MICHAELHOUSE

Mathematics Department

A BLOCK EXAMINATION
CORE MATHEMATICS PAPER 2
SEPTEMBER 2014

Examiner: LH, PS, AAC — Moderator: Mr S B Coxon
Time: 3 hours — Marks: 150

PLEASE READ THE INSTRUCTIONS CAREFULLY

1. This question paper consists of 15 page(s) and an Information Sheet. Please check that your paper is complete.
2. The following questions must be answered on the space provided in the Answer Book:
   Question 2(a); Question 8(d); Question 5; Question 6; Question 12; Question 13 and Question 14.
   The rest of the questions are to be answered on the line paper provided.
3. Read the questions carefully.
4. Answer all the questions.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. All the necessary working details must be clearly shown, giving an answer only will not necessarily give you full marks.
7. It is in your own interest to write legibly and to present your work neatly.
8. Round all answers to **1 decimal place** unless told to do otherwise.
9. Ensure that your calculator is in **DEGREE** mode.

NAME: ____

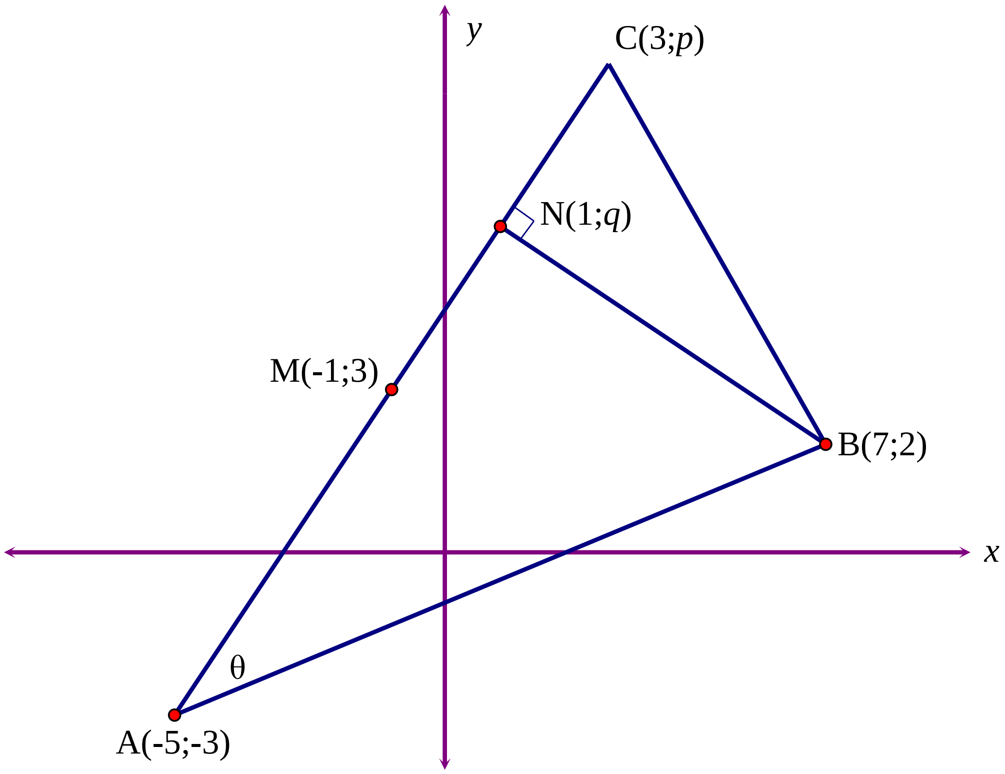
MATHS TEACHER **MEMO**

SECTION A

QUESTION 1

Refer to the sketch below:

A(– 5 ; – 3), B(7 ; 2) and C(3 ; p) are the vertices of ΔABC in the Cartesian plane.
$BN \perp CA$, M(– 1 ; 3) is the midpoint of AC and the co-ordinates of N are (1 ; q).



(a) Find the value of p. (2)

$$\frac{-3+p}{2}=3$$

$$-3+p=6$$

$$\therefore p=9$$

(b) Calculate the gradient of AB. (2)

$$m_{AB}=\frac{2+3}{7+5}$$

$$m_{AB}=\frac{5}{12}$$

(c) Calculate the length of AC. (Leave your answer in surd form.) (2)

$$AC = \sqrt{(-3-9)^2 + (-5-3)^2}$$

$$AC = \sqrt{(-12)^2 + (-8)^2}$$

$$AC = \sqrt{144+64}$$

$$AC = \sqrt{208}$$

$$AC = 4\sqrt{13}$$

(d) Find q if it given that NB = $2\sqrt{13}$. (4)

$$2\sqrt{13} = \sqrt{(1-7)^2 + (q-2)^2}$$

$$52 = 36 + q^2 - 4q + 4$$

$$q^2 - 4q - 12 = 0$$

$$(q-6)(q+2) = 0$$

$$\therefore q = 6 \quad or \quad q = -2$$

$$\therefore q = 6$$

(e) Calculate the area of ΔABC. (2)

$$Area = \frac{1}{2}\left(4\sqrt{13}\right)\left(2\sqrt{13}\right)$$

$$Area = 52 \ square \ units$$

(f) Calculate the measure of θ correct to 1 decimal place. (5)

$$\tan x = m_{AB}$$

$$\tan x = \frac{5}{12}$$

$$\therefore x = 22{,}619........^\circ$$

$$\tan y = m_{AC}$$

$$\tan y = \frac{12}{8}$$

$$\therefore y = 56{,}309........^\circ$$

$$\therefore \theta = 22{,}619........^\circ - 56{,}309......^\circ$$

$$\therefore \theta = 33{,}7^\circ$$

[17 marks]

QUESTION 2

The data below shows the marks of the Grade 12 trial examination the corresponding final examination marks for 11 learners.

