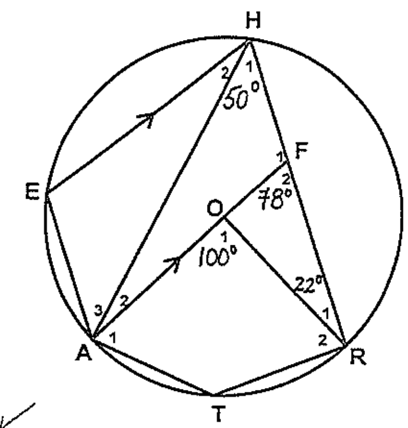
[6]

QUESTION 6

(a) Complete the following statement:

The exterior angle of a triangle is equal to the sum of the interior opposite angles ✓A (1)

(b) In the diagram O is the centre of the circle HEATR.
AOF is parallel to EH
HATR is a cyclic quadrilateral.
$\hat{F}_2 = 78^\circ$ and $\hat{R}_1 = 22^\circ$



Calculate, with reasons, the size of:

(1) $\hat{O}_1$ $= 78^\circ + 22^\circ$ (2)
$= 100^\circ$ ✓ ext angle of a Δ ✓

(2) $\hat{H}_1$ $= 50^\circ$ ✓ angle at centre is twice angle at circumference ✓ (2)

(3) $\hat{T}$ $= 130^\circ$ ✓ opp angles of cyclic quad are supplementary ✓ (2)

(4) $\hat{H}_2$ $H_{1+2} = 78^\circ$ ✓ corresponding angles EH//AF ✓ (3)
must have parallel lines
$\therefore \hat{H}_2 = 28^\circ$ ✓

[10]

QUESTION 7

Determine, correct to one decimal place, the general solution of : $\sin\theta + 2\cos\theta = 0$

$\sin\theta = -2\cos\theta$
$\tan\theta = -2$
KA: $63,4^\circ$
$\theta = 116,6^\circ + k.180^\circ;\ k \in \mathbb{Z}$ ✓A ✓A ✓A

OR

$\sin\theta = -2\cos\theta$
$\tan\theta = -2$
$\theta = -63,4^\circ + k.180^\circ;\ k \in \mathbb{Z}$ ✓A ✓A ✓A

[3]

QUESTION 8

If $\sin 80^\circ = m$, express the following in terms of m

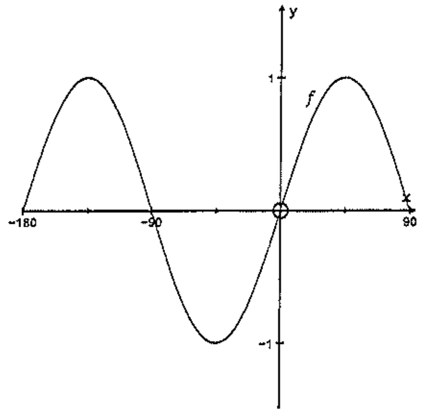
(a) $\cos(-10^\circ)$ (2)
$= \cos 10^\circ$
$-\sin 80^\circ$ ✓A
$= m$ ✓A

(b) $\cos 160^\circ$ (2)
$\cos 2(80^\circ)$
$1 - 2\sin^2 80^\circ$ ✓A
$1 - 2(\sin 80^\circ)^2$
$1 - 2m^2$ ✓A

[4]

QUESTION 9

The graph of $f(x)=\sin 2x$ for $x\in[-180^\circ; 90^\circ]$ is shown below.



