SOLUTION PRELIM PAPER ONE AUGUST 2014

1 (a) $2x^2 - 5x = 12$

$2x^2 - 5x - 12 = 0$ ✓

$(2x+3)(x-4) = 0$ ✓

$x = -\frac{3}{2}$ or $x = 4$ ✓ (3)

(b) $2 + \sqrt{5x-4} = x$

$\sqrt{5x-4} = x - 2$ ✓

$5x - 4 = (x-2)^2$ ✓

$5x - 4 = x^2 - 4x + 4$

$0 = x^2 - 9x + 8$ ✓

$0 = (x-1)(x-8)$

$x = 1$ or $x = 8$ (5)

invalid ✓ ✓

(c) $4x^2 > 100x$

$4x^2 - 100x > 0$ ✓

$4x(x-25) > 0$ ✓

$x \in (-\infty; 0) \cup (25; \infty)$ ✓ ✓ (4)

2 (a) $\sum_{k=4}^{n} [4 + 2(k+1)] = 140$

$14 + 16 + 18 + \ldots + (2n+6) = 140$ ✓

no of terms $= n - 3$ ✓

$S_n = \frac{n}{2}[a + l]$

$140 = \frac{n-3}{2}[14 + 2n + 6]$ ✓

$140 = (n-3)(n+10)$

$0 = n^2 + 7n - 170$ ✓

$0 = (n+17)(n-10)$

$\therefore \underline{n = 10}$ ✓ (5)

(b)(i) $20^2 = 400$ ✓ (1)

(b)(ii) $3; 7; 13; 21; 31; \ldots$

1st diff: $4 \quad 6 \quad 8 \quad 10$ ✓

2nd diff $2 \quad 2 \quad 2$ $\therefore a = \frac{1}{2}(2) = 1$ ✓

$T_0 = 1 \quad \therefore c = 1$ ✓

$\therefore T_n = an^2 + bn + c$

$T_n = n^2 + bn + 1$ ✓

$3 = 1^2 + b(1) + 1$

$1 = b$ ✓

$\therefore T_n = n^2 + n + 1$ ✓ (6)

(c) $15; \frac{1}{4}; 12; \frac{1}{2}; 9; 1; \ldots$

2 sequences:

$15; 12; 9; \ldots$ ✓ $\frac{1}{4}; \frac{1}{2}; 1; \ldots$

$S_{13} = \frac{13}{2}[2(15) + 12(-3)]$ ✓ $S_{12} = \frac{\frac{1}{4}(2^{12}-1)}{2-1}$ ✓

$= -39$ ✓ $= \frac{4095}{4}$ ✓

$\therefore S_{25} = \frac{3939}{4}$

$= 984{,}8$ ✓ (6)

(d) $10; 0{,}96 \times 10; (0{,}96)^2 \times 10; \ldots$ ✓

$S_\infty = \frac{a}{1-r} = \frac{10}{1-0{,}96}$ ✓ $= 250$cm.

$\therefore 2{,}5$m is sufficient ✓ (3)

3 $f(x) = 2x^3 + ax^2 - b^2x - 3$

$f(-1) = -2 + a - b^2(-1) - 3$ ✓

$9 = -5 + a + b^2 \quad \therefore a = 14 - b^2$ ✓

$f(1) = 2 + a - b^2 - 3$ ✓

$-5 = -1 + a - b^2 \quad \therefore a = -4 + b^2$ ✓

Equating: $14 - b^2 = -4 + b^2$ ✓

$18 = 2b^2$

$9 = b^2$

$\pm 3 = b$ ✓

$\therefore a = 5$ ✓ (7)

4 (a) $A = P(1-i)^n$

$x = 2x(1 - 0{,}13)^n$ ✓

$\frac{1}{2} = (0{,}87)^n$ ✓

$n = \log_{0{,}87}(0{,}5)$ ✓

$= 4{,}977\ldots$ ✓

$\therefore 5$ years. ✓ (5)

(ii) R170000 ✓ (1)

(b)(i) $P = x\left[\frac{1-(1+i)^{-n}}{i}\right]$

$170000 = x\left[\frac{1-(1+\frac{0{,}12}{12})^{-48}}{\frac{0{,}12}{12}}\right]$ ✓ ✓

$x = R4476{,}75$ ✓✓ (4)

(iii) Bal of Loan $= 4476{,}75\left[\frac{1-(1{,}01)^{-28}}{0{,}01}\right]$ ✓

$= R108858{,}69$ ✓✓ (3)