Trial examination marks	80	68	94	72	74	83	56	68	65	75	88
Final examination marks	72	71	96	77	82	72	58	83	78	80	92

(a) Draw a scatter plot of the above data on the grid provided on the DIAGRAM SHEET. (3)

(b) Find the equation of the best fit line for the *Final examination mark (y)* against the *Trial examination mark (x)* in the form $y = a + bx$. Determine the values of a and b rounded to 2 decimal digits. (2)

$a = 25{,}38$

$b = 0{,}71$

$y = a + bx$

$y = 25{,}38 + 0{,}71x$

(c) Sketch the line of best fit on the same set of axes (on the DIAGRAM SHEET) showing at least two significant points. (3)

(d) Calculate the correlation coefficient for the above data. What does this tell us about the students trials and final examination marks? (3)

$r = 0{,}7$ Moderately strong, positive correlation

(e) What will the predicted final examination mark be for a learner averaging 50 in the trial examination? (2)

$y = 25{,}38 + 0{,}71x$

$y = 25{,}38 + 0{,}71(50)$

$y = 60{,}9\%$

[13 marks]

QUESTION 3

(a) If $\sin 61^{\circ} = \sqrt{p}$, determine the following in terms of p:

(1) $\sin 241^{\circ}$ (2)

$= \sin(180 + 61)$

$= -\sin 61$

$= -\sqrt{p}$

(2) $\cos 61^{\circ} = \sqrt{1-p}$ (2)

(3) $\cos 73^{\circ} \cos 44^{\circ} + \sin 73^{\circ} \sin 44^{\circ}$ (3)

$= \cos 29$

$= \sin 61$

$= \sqrt{p}$

(b) Prove the identity: $\dfrac{\cos 2x - 1}{\sin 2x} - \tan x = -2\tan x$ (5)

$$LHS = \frac{1 - 2\sin^2 x - 1}{2\sin x \cos x} - \tan x$$

$$= \frac{-\sin x \sin x}{\sin x \cos x} - \tan x$$

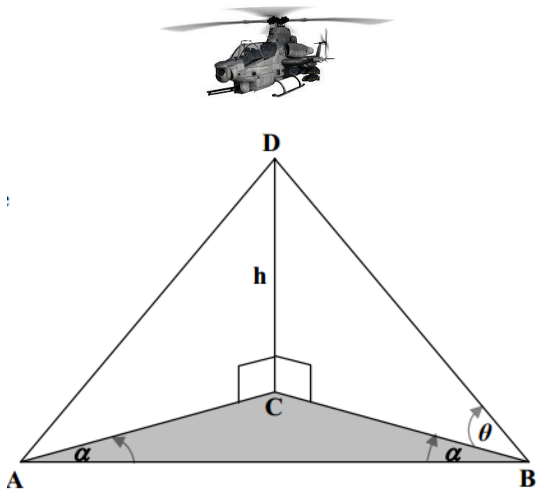
$$= -2\tan x$$

$$= RHS$$

[12 marks]

QUESTION 4

In the diagram below, A, B and C lie in the same horizontal plane. An observer at B sights a helicopter at D, which is *h* metres directly above point C. The angle of elevation of D from B is θ. $A\hat{B}C = B\hat{A}C = \alpha$.



(a) Express BC in terms of *h* and θ. (1)

$$\tan\theta = \frac{h}{BC}$$

$$\therefore BC = \frac{h}{\tan\theta}$$

(b) Hence, show that: Area of $\Delta ABC = \dfrac{h^2 \sin\alpha\cos\alpha}{\tan^2\theta}$. (5)