(a) Write down the range of f. (1)

$y \in [-1; 1]$ ✓ Absolutely correct to get the mark

(b) Determine the period of $f\left(\frac{3}{2}x\right)$. (1)

$f\left(\frac{3}{2}x\right) = \sin 2\left(\frac{3}{2}x\right) = \sin 3x$

$\frac{360}{3} = 120^\circ$ ✓

(c) Describe the transformation that graph f has to undergo in order to produce the function $y=\sin 2(x+30^\circ)$. (2)

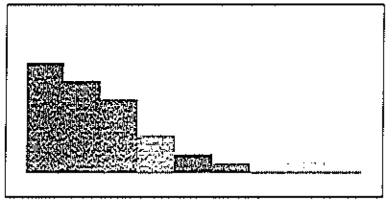
Translation/shift ✓ 30° to the left ✓

[4]

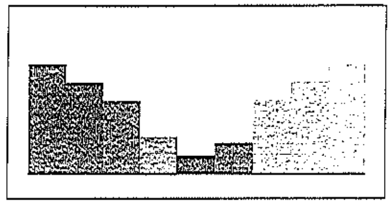
QUESTION 10

Four histograms are given below:

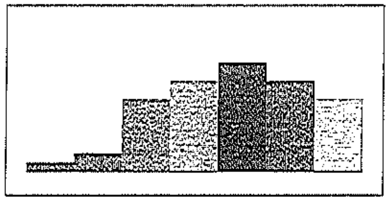
(a)



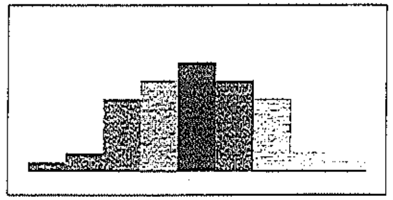
(b)



(c)

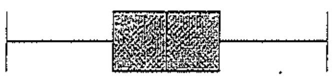


(d)



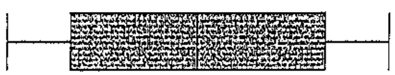
Match the correct box-and-whisker plot to each of the histograms above.

(1)



(2)

(3)



(4)

(a) 2 ✓

(b) 3 ✓

(c) 4 ✓

(d) 1 ✓

[4]

SECTION C (26 Marks)

Examination Number: ______________ Maths Teacher: ______________
(Govender, Owen, Botha, Willows, Ungerer)

QUESTION 11

Consider the expression: $\cos\theta + 2\cos(\theta + 240°)$

(a) Simplify the expression and leave the answer in terms of $\sin\theta$ (5)
[Show all your working details]

$\cos\theta + 2[\cos\theta\cos 240° - \sin\theta\sin 240°]$ ✓A expanding

$= \cos\theta + 2\cos\theta\cos(180°+60°) - 2\sin\theta\sin(180°+60°)$

$= \cos\theta - 2\cos\theta\cos 60° + 2\sin\theta\sin 60°$ ✓ sign ✓ sign

$= \cos\theta - 2\cos\theta\cdot\frac{1}{2} + 2\sin\theta\cdot\frac{\sqrt{3}}{2}$ ✓A correct substitution

$= \sqrt{3}\sin\theta$ ✓

(b) Determine the minimum value of the expression (1)

$y = -\sqrt{3}$ ✓

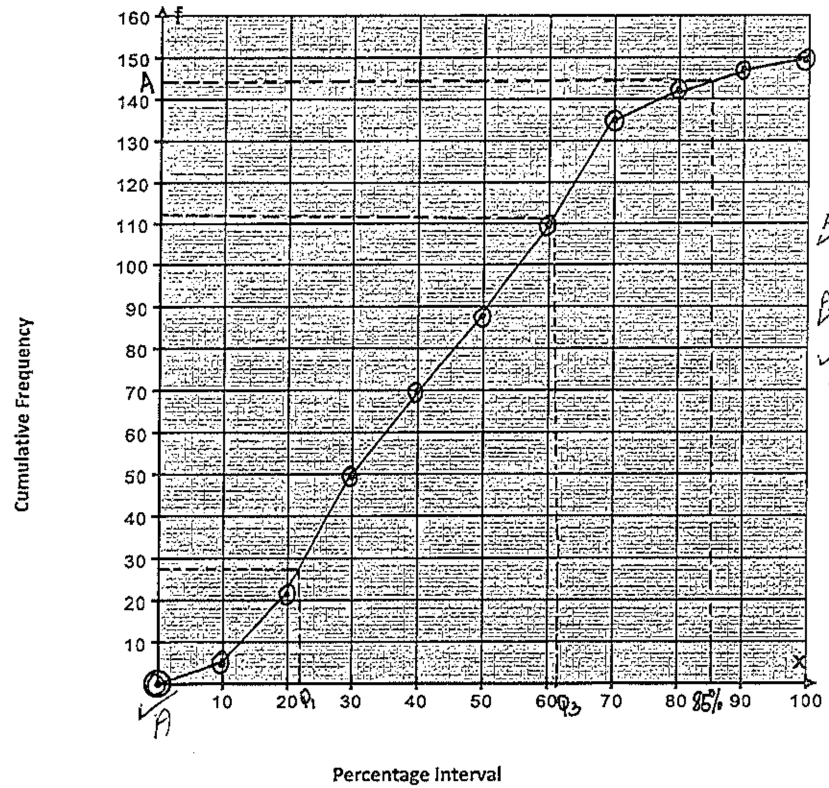
[6]