(c) $F = (1+i)\left[\frac{(1+i)^n - 1}{i}\right]$ ✓

$F = 1500(1 + \frac{0{,}105}{12})\left[\frac{(1+\frac{0{,}105}{12})^{36} - 1}{\frac{0{,}105}{12}}\right]$ ✓ ✓

$\underline{F = R63703{,}97}$ ✓ (4)

5 (a) $x = -4$ ✓ (1)

(b) $f(x) = -2x^2 - 4x + 16$

$= -2[x^2 + 2x + 1 - 1 - 8]$ ✓

$= -2(x+1)^2 + 18$ ✓✓

$\therefore M = (-1; 18)$ ✓

At N: $y = \frac{12}{-1+4} + 6 = 10$ ✓

$\therefore MN = 8$ units. ✓ (6)

(c) $y = -2(x-2)^2 + 16$

(d) $\frac{12}{x+4} \leq 4$

$\therefore \frac{12}{x+4} + 6 \leq 10$ ✓

$\therefore x \in (-\infty; -4) \cup (-1; \infty)$ ✓ ✓ (3)

6 (a) $f(x) = b^x$ $\qquad g(x) = \frac{k}{x}$

$9 = b^2 \qquad 9 = \frac{k}{2}$

$b = 3$ ✓ $\qquad k = 18$ ✓ (2)

(b) $f^{-1}(x) = \log_3 x$ ✓✓ (2)

(c) $x \in (0; 2]$ ✓ ✓ (2)

7

(a) Yes ✓ Each x value is only used once. ✓ (2)

(b) $y \in (-6; 6]$ ✓ ✓ (2)

(c) No. ✓ $x = 2$ will be a repeated x value ✓ (2)

(d) $k(3) = 2$ ✓ (1)

8 $f(x) = \frac{2}{x} \qquad f(x+h) = \frac{2}{x+h}$ ✓

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h}$

$= \lim_{h \to 0} \frac{\frac{2x - 2(x+h)}{x(x+h)}}{h}$ ✓

$= \lim_{h \to 0} \frac{-2h}{x(x+h)} \cdot \frac{1}{h}$ ✓

$= \lim_{h \to 0} \frac{-2}{x(x+h)}$ ✓

$= -\frac{2}{x^2}$ ✓ (5)

9 (a) $P - 3V = 2V^3$

$P = 2V^3 + 3V$ ✓

$\frac{dP}{dV} = 6V^2 + 3$ ✓ ✓ (3)

(b) $D_x\left[\frac{x^2 - 4x + 3}{x}\right]$

$= D_x[x - 4 + 3x^{-1}]$ ✓

$= 1 - 3x^{-2}$ ✓ ✓ (3)

(c) $f(x) = 2\sqrt{x} + \frac{1}{x^3} - \sqrt{2}x$

$= 2x^{\frac{1}{2}} + x^{-3} - \sqrt{2}x$

$f'(x) = x^{-\frac{1}{2}} - 3x^{-4} - \sqrt{2}$ (4)

$= \frac{1}{x^{\frac{1}{2}}} - \frac{3}{x^4} - \sqrt{2}$

10. $S = 2\pi rh + 2\pi r^2$ $\quad V = \pi r^2 h$

$375 = \pi r^2 h$

$\frac{375}{\pi r^2} = h$

$\therefore S = 2\pi r \cdot \frac{375}{\pi r^2} + 2\pi r^2$

$= \frac{750}{r} + 2\pi r^2$ (4)

(b) Let $S(r) = 750r^{-1} + 2\pi r^2$

$S'(r) = -\frac{750}{r^2} + 4\pi r$

For min S, $S'(r) = 0$

$\therefore 0 = -\frac{750}{r^2} + 4\pi r$

$750 = 4\pi r^3$

$\sqrt[3]{\frac{375}{2\pi}}$ cm $= r$ (4)

11. (a) $f(x) = a(x+2)^2(x-4)$

$-16 = a(0+2)^2(0-4)$

$1 = a$

$\therefore f(x) = (x+2)^2(x-4)$

$= (x^2+4x+4)(x-4)$

$= x^3 + 4x^2 + 4x - 4x^2 - 16x - 16$

$= x^3 - 12x - 16$ (5)

$\therefore a = 1;\ b = 0;\ c = -12;\ d = -16$

(b) $f'(x) = 3x^2 - 12$

$f'(0) = 3(0)^2 - 12$

$= -12$

$\therefore$ Eqn of tang: $y = -12x - 16$ (3)

