KEARSNEY COLLEGE
Founded in 1921

TRIALS EXAMINATION

WEDNESDAY 27th AUGUST 2014

TIME: 3 HOURS **MARKS: 150**

MATHEMATICS
PAPER I

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY:

1. This question paper consists of **21 pages** with **5 sections**. Please check that your paper is complete. Formulae have been printed on the inside of the cover sheet.
2. Write down your Examination number and Maths teacher's name on each section in the space provided.
3. Read the questions carefully.
4. Answer all the questions in the space provided.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated. Ensure that your calculator is in **DEGREE** mode.
6. Round off your answers to **one decimal digit** where necessary.
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.

ANSWERS ONLY NO MARKS!

SECTION A (29 Marks)

Examination Number: ______________________________ **Maths Teacher:**__________________
(Govender, Owen, Botha, Willows, Ungerer)

QUESTION 1

(a) Solve for x if $2x^2 + 9x = 5$ (3)

(b) Given: $(2x - 1)(3 - x) \geq 0$

(1) Solve for x: (3)

(2) Hence, find the sum of all the integers that satisfy the inequality. (2)

(c) Simplify the expression $\sqrt{\frac{3^{x+1}-3^{x-1}}{3^{x-2}}+1}$ (5)

(d) Solve for y if $3^{2y+1}-8.3^{y}-3=0$ (5)

(e) If $g(x) = x - 3$ is a factor of $f(x) = x^3 + (mx)^2 - 11x - 15m,$ determine the value(s) of m. (5)

(f) The solution to a quadratic equation is $x = 4 + \sqrt{1 - 3p}$. Complete the table below by finding a value for p in each case where $p \in \mathbb{R}$. (3)

p	x **is :**
	Rational
	Irrational
	Non-real

(g) Without solving the equation, describe the roots of the equation $k^2x^2 - kx + \frac{1}{4} = 0$. (3)

[29]

SECTION B (30 Marks)

Examination Number: ______________________________ **Maths Teacher:**__________________
(Govender, Owen, Botha, Willows, Ungerer)

QUESTION 2

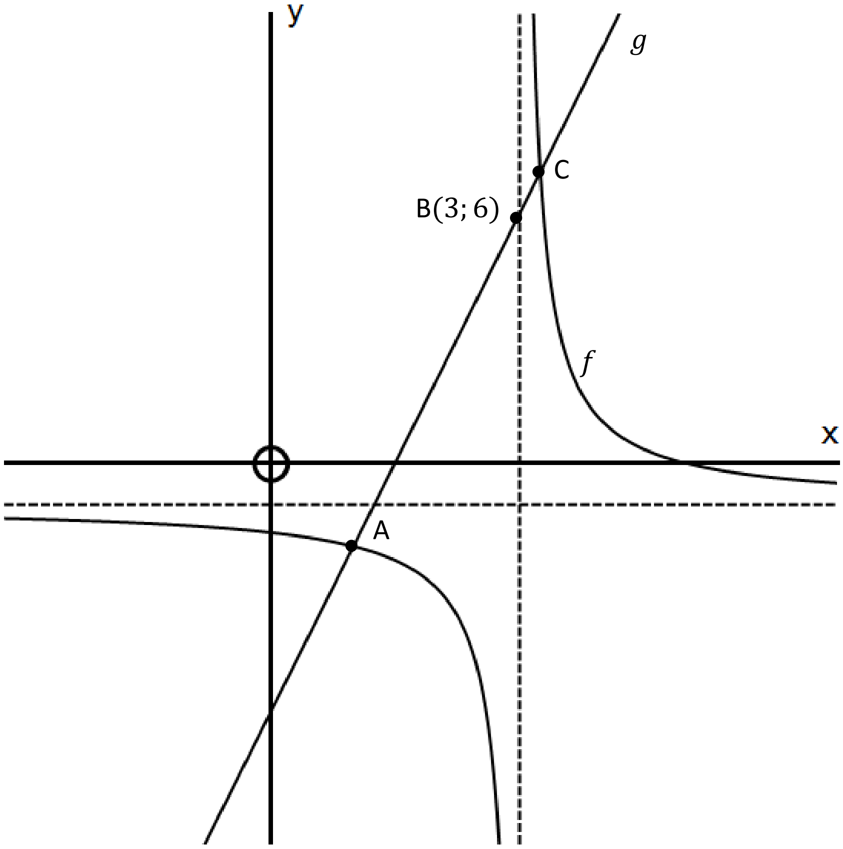
(a) Given: $f(x) = -\frac{4}{x}$

Determine $f'(x)$ from first principles. (4)

(b) Evaluate $\frac{dy}{dx}$ if $y = \frac{x^3 - 8\sqrt{x^3}}{4x}$. Leave answers with positive exponents. (5)

QUESTION 3

The sketch below represents the graphs of $f(x) = \frac{2}{x-3} - 1$ and $g(x) = dx + e$. Point $B\ (3;\ 6)$ lies on the graph of g and the two graphs intersect at points A and C.