$$Area\ of\ \ ABC = \frac{1}{2}BC.AC.\sin(180-2\alpha)$$

$$= \frac{1}{2}\left(\frac{h}{\tan\theta}\right)^2 \sin 2\alpha$$

$$= \frac{1}{2}\left(\frac{h}{\tan\theta}\right)^2 2\sin\alpha\cos\alpha$$

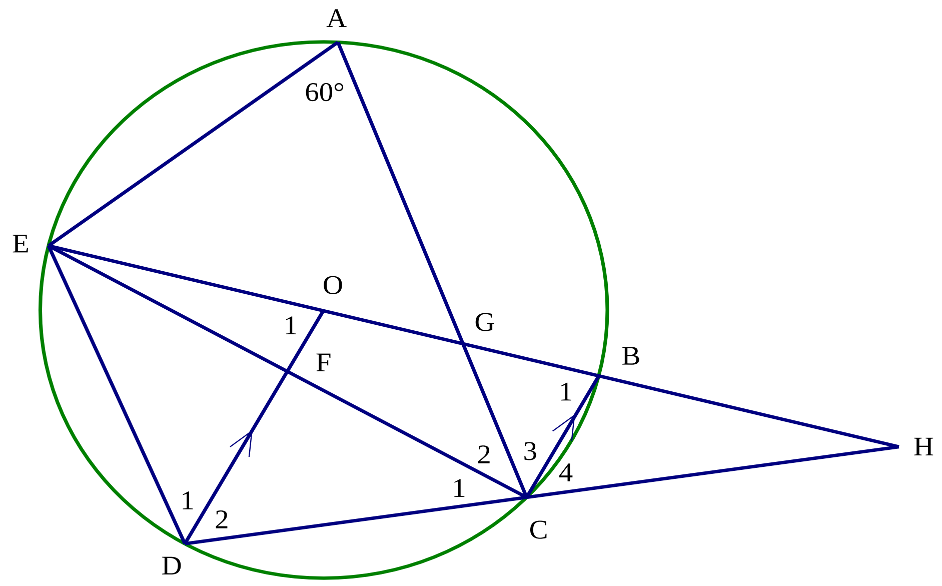
$$= \frac{h^2\sin\alpha\cos\alpha}{\tan^2\theta}$$

(c) If $\theta = 2\alpha = 30^\circ$ and $h = 10m$, find the area of ΔABC. (2)

$$Area = \frac{100.\sin 15\cos 15}{\tan^2 30} = 75units^2$$

[8 marks]

QUESTION 5



In circle ABCDE, centre O, OD//BC and diameter EB is produced at H.
$\hat{A} = 60^o$.
Find, with reasons, the size of:

(a) $\hat{B}_1 = 60^o$ (< s in same seg) (1)

(b) $\hat{O}_1 = 60^o$ (corr < s , OD//BC) (1)

(c) $\hat{C}_1 = 30^o$ (< at circum = ½ < at centre) (1)

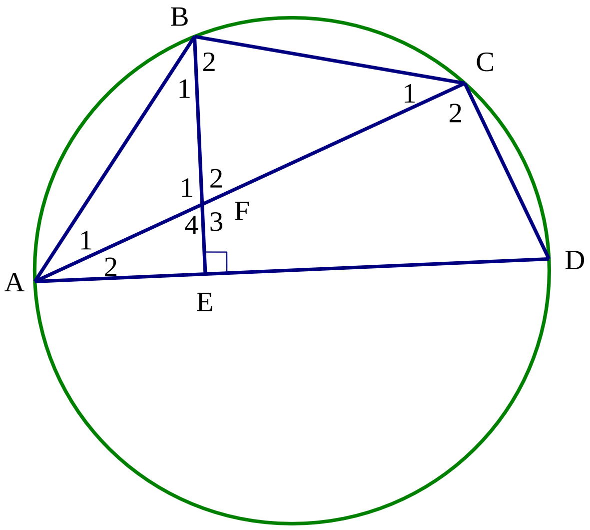
(d) $\hat{D}_{1+2} = 120^o$ (opp < s cyc quad = 180^o) (1)

(e) $C_{2+3} = 90^o$ (< in semi-circle)
$\hat{C}_4 = 60^o$ (< s on a st line) (2)

[6 marks]

QUESTION 6

AD is diameter and BE ⊥ AD. (Hint: let $\hat{A}_1 = x$ and $\hat{A}_2 = y$)



(a) $\hat{B}_1 = 90^o - (x + y)$ (sum < s Δ) (5)

$\hat{C}_2 = 90^o$ (< in semi – circle)

$\hat{C}_1 = 90^o - (x + y)$ (opp < s cyc quad = 180^o)

$\hat{B}_1 = \hat{C}_1$

(b) In $\Delta BAF\ ///\ \Delta CAB$ (4)

① $\hat{B}_1 = \hat{C}_1$ (proved above)

② $\hat{A}_1 = \hat{A}_1$ (common)

③ $\hat{F}_1 = \hat{B}_{1+2}$ (3^{rd} < in Δ)

$BAF\ ///\ \Delta CAB$ AAA

(c) $\frac{BA}{AF} = \frac{CA}{AB}$ $(BAF\ ///\ \Delta CAB)$

$AB^2 = AF \cdot AC$ (2)