(c) $x \in (-\infty; -2] \cup [2; 4]$ (3)

12. (a) $x = -2$; $x = 3$ (2)

(b) $x \in (-2; 3)$ (2)

(c) -4 (1)

(d) A.S. of $f'(x)$: $x = \frac{1}{2}$

$\therefore x$ value $= 1$ (2)

13. (a) $\boxed{3\,|\,3\,|\,2\,|\,1}$

$= 18$ ways of arranging

$\therefore P(2014) = \frac{1}{18}$ (2)

(b) $\boxed{2\,|\,2\,|\,1\,|\,1}$

cannot use zero; 1 if odd

$\therefore P(\text{odd}) = \frac{4}{18} = \frac{2}{9}$ (3)

14. (a) $P(\text{eggs} \cup \text{pasta}) = 0,3 \times 0,25$

$= 0,075$ (1)

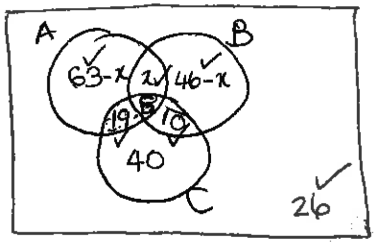
(b) $P(\text{eggs} \cup \text{pasta}) = P(\text{egg}) + P(\text{pasta}) - P(E \cap P)$

$0,075 = 0,3 + 0,25 - P(E \cap P)$

$\therefore P(E \cap P) = 0,55 - 0,075$

$= 0,475$ (3)

15. (a)



(6)

(b) $90 + 46 - x + 50 + 26 = 200$

$\underline{x = 12}$ (3)

16. A = 1002001

B = 7

C = 1003003001

D = 10 (2)

(ii) $4; 7; 10; \ldots$

$T_n = a + (n-1)d$

$T_{11} = 4 + 10(3)$

$= 34$ (2)

(b) $\frac{3^{2014} + (3^2)^{1007}}{(3^3)^{671}}$

$= \frac{3^{2014} + 3^{2014}}{3^{2013}}$

$= \frac{2 \cdot 3^{2014}}{3^{2013}}$

$= 2 \cdot 3$

$= 6$ (4)

SECOND PRELIM PAPER 2 2014

1 (a) $m_{DE} = \frac{3-5a-3}{2+4a-2} = \frac{-5a}{4a} = -\frac{5}{4}$

$m_{DF} = \frac{3-5b-3}{2+4b-2} = -\frac{5b}{4b} = -\frac{5}{4}$

$\therefore$ D, E and F are collinear

(b) The equation is

$y - 1 = -\frac{5}{4}(x-8)$

$y = -\frac{5}{4}x + 11$

(or $4y + 5x - 44 = 0$).

2 (a) $A(-2;0)$ $B(10;4)$

$AB^2 = (-2-10)^2 + (0-4)^2$

$AB^2 = 144 + 16 = 160$

$AB = \sqrt{160} = 4\sqrt{10}$

$\therefore r = 2\sqrt{10}$ & $r^2 = 40$.

Midpt AB $\left(\frac{-2+10}{2}; \frac{0+4}{2}\right)$

$= (4;2)$

$\therefore (x-4)^2 + (y-2)^2 = 40$

is the eqn of the circle.

(b) $m_{AB} = \frac{4-0}{10-(-2)} = \frac{1}{3}$

$\therefore m_{TANG} = -3$ (radius ⊥ tang)

$\therefore$ Eqn of tang is

$y - 4 = -3(x-10)$

$y = -3x + 34$

At C, $y=0$

$\therefore 0 = -3x + 34$

$\therefore C\left(\frac{34}{3};0\right)$.

(c) $BC^2 = \left(\frac{34}{3} - 10\right)^2 + (0-4)^2$

$= \frac{16}{9} + 16 = \frac{160}{9}$

$\therefore BC = \frac{4}{3}\sqrt{10}$ $AB = 4\sqrt{10}$

$\therefore$ Area $\triangle ABC = \frac{1}{2} AB.BC$

$= \frac{1}{2} \cdot \frac{4}{3}\sqrt{10} . 4\sqrt{10}$

(d) At A and D, $y=0$

$\therefore (x-4)^2 + (0-2)^2 = 40$

$(x-4)^2 = 36$

$x - 4 = \pm 6$

$x = 10$ or $x = -2$

$\therefore D(10;0)$

(e) $m_{AM} = \frac{1}{3}$

$\therefore$ inclination of AM $= 18,43°$

$m_{MD} = \frac{2-0}{4-10} = -\frac{1}{3}$

$\therefore$ inclination of MD $= 161,57°$

$\therefore A\hat{M}D = 143,14°$.

