PAPER 1 MEMO

QUESTION 1

(a) (1) $x(x+3)=0$

$\therefore x=0 \text{ or } x=-3$

$x(x+3)=x+3$

$x^2+3x-x-3=0$

(2) $x^2+2x-3=0$

$(x+3)(x-1)=0$

$\therefore x=-3 \text{ or } x=1$

$2^{6\log x}=2^6$

(b) (1) $\rightarrow 6\log x=6$

$\log x=1$

$\therefore x=10$

$(x-3)^2<9$

$x^2-6x+9-9<0$

(2) $x^2-6x<0$

$x(x-6)<0$

$\therefore 0<x<6$

QUESTION 2

$$x=\frac{-20\pm\sqrt{400-48}}{24}$$

(a) $$x=\frac{-5\pm\sqrt{22}}{6}$$

$\therefore x=-0{,}1 \text{ or } -1{,}6$

$400-48h\geq 0$

(b) $-48h\geq -400$

$$h\leq\frac{25}{3}$$

$400-48h=PS$

(c) $\therefore h=8$

$\triangle=400-48(8)=16$

$\therefore x=0$

(d) (1) $\therefore h=0$

$$x=\frac{-20+20}{24}=0$$

(2) $$x=\frac{-20-20}{24}=\frac{-40}{24}=\frac{-5}{3}$$

QUESTION 3

(a) (1) $d = (3x-2)-(x+1)$

$\therefore d = 2x-3$

(2) $T_6 = (x+1)+(5)(2x-3)$

$\therefore T_6 = 11x-14$

$105 = \frac{6}{2}\left[(x+1)+(11x-14)\right]$

$105 = 3(12x-13)$

(3) $36x-39 = 105$

$36x = 144$

$x = 4$

$1+2+3+.....+x = 55$

$55 = \frac{n}{2}\left[2(1)+(n-1)(1)\right]$

$110 = n(n+1)$

(b) $n^2+n-110 = 0$

$(n-10)(n+11) = 0$

$\therefore n = 10$

$\therefore 55 = \frac{10}{2}(1+x)$

$x = 10$

QUESTION 4

(a)

$$f'(x) = \lim_{h\to 0}\frac{\frac{1}{2}(x+h)^2-9-(\frac{1}{2}x^2-9)}{h}$$

$$= \lim_{h\to 0}\frac{\frac{1}{2}x^2+xh+\frac{1}{2}h^2-9-\frac{1}{2}x^2+9}{h}$$

$$= \lim_{h\to 0}\frac{h(x+\frac{1}{2}h)}{h}$$

$$= \lim_{h\to 0}(x+\frac{1}{2}h)$$

$$= x$$

(b) (1) $f(x) = \frac{(x+2)(x-2)}{-(x-2)} = -x-2$

$\therefore f'(x) = -1$

$D_x\left[x^{\frac{1}{2}}+2kx-k\right]$

(2) $= \frac{1}{2}x^{\frac{-1}{2}}+2k$

$= \frac{1}{2\sqrt{x}}+2k$

$f(3) = (3)^3-20(3)+30 = -3$

$f'(3) = 3(3)^2-20 = 7$

$\therefore y = 7x+c$

(c) $sub\ (3;-3)$

$-3 = 7(3)+c$

$\therefore c = -24$

$\therefore y = 7x-24$

QUESTION 5

$F = 800000(1+0,08)^5$

$F = 1175462,46$

(a) (1) $\therefore$ *He will need*

$1175462,46 - 200000$

$= R\,975462,46$

(2) $975462,46 = \dfrac{x\left[(1,01)^{60} - 1\right]}{0,01}$

$\therefore x = R\,11944,00$

(b)

$$1250000 = \frac{15000\left[1-\left(1+\frac{0,056}{12}\right)^{-n}\right]}{\frac{0,056}{12}}$$

$$\frac{11}{18} = \left(1+\frac{0,056}{12}\right)^{-n}$$

(1) $$-n = \frac{\log\frac{11}{18}}{\log 1,046}$$

$n = 105,8$

$106 / 105$ *months*

Question 5 (b)((2)

If 105 *months* $\rightarrow n = 21$

$$\therefore P = \frac{15000\left[1-\left(1+\frac{0,056}{12}\right)^{-21}\right]}{\frac{0,056}{12}}$$

$P = R\,299392,70$

If 106 *months* $\rightarrow n = 22$

$$\therefore P = \frac{15000\left[1-\left(1+\frac{0,056}{12}\right)^{-22}\right]}{\frac{0,056}{12}}$$

$P = R\,312932,35$

Correct Method :

$$BO = 1250000\left(1+\frac{0,056}{12}\right)^{84} - \frac{150000\left[\left(1+\frac{0,056}{12}\right)^{84} - 1\right]}{\frac{0,056}{12}}$$