(a) Write down the equations of the asymptotes of f. (2)

(b) Show that the equation of $g(x)$ is $y = 4x - 6$, if the graph of g has an angle of inclination of $76°$ with the $x -$ axis. (3)

(c) Determine the coordinates of A and C. (5)

(d) For what values of x is $g(x) \geq f(x)$ (3)

(e) Determine the equations of the axes of symmetry of f. (3)

[16]

QUESTION 4

Find the equation of the tangent to the curve of $f(x) = \sqrt{x}$ at $x = 9$. (5)

[14]

SECTION C (29 Marks)

Examination Number: ______________________________ **Maths Teacher:**__________________
(Govender, Owen, Botha, Willows, Ungerer)

QUESTION 5

(a) A geometric sequence has $T_3 = 20$ and $T_4 = 40$. Determine the general term of the sequence. (4)

(b) Given the sequence: $\frac{2}{1};\ -\frac{1}{5};\ -\frac{4}{25}\ldots$

The terms in the numerator form an arithmetic sequence while the terms in the denominator form a geometric sequence.

(1) Write down the fourth term. (1)

(2) Determine the general term of the sequence. (3)

(3) Which term will be the first in the sequence to have a **NUMERATOR** smaller than -59? (2)

[10]

QUESTION 6

One hundred and seventy-five movie critics were invited to preview a new movie. After seeing the movie, a survey was conducted and the results were recorded in a two-way contingency table.

	Age < 40	**Age $\geq$ 40**	**Totals**
Liked the movie	65	37	102
Did not like the movie	A	31	B
Totals	C	D	175

(a) Find the values of A, B, C and D. (4)

(b) A critic was chosen at random. What is the probability that the critic was less than 40 years old **and** did not like the movie. (2)

(c) Given that the critic **liked** the movie, what is the probability that he/ she was $\geq$ 40 years old? (3)

[9]

QUESTION 7

Given: $P(W) = 0{,}4$; $P(T) = 0{,}35$ and $P(T \cap W) = 0{,}14$

(a) Are the events W and T mutually exclusive? Give a reason for your answer. (2)

(b) Are the events W and T independent? Give a reason for your answer. (3)

[5]

QUESTION 8

The arithmetic sequence $4; 10; 16;$ ... is the sequence of first differences of a quadratic sequence with a first term equal to 3. Determine the 50^{th} term of the sequence.

[5]

SECTION D (32 Marks)

Examination Number: ______________________________ **Maths Teacher:**__________________
(Govender, Owen, Botha, Willows, Ungerer)

QUESTION 9

In a geometric series, the sum of the first n terms is given by $S_n = p\left(1 - \left(\frac{1}{2}\right)^n\right)$ and $S_\infty = 10$ and $r = \frac{1}{2}$

(a) Calculate the value of p. (3)

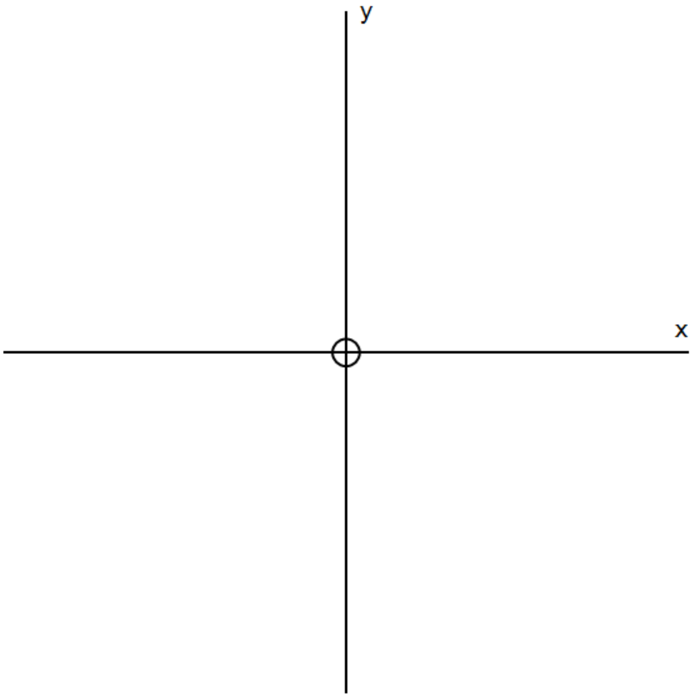
(b) Determine the second term of the series. (3)

[6]

QUESTION 10

Sketch the graph of $h(x) = ax^2 + bx + c$ if it is also given that:

- $y \in (-\infty; 7]$
- $a \neq 0$
- $b < 0$
- One root of h is negative and the other is positive.