3 (a) $x^2 + y^2 - 8x - 10y = a$

$x^2 - 8x + y^2 - 10y = a$

$(x-4)^2 + (y-5)^2 = a + 16 + 25$

$(x-4)^2 + (y-5)^2 = a + 41$

Centre $(4;5)$

$(x-1)^2 + (y-1)^2 = 4$

centre $(1;1)$

(b) distance betwn centres

$= \sqrt{(4-1)^2 + (5-1)^2}$

$= 5$

$\therefore$ $5 = \sqrt{a+41} + 2$ (distance = radius$_1$ + radius$_2$)

$3 = \sqrt{a+41}$

$9 = a + 41$

$-32 = a$

5 (a) $r = 0,67$

positive correlation

(b) $\bar{x} = 93,75$mm

(c) decrease by 10mm

(ii) no change in σ.

(d) $y = 17,16 + 0,12x$

(e)(i) $y = 17,16 + 0,12(40)$

$= 21,96$ tons

(ii) Not necessarily valid as outside the range of given rainfalls (extrapolation).

QUESTION 4 **ANSWER THIS QUESTION ON THIS PAGE**

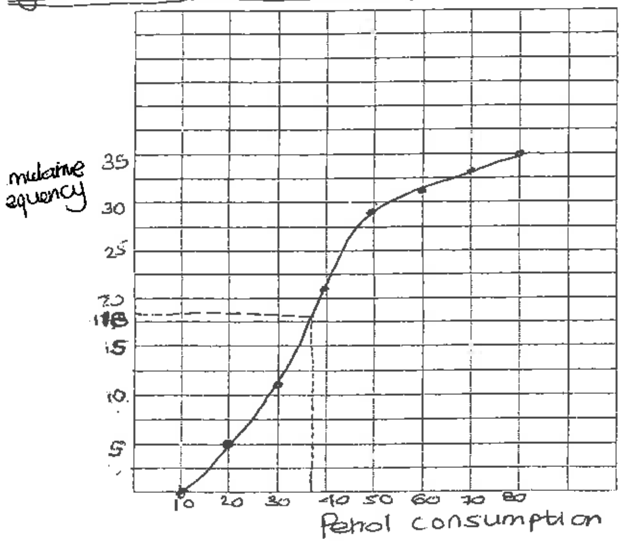
A survey conducted on the petrol consumption (in litres) in a month for a group of 35 King David matric pupils yielded the following results:

Interval (litres)	Frequency	Cumulative frequency
(10 ; 20]	5	5
(20; 30]	7	12
(30 ; 40]	10	22
(40 ; 50]	7	29
(50 ; 60]	2	31
(60; 70]	2	33
(70 ; 80]	2	35

a) Complete the 3rd column of the table (2)

b) Draw the ogive for the data on the grid provided below. (4)

Ogive showing Petrol consumption in litres by KD pupils



c) Determine the median of the data from the ogive, clearly showing where the reading is taken. (2)

median ≈ 38/39 ℓ.

QUESTION 5

The table shows the rainfall in a particular region over a number of years and the corresponding yield in tons of maize.

Rainfall (mm)	Yield (tons)	Rainfall (mm)	Yield (tons)
120	36	75	26
115	32	115	27
50	24	60	29
80	19	135	34

a) Write down the correlation coefficient and discuss the correlation between the two variables. (3)

b) Write down the mean rainfall over the 8 – year period. (1)

c) If the rainfall had been 10 mm less per year, what would the effect be on
   (i) the mean? (1)
   (ii) the standard deviation? (1)

d) Write down the equation of the line of best fit. (2)

e) (i) Estimate the maize yield if the rainfall is 40mm. (2)
   (ii) Discuss the validity of your answer. (2)

KING DAVID HIGH SCHOOL LINKSFIELD

MATHEMATICS GRADE 12

PRELIMINARY EXAMINATION

SEPTEMBER 2014

TRIGONOMETRY PAPER 2

ANSWER BOOKLET

NAME:____________________________________

THIS ANSWER BOOKLET CONSISTS OF 6 PAGES INCLUDING THIS FRONT COVER

QUESTION 6

a) If $\sin 40° = a$, express $\dfrac{2-2\cos^2 40°}{2\sin 20° \cos 340°}$ in terms of a (5)