$= R\,309914,71$

QUESTION 6

(a)
$$x(13-x)=12$$
$$x^2-13x+12=0$$
$$(x-12)(x-1)=0$$
$$\therefore x=12 \text{ or } x=1$$
$$\therefore A(12;1)$$

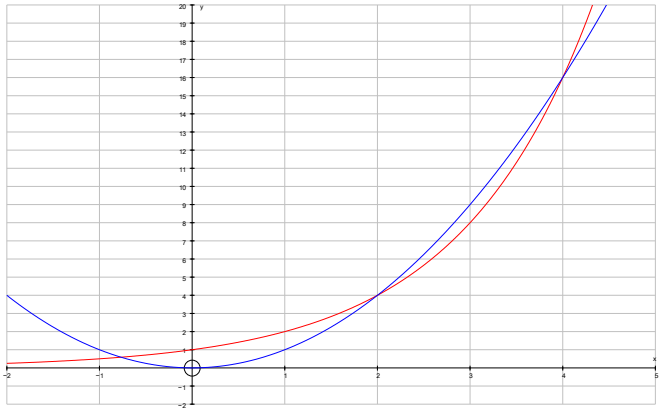
(b)
$$13-x-\frac{12}{x}=6$$
$$13x-x^2-12=6x$$
$$x^2-7x+12=0$$
$$(x-4)(x-3)=0$$
$$\therefore x=4 \text{ or } x=3$$
$$\therefore CN=\frac{12}{4}=3 \text{ units}$$
$$\text{or } CN=\frac{12}{3}=4 \text{ units}$$

(c)
$$BC=13-x-\frac{12}{x}$$
$$Max: \frac{dBC}{dx}=0$$
$$-1+\frac{12}{x^2}=0$$
$$x^2=12$$
$$\therefore x=\sqrt{12} \text{ units}$$

QUESTION 7

(a) $x=2;4$

(b)



(c) $A\approx -0,7 / -0,8$

(d) $2<x<4$

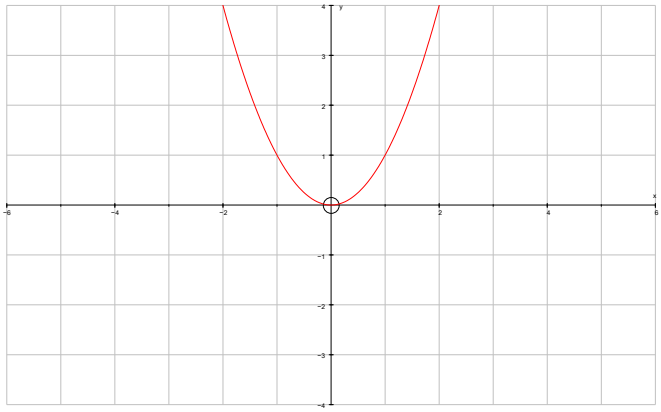
(e)
$$y=2^x \rightarrow x=2^y$$
$$\therefore f^{-1}(x)=\log_2 x$$
$$y=x^2 \rightarrow x=y^2$$
$$\therefore g^{-1}(x)=\pm\sqrt{x};\ x\geq 0$$

(f)

No, $g(x)$ is a many to one function
so $g^{-1}(x)$ is a one to many – not a function

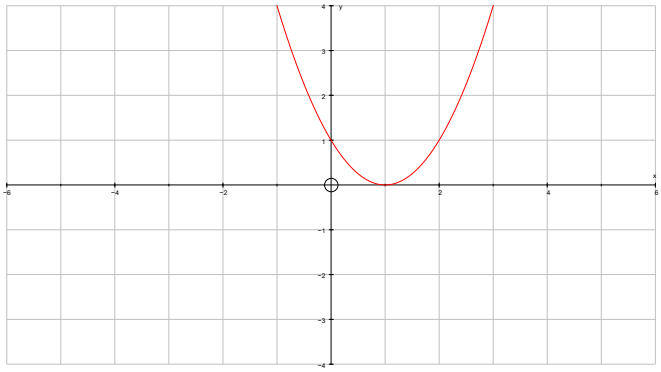
(g)
$$x^2=x$$
$$\therefore x^2-x=0$$
$$x(x-1)=0$$
$$\therefore x=0 \text{ or } x=1$$

(h) (1) $g(-x) = g(x)$

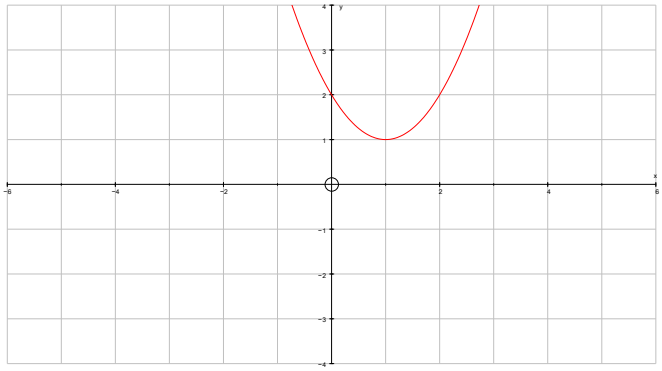


(2)