QUESTION 12

The average percentage of 150 learners for all their subjects is summarised in the cumulative frequency table below.

PERCENTAGE INTERVAL	CUMULATIVE FREQUENCY
$0 < x \le 10$	5
$10 < x \le 20$	21
$20 < x \le 30$	50
$30 < x \le 40$	70
$40 < x \le 50$	88
$50 < x \le 60$	110
$60 < x \le 70$	135
$70 < x \le 80$	142
$80 < x \le 90$	147
$90 < x \le 100$	150

(a) Draw the ogive (cumulative frequency graph) representing the above data (4)



✓A accuracy of points
✓A Q1 ; Q3
✓A 85%

(b) Use the ogive to approximate the following: (Show on the graph where you will find the values used)

(1) the number of learners who scored less than 85%. (2)

at A on graph ✓
(85%; ± 144)
accept 144/145 ✓

(2) the interquartile range (Show ALL calculations) (3)

$Q_1 = 25/26/27$ ✓ must show roughly on graph
$Q_3 = 61/62/63$ ✓ must show roughly on graph

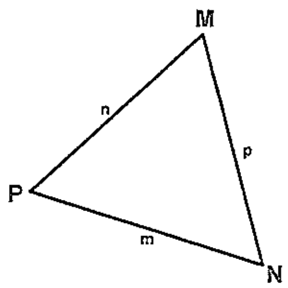
$IQR = 35/36/37$ ✓

[9]

QUESTION 13

In the figure, MNP is any triangle.
Use the figure to show the following:

$$m = \frac{p.\sin M}{\sin(M+N)}$$



✓ using sin rule correctly

$\frac{m}{\sin M} = \frac{p}{\sin P}$ But $P = 180^\circ - (M+N)$ ✓

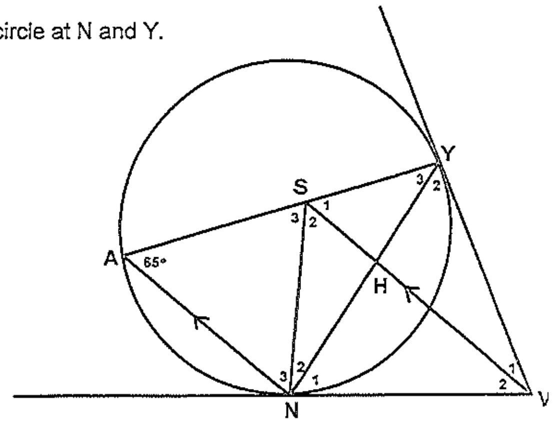
$\frac{m}{\sin M} = \frac{p}{\sin[180^\circ - (M+N)]}$

$m = \frac{p.\sin M}{\sin(M+N)}$ ✓

[3]

QUESTION 14

In the diagram, VN and VY are tangents to the circle at N and Y.
NA // VS and $\hat{A} = 65^\circ$



(a) Write down, with reasons, three other angles also equal to 65° (6)

$\hat{N}_1 = 65^\circ$ ✓A tan-chord theorem ✓ / 2 tangents from same point. angles opp equal sides VY = VN

$\hat{S}_1 = 65^\circ$ ✓A corresponding angles ✓ AN // SV

$\hat{Y}_2 = 65^\circ$ ✓A tan-chord theorem ✓ / 2 tangents from same point. angles opp equal sides VY = VN

(b) Prove that VYSN is a cyclic quadrilateral (2)

$\hat{S}_1 = 65^\circ$
$N_1 = 65^\circ$ } proved above ✓A

∴ VYSN is a cyclic quadrilateral – converse ✓A angles in the same segment

A must have the word.