$\dfrac{2(1-\cos^2 40°)}{2\sin 20° \cos 20°}$

$= \dfrac{2\sin^2 40°}{\sin 40°}$

$= 2\sin 40°$

$= 2a.$

b) (i) Prove that $\dfrac{\cos x - \cos 2x + 2}{3\sin x - \sin 2x} = \dfrac{1+\cos x}{\sin x}$ (5)

LHS $= \dfrac{\cos x - (2\cos^2 x - 1) + 2}{3\sin x - 2\sin x \cos x}$

$= \dfrac{\cos x - 2\cos^2 x + 3}{\sin x(3 - 2\cos x)}$

$= \dfrac{3 + \cos x - 2\cos^2 x}{\sin x(3-2\cos x)}$

$= \dfrac{\cancel{(3-2\cos x)}(1+\cos x)}{\sin x\cancel{(3-2\cos x)}} = \dfrac{1+\cos x}{\sin x} =$ RHS

(ii) For what values of x is the identity NOT defined? (3)

$\sin x = 0$ $\quad 3 - 2\cos x = 0$

$\cos x = 3/2$

$\therefore x = 180°k;\ k \in Z$ $\quad$ No soln

QUESTION 7

a) Show that the equation $2\cos\theta = \sin(\theta + 30°)$ is equivalent to $\sqrt{3}\sin\theta = 3\cos\theta$.
Hence, solve the equation for of $\theta \in (-180°; 180°)$. (6)

$2\cos\theta = \sin(\theta + 30°)$

$2\cos\theta = \sin\theta\cos 30° + \cos\theta\sin 30°$

$2\cos\theta = \sin\theta \cdot \frac{\sqrt{3}}{2} + \cos\theta \cdot \frac{1}{2}$

$2\cos\theta = \frac{1}{2}(\sqrt{3}\sin\theta - \cos\theta)$

$4\cos\theta = \sqrt{3}\sin\theta - \cos\theta$

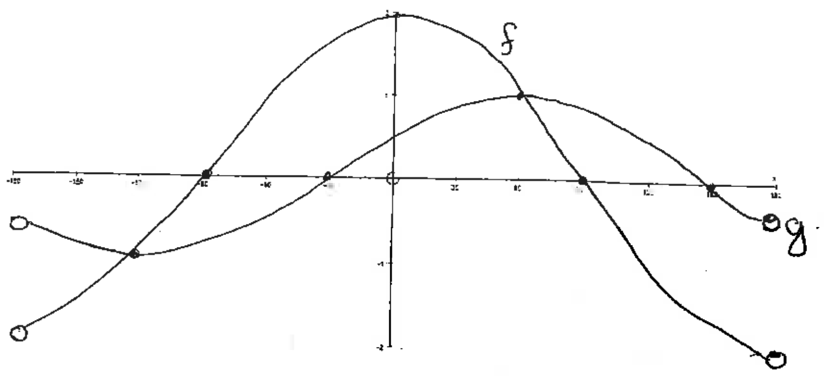
$3\cos\theta = \sqrt{3}\sin\theta$

$\tan\theta = \frac{3}{\sqrt{3}} = \sqrt{3}.$

Gen soln: $\theta = 60° + 180°k;\ k \in Z$

$\theta \in (-180°; 180°)$: $\theta = 60°; -120°$

b) On the same set of axes below, sketch the graphs of $f(\theta) = 2\cos\theta$ and $g(\theta) = \sin(\theta + 30°)$ for $\theta \in (-180°; 180°)$, clearly indicating all intercepts with the axes and the end-points. (5)



c) Using the graphs drawn and your answers to (a), determine:

(i) the values of $\theta \in (0°; 180°)$ so that $2\cos\theta.\sin(\theta + 30°) > 0$. (2)

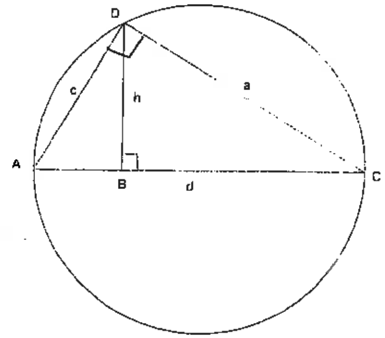
$\theta \in (0°; 90°) \cup (150°; 180°)$

(ii) the values of θ so that $\sin(\theta + 30°)$ increases as θ increases. (1)

$\theta \in (-120°; 60°)$

QUESTION 8

AC is a diameter of the circle and D is a point on the circumference as shown in the figure. DB ⊥ AC.