(d) $AB^2 = AF \cdot AC$

$49 = 5 \cdot AC$

$\frac{49}{5} = AC$

$CF = \frac{49}{5} - 5$

$FC = \frac{24}{5}$

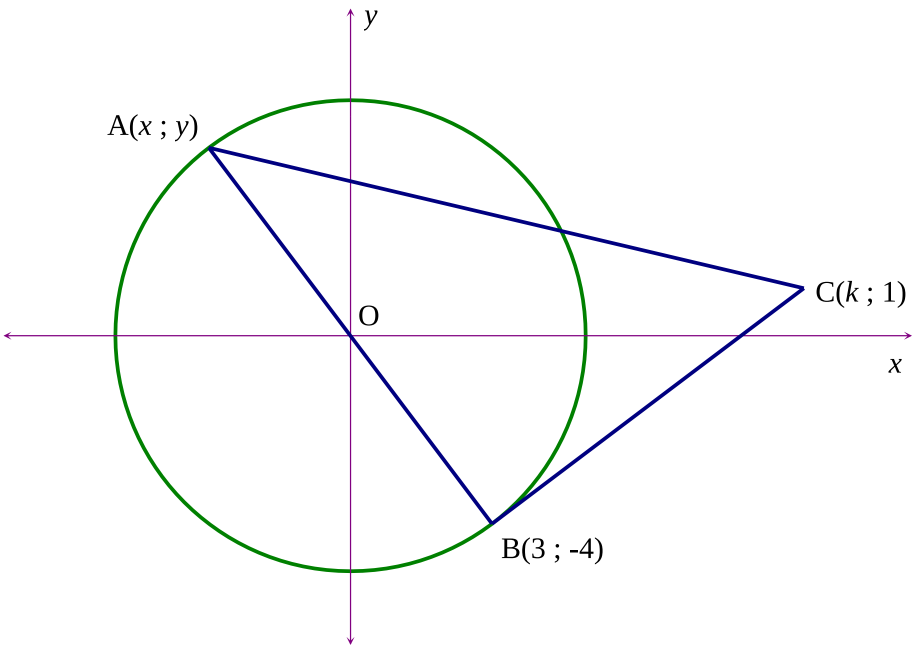
(3)

[14 marks]

SECTION B

QUESTION 7

In the figure below, the origin is the centre of the circle.
A(x ; y) and B(3 ; – 4) are two points on the circle.
AB is a diameter of the circle and BC is a tangent to the circle at B.
C is the point (k ; 1).



(a) Determine the equation of the circle with centre O. (2)

$OB=\sqrt{(0+4)^2+(0-3)^2}$

$OB=\sqrt{25}$

$OB^2=25$

$\therefore x^2+y^2=25$

(b) Show that the length of AB is 10. (2)

radius = 5 units, therefore AB = 10 units

(c) Write down the equation of the circle with centre B and radius AB in the form $\mathrm{A}x^2+\mathrm{B}x+Cy^2+Dy+E=0$. (3)

$(x-3)^2+(y+4)^2=10^2$

$x^2-6x+9+y^2+8y+16=100$

$x^2-6x+y^2+8y-75=0$

(d) Explain why the coordinates of the point A are (– 3 ; 4). (1)

A is the image of B when B is rotated through an angle of 180° about the origin/Symmetry.

(e) Calculate the gradient of line AB. (2)

$$m_{AB} = \frac{-4-0}{3-0}$$

$$= -\frac{4}{3}$$

(f) Determine the equation of the tangent BC. (4)

$m_{BC} = \frac{3}{4}$ ... tangent $\perp$ radius

Substitute (3 ; – 4)

$$-4 = \frac{3}{4}(3) + c$$

$$-4 = \frac{9}{4} + c$$

$$-16 = 9 + 4c$$

$$c = -\frac{25}{4}$$

$$\therefore y = \frac{3}{4}x - \frac{25}{4}$$

(g) Determine the value of k. (2)