[3]

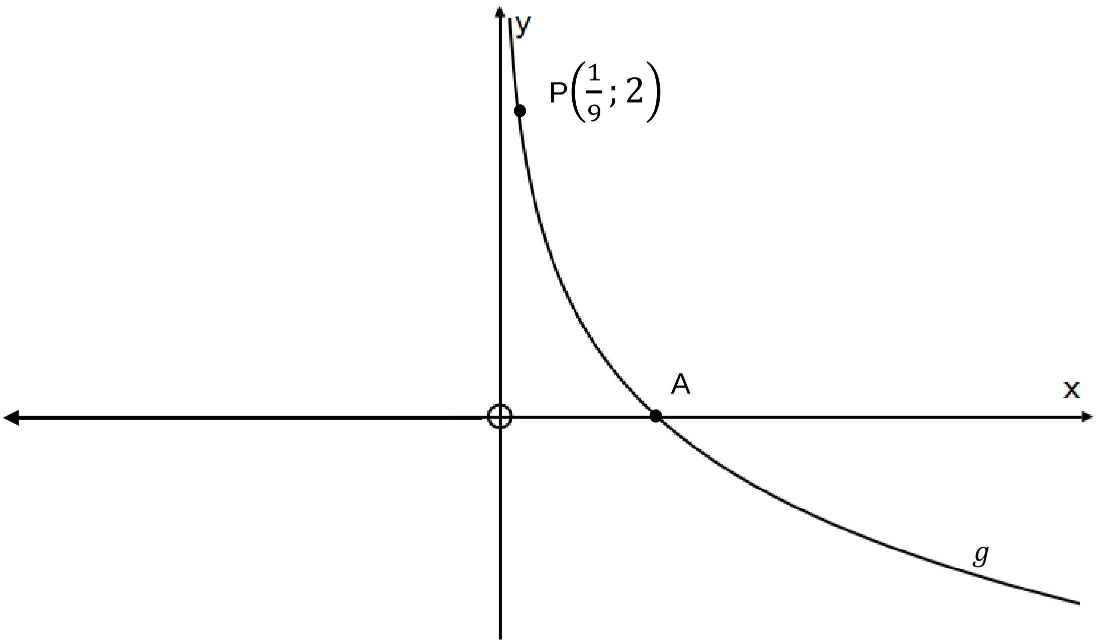
QUESTION 11

(a) Given $f(x) = 3x^2$

(1) Write down the domain and range of $f^{-1}(x)$. (2)

(2) Simplify the following expression: $f(x) + f\left(\frac{1}{x}\right) - \frac{1}{f'(x)}$ to a single term. (4)

(b) Given the graph of $g(x) = \log_{\frac{1}{3}} x$. A is the x –intercept of g and P$\left(\frac{1}{9}; 2\right)$ lies on the graph.



(1) Write down the coordinates of A. (1)

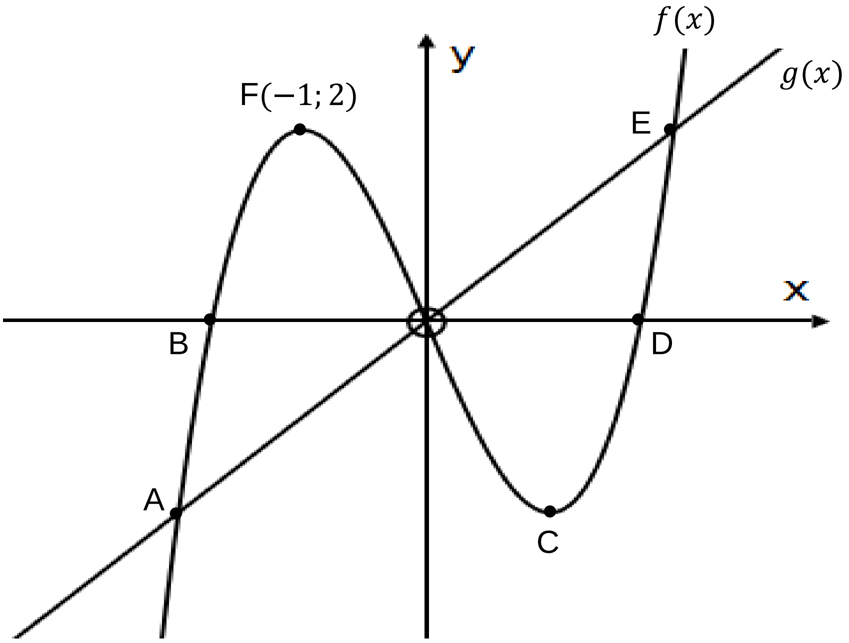
(2) Sketch the graph of $g^{-1}(x)$ on the axes above, indicating any intercepts with the axes and one other point. (3)