AC = d; AD = c; DC = a and DB = h.

a) Write down 2 ratios for sinA: (2)

In ΔABD: $\sin A = \frac{h}{c}$

In ΔADC: $\sin A = \frac{a}{d}$

b) Prove that $h = \frac{ac}{d}$ (2)

$\frac{h}{c} = \frac{a}{d}$

$h = \frac{ac}{d}$

c) Hence deduce that $\frac{1}{h^2} = \frac{1}{a^2} + \frac{1}{c^2}$ (4)

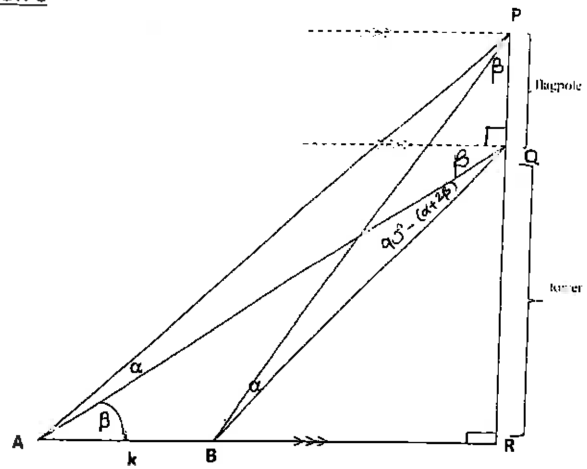
$\frac{1}{h^2} = \frac{d^2}{a^2c^2}$

$= \frac{a^2+c^2}{a^2c^2}$ (By Pythag in ΔADC: $d^2 = a^2 + c^2$)

$= \frac{a^2}{a^2c^2} + \frac{c^2}{a^2c^2}$

$= \frac{1}{c^2} + \frac{1}{a^2}$

QUESTION 9



In the figure, A, B and R are 3 points in a straight line on the horizontal plane. A flagpole PQ on top of a vertical tower QR subtends an angle α both at A and B (as in the diagram). BQ subtends an angle β at A.

a) Why is PABQ a cyclic quadrilateral? (1)

Equal angles subtended at A and B by PQ.

b) Determine $P\hat{Q}A$ and $Q\hat{P}B$ in terms of β. (No reasons required.) (2)

$P\hat{Q}A = 90° + \beta$

$Q\hat{P}B = \beta$ (∠s subtended by BQ)

c) If $A\hat{P}B = 90° - (\alpha + 2\beta)$, write down another angle equal to $[90° - (\alpha + 2\beta)]$. (1)

$A\hat{Q}B$

d) If AB is k, prove that $PQ = \dfrac{k \sin\alpha}{\cos(\alpha + 2\beta)}$ (6)

In ΔABP:

$$\frac{PB}{\sin(\alpha+\beta)} = \frac{k}{\sin[90° - (\alpha+2\beta)]}$$

$$PB = \frac{k \sin(\alpha+\beta)}{\cos(\alpha+2\beta)}.$$

In ΔPQB

$$\frac{PQ}{\sin\alpha} = \frac{PB}{\sin[90° + \beta + 90° - \alpha - 2\beta]}$$

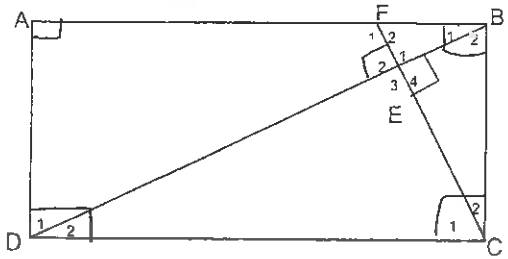
$$PQ = \frac{PB}{\sin[180° - (\alpha+\beta)]}$$

$$= \frac{k \sin(\alpha+\beta)}{\cos(\alpha+2\beta) . \sin(\alpha+\beta)}$$

$$= \frac{k}{\cos(\alpha+2\beta)}.$$

QUESTION 10

In the figure, ABCD is a rectangle. CF ⊥ DB at E.



a) Why is AFED a cyclic quadrilateral? (1)

Opp. angles are suppl. (or ext ∠ equals int opp ∠)

b) Hence prove that BD . BE = BA . BF (5)

In $\triangle ABD$ and $\triangle BEF$

① $\hat{B}_1$ is common

② $\hat{A} = \hat{E}_1 = 90°$ (given)