[8]

TOTAL : 26 MARKS

SECTION D (34 Marks)

Examination Number: ______________ **Maths Teacher:** ______________
(Govender, Owen, Botha, Willows, Ungerer)

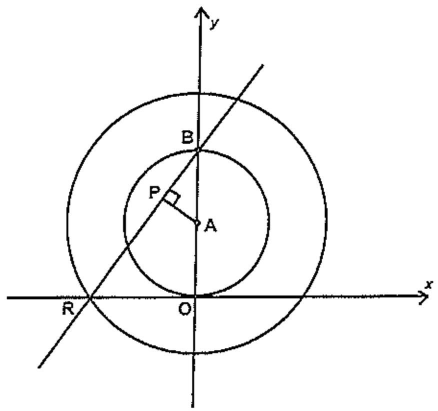
QUESTION 15

In the diagram, two concentric circles are drawn. Both circles have A as their centre and A lies on the y – axis.

The smaller circle cuts the y – axis at the origin O and the point B.

The line through B having equation $y = \frac{3}{2}x + 6$ cuts the x – axis at R.

The larger circle passes through R.



(a) Determine the equations of both circles. (6)

$B(0;6)$ ✓A $A(0;3)$ ✓A

Small circle: centre A:
$(x-0)^2+(y-3)^2=9$
$x^2+(y-3)^2=9$ ✓A

Large circle: centre A:
$(x-0)^2+(y-3)^2=r^2$
sub $(-4;0)$ $16+9=r^2$ ✓A

$\frac{3}{2}x+6=0$
$\frac{3}{2}x=-6$
$x=-4$
$R(-4;0)$ ✓A

$x^2+(y-3)^2=25$ ✓A

(b) Determine the co-ordinates of P, the point on the line passing through B and R, which is closest to A. (5)

$m_{AP}=-\frac{2}{3}$ ✓A

$y=-\frac{2}{3}x+C$ ✓A

But $A(0;3)$ $y=-\frac{2}{3}x+3$

$-\frac{2}{3}x+3=\frac{3}{2}x+6$ ✓A

$-4x+18=9x+36$

$-13x=18$

$x=\frac{-18}{13}$ ✓A

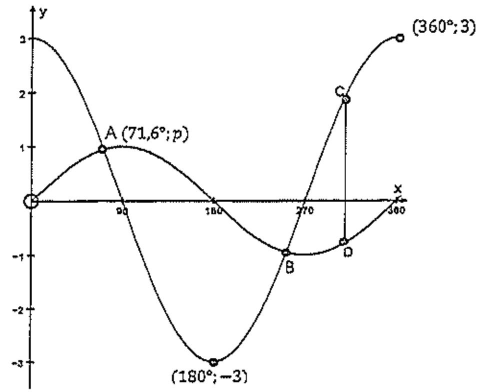
$y=\frac{51}{13}$ ✓A

$P\left(\frac{-18}{13};\frac{51}{13}\right)$

[11]

QUESTION 16

In the diagram, the graphs of $f(x)=3\cos x$ and $g(x)=\sin x$ for $x\in[0^\circ;360^\circ]$ intersect at $A(71,6^\circ;p)$ and B.