Substitute (k ; 1) into $y = \frac{3}{4}x - \frac{25}{4}$

$$1 = \frac{3}{4}(k) - \frac{25}{4}$$

$$4 = 3k - 25$$

$$29 = 3k$$

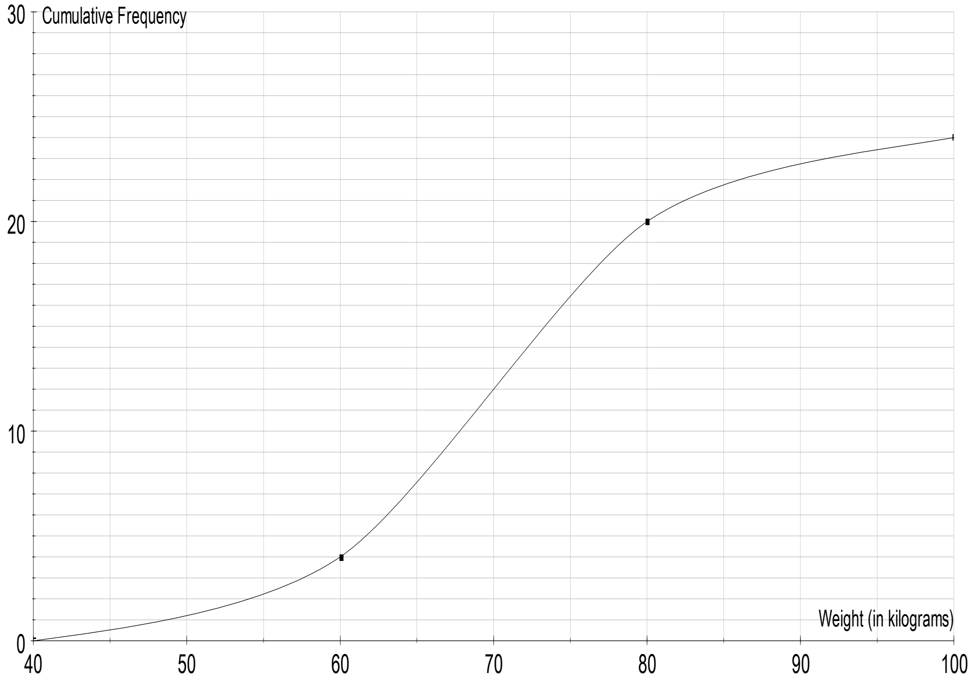
$$k = \frac{29}{3}$$

[16 marks]

QUESTION 8

The weights of a random sample of boys in Grade 12 were recorded. The cumulative frequency graph (ogive) represents the recorded weights.

Cumulative frequency curve showing weight of boys



(a) How many of the boys weighed between 80 and 100 kilograms? (1)

4 boys

(b) Estimate the median weight of the boys. (1)

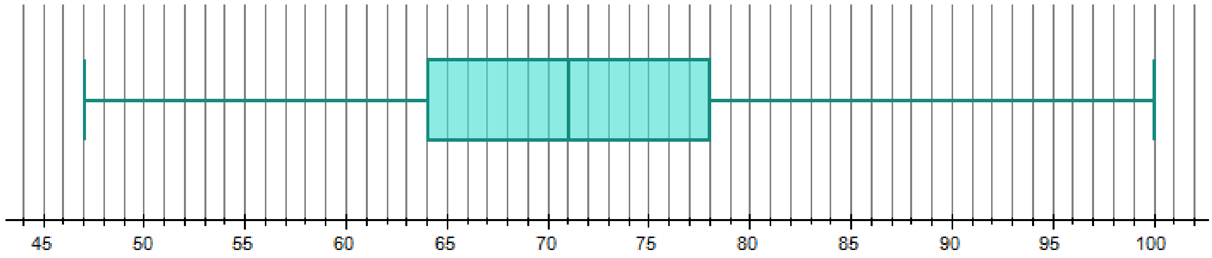
71kg

(c) If there were 250 boys in Grade 12, estimate how many of them would weigh less than 80 kilograms? (2)

$\frac{20}{24} \times 250$

$= 208$

(d) If the lowest recorded weight was 47 kilograms and the highest 100 kilograms, draw a box and whisker plot of the data from the ogive using the axes provided. (3)



(e) One of the weights recorded was 99 kilograms. By performing the necessary calculations, determine whether this value is an outlier or not. Use ($Q_1 - 1{,}5*IQR$; $Q_3 + 1{,}5*IQR$). (3)

$\left(64 - 1{,}5 \times 14; 78 + 1{,}5 \times 14\right)$

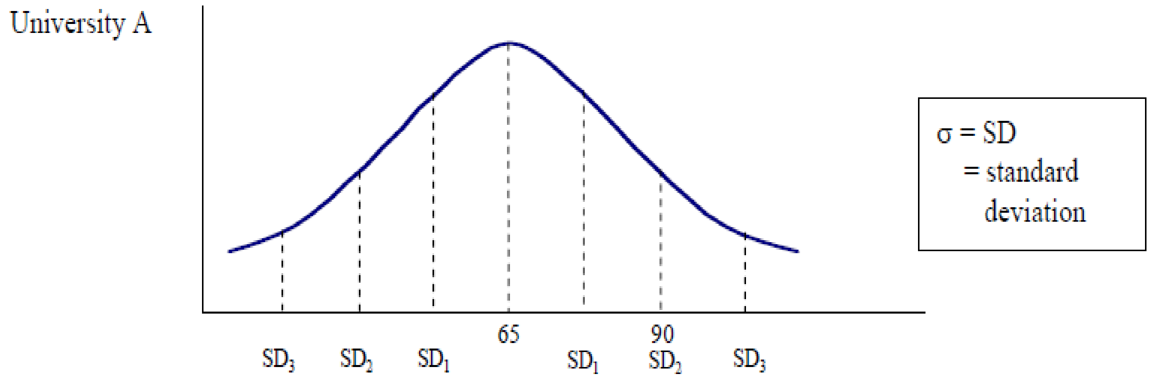
$\left(43; 99\right)$

$\therefore$ this weight is on the borderline of being an outlier.

[10 marks]

QUESTION 9

Two universities administer Mathematics entrance examination for prospective first-year students. Renata wrote the entrance examinations at both universities. The normal distribution graph below represents the results for University A.



University B revealed the following:

Mean $(\overline{x}) = 49$

Standard Deviation $(\sigma) = 5$

(a) Calculate the Standard Deviation for University A's entrance examination results. (2)

$$\sigma = \frac{90-65}{2}$$

$$\sigma = 12{,}5$$

(b) Renata's results were as follows:

University A : 78
University B : 60

In which entrance examination did she perform better in comparison with the other students who took the test? Motivate your answer with relevant calculations. (2)

University A:

78 - 65 = 13

Her result lies just over 1 standard deviation from the mean.