(3) Write down the domain of $g^{-1}(x)$. (1)

[11]

QUESTION 12

The graph below represents the functions f and g with $f(x) = ax^3 + bx$ and $g(x) = x$.
F$(-1; 2)$ and C are the turning points of f. The graphs of f and g intersect at A, O and E.
B, O and D are the x –intercepts of f.



(a) Show that $a = 1$ and $b = -3$. All working details must be shown. (5)

(b) Find the coordinates of C. (3)

(c) Show that AC is parallel to the x −axis (4)

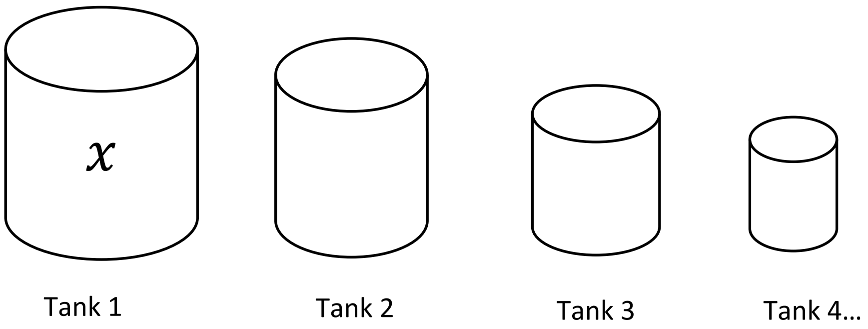
[12]

SECTION E (30 Marks)

Examination Number: ______________________________ **Maths Teacher:**__________________
(Govender, Owen, Botha, Willows, Ungerer)

QUESTION 13

Twenty water tanks are decreasing in size in such a way that the volume of each tank is half the volume of the previous tank. The first tank is empty, but the other 19 tanks are full of water. Let the volume of Tank 1 be x.



Determine whether it would be possible for the first water tank to hold all the water from the other 19 tanks. ***Show all working details.***

[5]

QUESTION 14

(a) The estates department decide to purchase a new machine to mark the fields for different sporting events. The machine costs R220 000. The value of the machine will depreciate by 9% per annum according to a reducing balance interest rate.

(1) Determine the scrap value of the vehicle at the end of 5 years. (2)

(2) After 5 years the machine will need to be replaced. During this time the inflation rate will remain at 7,5% per annum. Determine the cost of a new machine at the end of 5 years. (2)

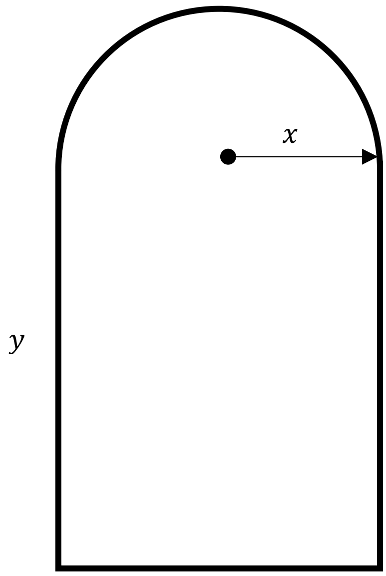
(3) The estates department estimates that they will need R165 000 by the end of 5 years. A sinking fund is set up with an interest rate of 8,5%p.a., compounded monthly. The first payment is made immediately and the last payment will be made at the end of the 5 year period. Calculate the monthly payment into the sinking fund. (5)

(b) Anthony takes out a loan of R900 000 to realise his dream to travel around the world. He plans to repay the loan on his return by making equal payments of R18 000 per month at an interest rate of 10,5% per annum compounded monthly. His first instalment is paid at the end of the first month back. How long will it take him to pay off the loan? (6)

[15]

QUESTION 15

The sketch represents the door frame inside the Kearsney College Chapel.



(a) Determine an expression for the area of the door in terms of x, y and π. (2)

(b) The total area enclosed by the door frame is $14m^2$. Show that the expression for the perimeter of the frame in terms of x is: $P = \frac{14}{x} + 2x + \frac{\pi x}{2}$ (4)

c) Hence show that the perimeter will be a minimum if $x = \sqrt{\frac{28}{4+\pi}}$ (4)

[10]