Use the graphs to answer the following questions

(a) Determine the value of p, rounded off to two decimal places. (1)

$p=0,95$ ✓

(b) Give the co-ordinates of B. (2)

$B(251,6^\circ;-0,95)$ ✓A ✓A

(c) For what values of x is $f(x)\le g(x)$. (2)

$x\in[71,6^\circ;251,6^\circ]$ ✓numbers ✓brackets

(d) Calculate the length of CD, correct to one decimal digit, given that CD is parallel to the y-axis and the y-co-ordinate of C is $2p$. (3)

C: $y=2(0,95)=1,90$ ✓A $C(x;1,9)$

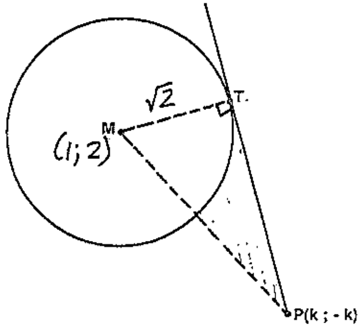
$3\cos x=1,9$
$\cos x=0,633$
$x=50,7^\circ$ OR $x=309,3^\circ$ ✓A

$CD=3\cos x-\sin x$
$=3\cos 309,3^\circ-\sin 309,3^\circ$
$=2,67$
$\approx 2,7$ units ✓A

[8]

QUESTION 17

M is the centre of a circle with equation $(x-1)^2+(y-2)^2=2$.
P is any point outside the circle with co-ordinates $(k;-k)$.
A tangent PT is drawn from P touching the circle at T.



(a) Show that $PT^2 = 2k^2+2k+3$ (5)

$PT \perp MT$ tangent ⊥ radius ✓A

$PM^2 = MT^2 + TP^2$ Pythag ✓A

$(k-1)^2 + (-k-2)^2 = 2 + TP^2$ ✓A ✓correct substitution

$k^2-2k+1+k^2+4k+4 = 2+TP^2$ ✓A

$2k^2+2k+3 = PT^2$

(b) Determine the length, rounded off to one decimal digit, of the shortest possible tangent that can be drawn from P to the circle.

$PT = \sqrt{2k^2+2k+3}$

let $y = 2k^2+2k+3$

$\left.\begin{array}{l}\frac{dy}{dk} = 4k+2 \\ 4k+2=0\end{array}\right\}$ ✓

$k = -\frac{1}{2}$ ✓

$PT = \sqrt{2\left(\frac{1}{4}\right)+2\left(-\frac{1}{2}\right)+3}$ ✓substitution

$= \sqrt{\frac{1}{2}-1+3}$ taking square root

$= \sqrt{\frac{5}{2}}$ ✓ $= 1,6$ ✓

OR $2\left[k^2+k+\frac{3}{2}\right]$

$2\left(k^2+k+\frac{1}{4}+\frac{3}{2}-\frac{1}{4}\right)$ ✓✓✓

$2\left[\left(k+\frac{1}{2}\right)^2+\frac{5}{4}\right]$

$2\left(k+\frac{1}{2}\right)^2+\frac{5}{2}$

$\therefore$ Shortest $PT = \sqrt{\frac{5}{2}}$ ✓

$= 1,6$ ✓

(5)

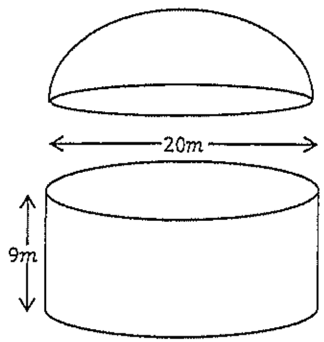
[10]

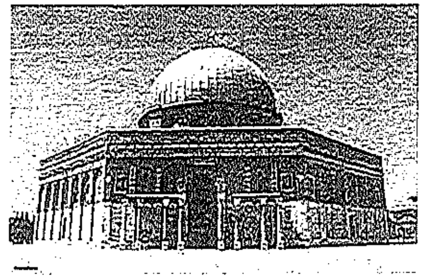
QUESTION 18

The building known as **The Dome of The Rock** in Jerusalem has a cylindrical portion above the main building and a gold hemispherical dome on top of the cylindrical section.

The diameter of the **inside of the dome** is $20\,m$ and **the inside height** of the cylindrical section is $9\,m$.