University B:

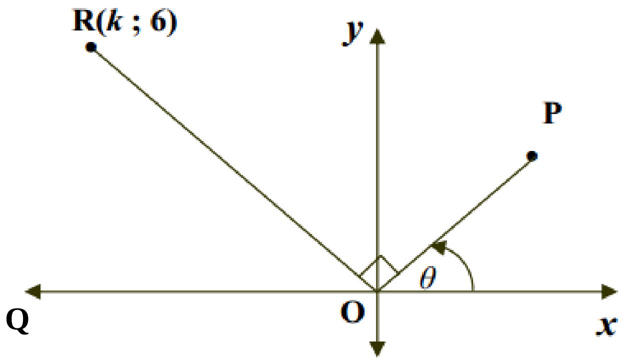
$\bar{x} + \sigma = 54$
$\bar{x} + 2\sigma = 59$

Her result lies just over 2 standard deviations from the mean.

Her result for University B is better.

[4 marks]

QUESTION 10



In the diagram above, P is a point such that $5\sin(90^\circ - \theta) - 3 = 0$ and $R(k;6)$ is a point in the second quadrant such that $P\hat{O}R = 90^\circ$.

Determine the value of:

(a) $\tan\theta$ (3)

$\sin(90^\circ - \theta) = \frac{3}{5}$

$\therefore \cos\theta = \frac{3}{5}$

$\therefore \tan\theta = \frac{4}{3}$ $\quad (y = 4 \text{ by Pyth})$

(b) k without determining the size of θ (5)

$\tan(90^\circ + \theta) = \frac{6}{k}$

$but\ \tan(90^\circ + \theta) = \cot\theta = -\frac{3}{4}$ Or can be done with $\frac{\sin}{\cos}$ *and* $\frac{\cos}{\sin}$

$\therefore \frac{6}{k} = -\frac{3}{4}$

$\therefore k = -8$

[8 marks]

QUESTION 11

(a) Determine the general solution to the equation:

$$\tfrac{1}{2}\tan x = \sin 2x .$$ (7)

$\dfrac{\sin x}{2\cos x} = 2\sin x\cos x$

$\sin x = 4\sin x\cos^2 x$

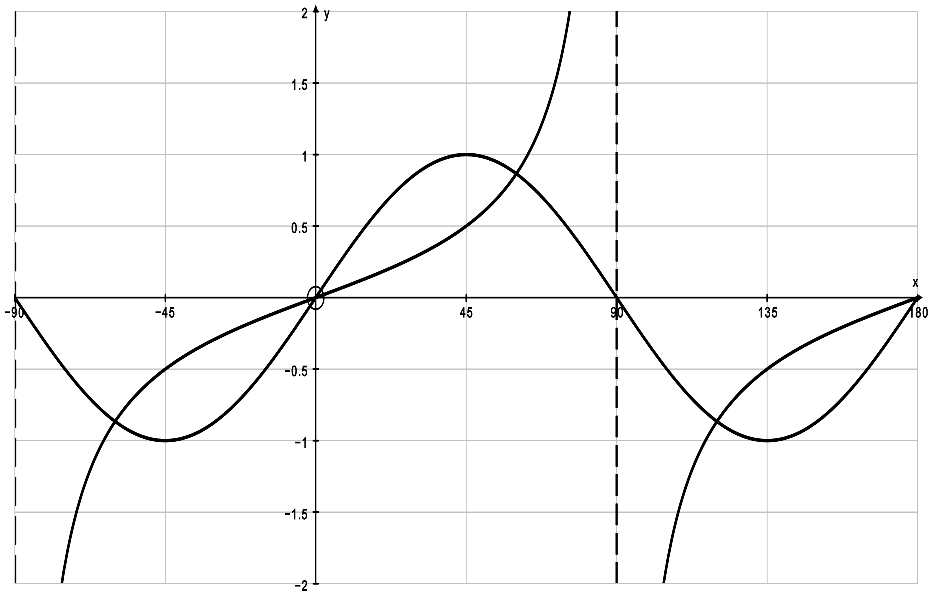
$\sin x(4\cos^2 x - 1) = 0$

$\therefore \sin x = 0$ *or* $\cos^2 x = \dfrac{1}{4}$

$\therefore x = 0^\circ + k.360^\circ$ *or* $x = 180^\circ + k.360^\circ$ $\cos x = \pm\dfrac{1}{2}$

$x = \pm 60^\circ + k.360^\circ$ *or* $x = \pm 120^\circ + k.360^\circ$

(b) Sketched below are the graphs of $f(x) = \tfrac{1}{2}\tan x$ and $g(x) = \sin 2x$:



Use your solutions obtained in question (a) above as well as the graphs to determine the value(s) of x for $x \in [-90^\circ ; 180^\circ]$ for which:

(1) $f(x) > g(x)$. (4)

$-60^\circ < x < 0^\circ$ *or* $60^\circ < x < 90^\circ$ *or* $120^\circ < x < 180^\circ$