∴ ③ $\hat{D}_1 = \hat{F}_2$ (3rd ∠s of △s)

∴ $\triangle BAD \,|||\, \triangle BEF$ (A.A.A)

∴ $\frac{BA}{BE} = \frac{BD}{BF}$ (△s |||)

∴ $BA.BF = BD.BE$

c) Prove △ EBC ||| △ ADB, and write down an expression for BD . BE. (4)

① $\hat{E}_4 = \hat{A} = 90°$ (given)

② $\hat{B}_2 = \hat{D}_1$ (alt ∠s = ; AD ∥ BC)

∴ ③ $\hat{C}_2 = \hat{B}_1$ (3rd ∠s of △s)

∴ $\triangle EBC \,|||\, \triangle ADB$ (A.A.A)

∴ $\frac{BC}{DB} = \frac{EB}{AD}$ (△s |||)

∴ $DB.EB = BC.AD$

d) Hence deduce that $BC^2 = BF . BA$ (3)

∴ $BA.BF = BC.AD$ (both = $BD.BE$)

but $BC = AD$ (opp sides of rectangle are =)

∴ $BA.BF = BC.BC$

i.e. $BC^2 = BF.BA$

e) Now show that $\frac{BC^2}{BD^2} = \frac{BE}{BD}$ (2)

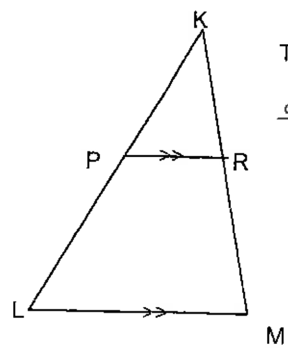
$\frac{BC^2}{BD^2} = \frac{BF.BA}{BD^2}$ and $BE = \frac{BA.BF}{BD}$ (from ⓐ)

$= \frac{BE.BD}{BD^2}$

$= \frac{BE}{BD}$.

QUESTION 11

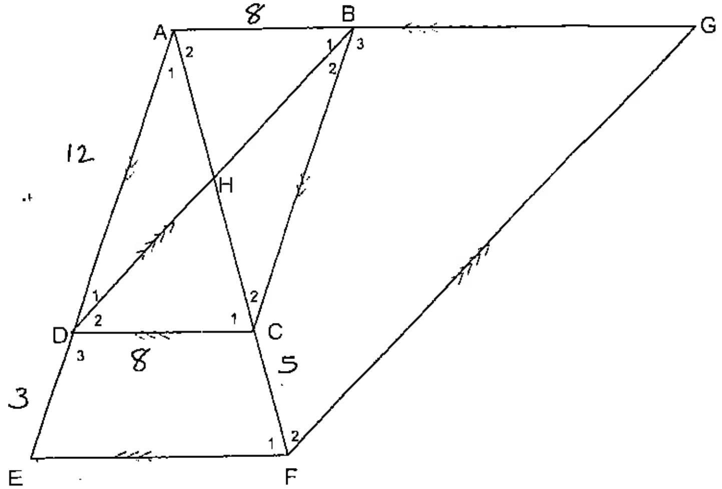
a) Use the figure below to complete the statement. (1)



The line drawn parallel to one side of a triangle

divides the other two sides in proportion.

b) In the figure below, ADCB is a parallelogram. AC and DB intersect at H. ADE, ACF and ABG are straight lines so that DC // EF and DB // FG.



AD = 12 units; DE = 3 units; CF = 5 units and DC = 8 units. Calculate with reasons the lengths of:

(i) AC (2)

In $\triangle AEF$: $DC \parallel EF$ (given)

$\therefore \frac{AC}{CF} = \frac{AD}{DE}$ (prop thm)

$\therefore \frac{AC}{5} = \frac{12}{3}$

$AC = 20$ units

(ii) BG (5)

$AB = DC = 8$ (opp sides of parm =)

In $\triangle AFG$: $BH \parallel FG$ (given)

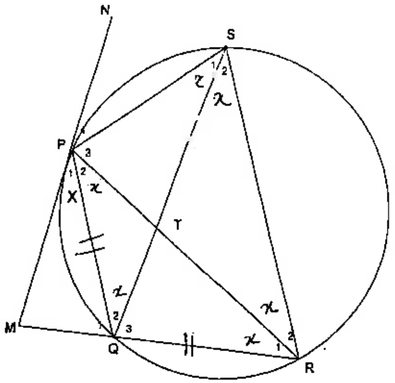
$\therefore \frac{AH}{HF} = \frac{AB}{BG}$ (prop thm)

$\frac{10}{15} = \frac{8}{BG}$ (AH = HC – diags of parm bisect each other).