Formulae you may need: πr^2 ; $2\pi r h$; $4\pi r^2$





Calculate the internal surface area of the cylindrical section and hemisphere. Assume they have the same radii.
[Leave your answer in terms of π]

Surface Area = SA floor + SA cylinder + SA dome

$= \pi r^2 + 2\pi r h + \frac{1}{2}(4\pi r^2)$ ✓ ✓using correct formulae

$= \pi(10)^2 + 2\pi(10)(9) + \frac{1}{2}(4\pi)(100)$ ✓correct substitution

$= 100\pi + 180\pi + 200\pi$ ✓

$= 480\pi\ m^2$ ✓

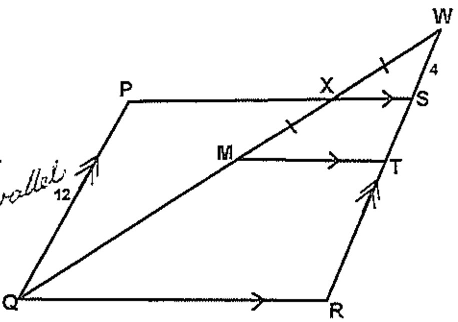
[5]

SECTION E (30 Marks)

Examination Number: ____________________ **Maths Teacher:** ____________________
(Govender, Owen, Botha, Willows, Ungerer)

QUESTION 19

In the diagram, PQRS is a parallelogram. Side RS is extended to W.
WQ intersects PS at X.
M is a point on XQ such that MX = XW.
MT // XS.
PQ = 12 cm. WS = 4 cm.



(a) Calculate the length of TR, giving reasons. (4)

In $\triangle WMT$:

$\frac{ST}{SW} = \frac{MX}{XW} = 1$ ✓ Prop Int Theorem ✓ XS//MT

must have parallel lines

$\frac{ST}{4} = 1$

$ST = 4$ ✓

$\therefore TR = 12 - 4 = 8cm$ ✓ opp sides of parm are equal in length

(b) Determine the value of $\frac{XM}{XQ}$, giving reasons (3)

In $\triangle WQR$

$\frac{WX}{XQ} = \frac{WS}{SR}$ Prop Int Theorem XS//QR ✓

must quote parallel lines

$\frac{WX}{XQ} = \frac{4}{12} = \frac{1}{3}$ ✓

But WX = XM (given)

$\frac{XM}{XQ} = \frac{1}{3}$ ✓

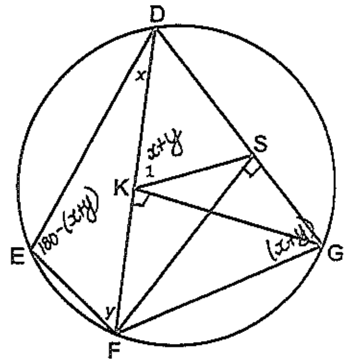
[7]

QUESTION 20

(a) Ring the correct answer

DEFG is a cyclic quadrilateral.

$GK \perp DF$ and $FS \perp DG$



$\hat{K}_1 =$ ____________

A: $180° - x + y$

B: $180° - x - y$

C: $x + y$ ✓✓✓

D: $x - y$

(3)

(b) For each of the following, **state the relationship that exists between m and n.** Show all your working. Reasons are **not** required.

(1) If $a + m = 90°$ and $2a + 2n = 180°$, then ......

$2a + 2m = 180°$

$\therefore 2n = 2m$

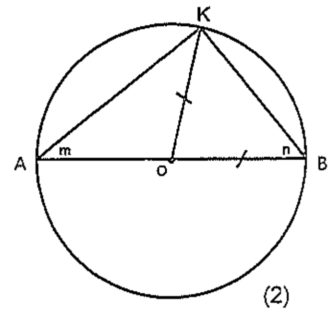
$n = m$ ✓✓

(2)

(2) Diameter AB passes through the centre of the circle O.
K is a point on the circumference.