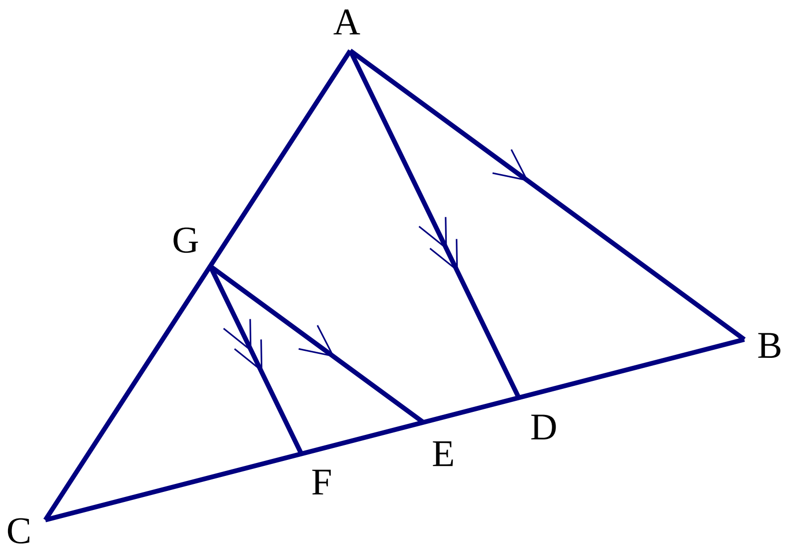
(2) both $f'(x) > 0$ and $g'(x) > 0$. (2)

$-45^\circ < x < 45^\circ$ *or* $135^\circ < x < 180^\circ$

[13 marks]

QUESTION 12

In ΔABC, AB//GE and AD//GF (not drawn to scale).
AG : GC = 5 : 3.



(a) AG : GC = 5 : 3. (AB//GE) (2)

BE : EF = 5 : 3

BE = $\frac{5}{8}$ X 40

(b) DF : FC = 5 : 3. (AD//GF) (2)

$\frac{15}{FC} = \frac{5}{3}$

FC = 9

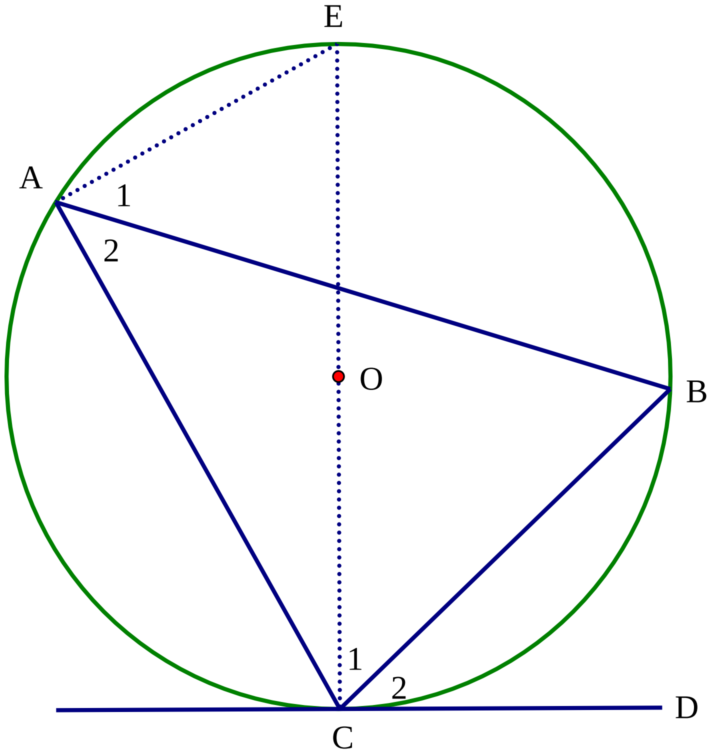
(c) $\frac{\text{Area } \Delta CFG}{\text{Area } \Delta CDA} = \frac{\frac{1}{2} X 3 X 3h}{1/2 X 8 X 8h}$. (3)

$= \frac{9}{64}$

[7 marks]

QUESTION 13

(a) Complete the statements below for the theorem, which states that 'the angle between the tangent and a chord is equal to the angle in the alternate segment'.



Given: Circle ABC, centre O, with tangent CD.