$BG = 12$ units.

QUESTION 12

In the figure below PQRS is a cyclic quadrilateral. Diagonals PR and QS intersect at T. Tangent MPN and the extension of chord RQ meet at M. PR bisects $Q\hat{R}S$, PQ = RQ.



a) If $\hat{P}_1 = x$, write down with reasons 6 other angles equal to x. (6)

$\hat{R}_1 = x$ (tan - chd thm)

$\hat{S}_1 = x$ (∠s subtended by same arc)

$\hat{P}_2 = \hat{R}_1 = x$ (∠s opp equal sides are =)

$\hat{S}_2 = \hat{P}_2 = x$ (∠s subtended by same arc)

$\hat{P}_4 = \hat{R}_2 = x$ (tan - chd thm)

$\hat{Q}_2 = \hat{R}_2 = x$ (∠s subtended by same arc).

b) Why is PQ // RS? (1)

alternate ∠s are equal.

c) If PQ = PM, calculate the size of $P\hat{S}R$. (4)

PQ = PM (given)

$\therefore \hat{M} = \hat{Q}_1$ (∠s opp equal sides are =)

but $\hat{Q}_1 = 2x$ (ext ∠ of cyclic quad = int. opp ∠)

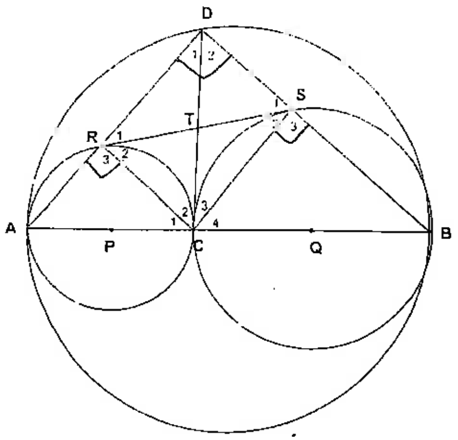
$\therefore \hat{M} + \hat{Q}_1 = 4x$ and $\hat{P}_1 = x$

$5x = 180°$ (sum of ∠s of $\triangle PMQ = 180°$)

$x = 36°$

QUESTION 13

In the figure AB is the diameter of the largest circle, and AC and CB are the diameters of the smaller circles. CD is a common tangent to the 2 smaller circles. DA and DB cut the smaller circles at R and S respectively.



a) Prove that DRCS is a rectangle. (3)

$\hat{D}_1+\hat{D}_2=\hat{R}_3=\hat{S}_3=90°$ (∠s subtended by diameter)

$\therefore \hat{R}_1+\hat{R}_2=\hat{S}_1+\hat{S}_2=90°$ (adj ∠s on str line suppl)

∴ DRCS is a rectangle. (all 4 ∠s = 90°)

b) Prove that RS is a tangent to the circle through R, A and C. (4)

$\hat{C}_2=\hat{A}$ (tan-chrd thm).

TR = TA (diags of a rectangle are equal in length and bisect each other)

$\therefore \hat{R}_2=\hat{C}_2$ (∠s opp equal sides are =)

$\therefore \hat{A}=\hat{R}_2$

∴ RS is a tang. to circle RAC (converse of ~~of~~ tan-chrd thm)

(∠ betwn line RS and chrd RC = ∠ in alt segment)

c) (i) Explain why ΔADC ||| Δ DBC ||| Δ ABD (2)

AC ⊥ DC (rad. ⊥ tang)

∴ Δs ||| because DC is ⊥ in right angled ΔADB

(ii) One of the properties of the diagonals of a rectangle is that they bisect. Write down ANOTHER property of the diagonals of a rectangle. (1)

They are equal in length.

(iii) If P and Q are the centres of the smaller circles, prove that $RS^2 = 4PC \cdot CQ$ (5)

Δ ADC ||| Δ DBC

$\therefore \frac{AC}{DC}=\frac{DC}{BC}$ (Δs |||)

$\therefore DC^2 = AC \cdot BC$

but DC = RS (diags of a rectangle are ~~equal in length~~)

$\therefore RS^2 = AC \cdot BC$

$= 2PC \cdot 2CQ$ (diam = 2 radius)

$= 4PC \cdot CQ$