$K = 90°$

$m + n = 90°$ ✓✓



(2)

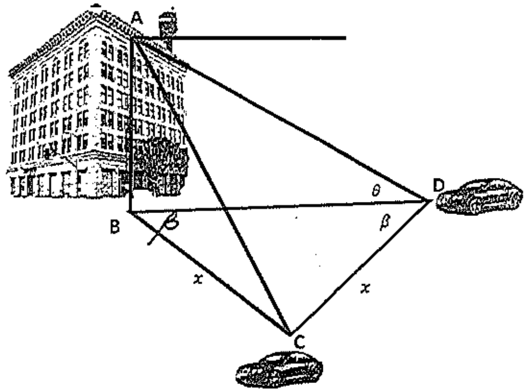
[7]

QUESTION 21

A surveillance camera, A, at the top of a security building shows 2 cars parked outside the Building (B).

The angle of elevation of A from D is θ. Car C is equidistant from Car D and the building B.

Let x denote the distance DC and $C\hat{D}B = \beta$



(a) Find BD in terms of x and $\cos 2\beta$ (4)

$BD^2 = x^2 + x^2 - 2x.x.\cos(180° - 2\beta)$ ✓A ✓A

$= 2x^2 - 2x^2(-\cos 2\beta)$

$= 2x^2[1 + \cos 2\beta]$ ✓A

$BD = x\sqrt{2(1+\cos 2\beta)}$ ✓A

(b) Prove that: $AB = 2x.\cos\beta.\tan\theta$ (3)

In $\triangle ABD$: $\tan\theta = \frac{AB}{BD}$ ✓A

$AB = BD.\tan\theta$

$= x\sqrt{2(1+2\cos^2\beta - 1)}.\tan\theta$ ✓A

$= x\sqrt{(2 + 4\cos^2\beta - 2)}.\tan\theta$

$= x\sqrt{4\cos^2\beta}.\tan\theta$

$= 2x\cos\beta.\tan\theta$

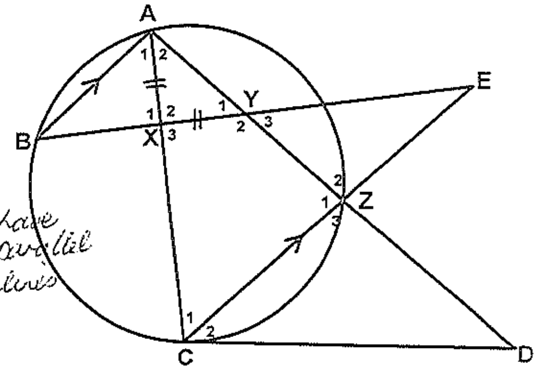
manipulating to get to actual answer ✓A

[7]

QUESTION 22

In the figure, CD is a tangent to the circle through A, B, C and Z.
Line segment BA is parallel to CE and AX = YX.
AC, AD, CE and BE are straight lines

Prove:



(a) $\hat{A}_{1+2} = \hat{C}_{1+2}$ (4)

$\hat{A}_1 = \hat{C}_1$ ✓A alternate angles BA∥CE ✓A must have parallel lines

$\hat{A}_2 = \hat{C}_2$ ✓A tan-chord theorem ✓A

$\therefore \hat{A}_{1+2} = \hat{C}_{1+2}$

(b) $\Delta BAY \mathbin{|||} \Delta DCA$ (3)

In ΔBAY and ΔDCA

$\hat{A}_{1+2} = \hat{C}_{1+2}$ proved

$Y_1 = A_2$ angles opp equal sides $AX = YX$ ✓A ✓A

$\hat{B} = D$ third angle

$\therefore \Delta AYB \mathbin{|||} \Delta CAD$ (a; a; a) ✓A must be correct order

(c) $AC = \frac{AY.CD}{AB}$ (2)

$\frac{AY}{CA} = \frac{YB}{AD} = \frac{AB}{CD}$ ✓A

$AY.CD = AB.AC$

$\frac{AY.CD}{AB} = AC$ ✓A

[9]

TOTAL : 30 MARKS