$g(-x+1) \rightarrow g(x)$ *has moved* 1 *unit to the R*



(3) $g(-x+1)+1 \rightarrow g(x)$ *has moved* 1 *unit to the R and* 1 *unit up*



QUESTION 8

(a) (1) $n = 5 \rightarrow$

$r = x^2 \therefore x^{10} = r^5$

(2)

$a = x\,;\, r = x^2\,;\, n = n$

$\therefore T_n = ar^{n-1} = x(x^2)^{n-1}$

$T_n = x.x^{2n-2}$

$T_n = x^{2n-1}$

(3)

$\frac{1}{3} = 3^{2n-1}$

$\therefore 3^{-1} = 3^{2n-1}$

$\Rightarrow -1 = 2n - 1$

$n = 0$

$T_1 = k - \frac{3}{2}$

$T_2 = \left(k - \frac{3}{2}\right)^2$

(b) (1) $\therefore r = k - \frac{3}{2}$

$Converge: -1 < k - \frac{3}{2} < 1$

$\therefore \frac{1}{2} < k < \frac{5}{2}$

$$S_\infty = \frac{k-\frac{3}{2}}{1-\left(k-\frac{3}{2}\right)}$$

(2) $$= \frac{k-\frac{3}{2}}{\frac{5}{2}-k}$$

$$= \frac{2k-3}{5-2k}$$

$k = 1 \rightarrow S_\infty = \frac{2(1)-3}{5-2(1)} = \frac{-1}{3}$

(3) $k = -1 \rightarrow$ *series does not converge*

$\because r < -1$

QUESTION 9

(a) $A(2\,;0)$

$0 = p(-1)^3 + q(-1) - 2$

$\therefore -p - q = 2 \rightarrow (1)$

$3px^2 + q = 0 \; at \; x = -1$

(b) $\therefore 3p + q = 0 \rightarrow (2)$

$\therefore 2p = 2$

$p = 1$

$\therefore q = -3$

$B: \; f'(x) = 0$

$\therefore 3x^2 - 3 = 0$

$3(x+1)(x-1) = 0$

$\therefore x = -1 \; or \; x = 1$

$at \; B: \; f(1) = -4$

$B(1\,;-4)$

$C: x^3 - 3x - 2 = x - 2$

$x^3 - 4x = 0$

$x(x-2)(x+2) = 0$

$\therefore x = 0 \; or \; x = 2 \; or \; x = -2$

$at \; C: g(-2) = -4$

$C(-2\,;-4)$

(c) $\therefore BC \; // \; x-axis \; same \; y \; value$

(d) $x \leq -1\,; 0 \leq x \leq 1$

QUESTION 10

(a)

$$P(0\,;0)\ Q(x\,;2x-3)$$
$$\therefore PQ^2=(x-0)^2+(2x-3-0)^2$$
$$=x^2+4x^2-12x+9$$
$$=5x^2-12x+9$$

$$PQ^2 \text{ min}$$
$$\therefore \frac{dPQ^2}{dx}=0$$
$$\therefore 10x-12=0$$

(b)

$$x=\frac{6}{5}$$
$$\therefore PQ=\sqrt{5\left(\frac{6}{5}\right)^2-12\left(\frac{6}{5}\right)+9}$$
$$\approx 1,3\ units$$

(c)

$$y=-\frac{1}{2}x$$

QUESTION 11

(a) $3,14159265$

(b)

$$T_1=\frac{1}{16^0}\left[\frac{4}{1}-\frac{2}{4}-\frac{1}{5}-\frac{1}{6}\right]=\frac{47}{15}$$
$$T_2=\frac{1}{16}\left[\frac{4}{9}-\frac{2}{12}-\frac{1}{13}-\frac{1}{14}\right]=\frac{53}{6552}$$
$$T_3=\frac{1}{16^2}\left[\frac{4}{17}-\frac{2}{20}-\frac{1}{21}-\frac{1}{22}\right]=0,0001649$$
$$\therefore S_3=\frac{47}{15}+\frac{53}{6552}+0,0001649$$
$$=3,14158739$$

(c) 4 decimals

QUESTION 12

(a)

$$LHS=35-\left[35^{\frac{1}{2}}\right]^2$$
$$=35-\left[5,9\right]^2$$
$$=35-5^2$$
$$=10$$
$$=RHS$$

(b)

$$35<x<36$$
$$46;\ 59;\ 74;\ 91;\ 110\ldots$$

(c)