RTP: $\hat{A}_2 = \hat{B}_2$

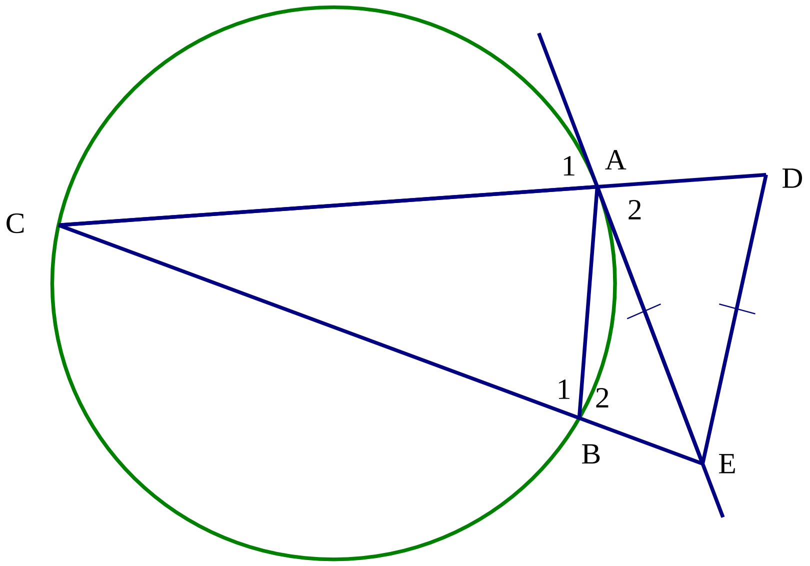
Const: Draw CE through the centre O
Join EA

Proof:

$\hat{A}_1 =$	$\hat{C}_1$	($\angle$s in same segment)
$\hat{A}_{1+2} =$	90^o	($\angle$ in semi – circle)
$\hat{C}_{1+2} =$	90^o	(diameter $\perp$ tangent)
$\therefore \hat{A}_2 =$	$\hat{C}_2$	

(5)

(b)



In circle ABC, CA is extended to D and CB is extended to E.
AE is a tangent to the circle at A. AE = DE.

(1) Prove that ABED is a cyclic quadrilateral. (5)

$\hat{A}_1 = \hat{A}_2$ (vert opp)

$\hat{A}_2 = \hat{D}$ (isos Δ)

$\hat{A}_1 = \hat{D}$

But $\hat{A}_1 = \hat{B}_1$ (tan chord th)

ABED is a cyclic quadrilateral (ext < = int opp <)

(2) Join BD and prove that DE is a tangent to the circle CBD. (4)

$B\hat{D}E = B\hat{A}E$ (< s in same segment)

$B\hat{A}E = \hat{C}$ (tan chord th)

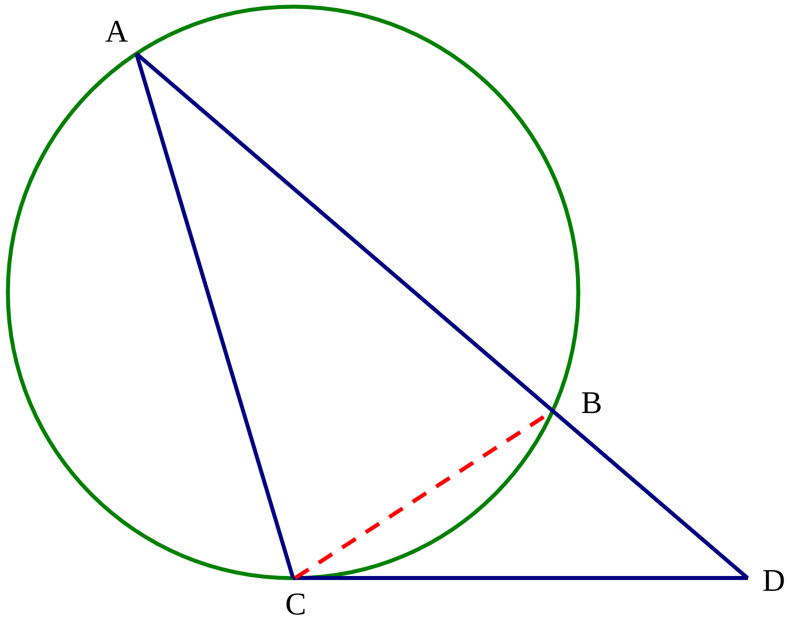
$B\hat{D}E = \hat{C}$

DE is a tangent to the circle CBD. (con tan chord th)

[14 marks]

QUESTION 14

In circle ABC, AB is produced at D and CD is a tangent.



Prove that $CD^2 = BD \cdot AD$ (5)

Const: Join BC

In ΔACD and ΔCBD

① $B\hat{C}D_1 = \hat{A}$ (tan chord th)

② $\hat{D} = \hat{D}$ (common)

③ $C\hat{B}D = A\hat{C}D$ (3^{rd} ∠ in Δ)

$\Delta ACD \; /// \; \Delta CBD$ AAA

$\frac{CD}{AD} = \frac{BD}{CD}$ ($\Delta ACD \; /// \; \Delta CBD$)

$CD^2 = BD \cdot AD$

[5 marks]

QUESTION 15

Determine the value of:

$$\tan 1^{o} \times \tan 2^{o} \times \tan 3^{o} \times \tan 4^{o} \times ..... \times \tan 87^{o} \times \tan 88^{o} \times \tan 89^{o} \qquad (3)$$

$= \tan 1^{o} \times \tan 2^{o} \times .... \times \tan 45^{o} \times ..... \times \tan(90^{o} - 2^{o}) \times \tan(90^{o} - 1^{o})$

$= \tan 1^{o} \times \tan 2^{o} \times .... \times \tan 45^{o} \times ..... \times \cot 2^{o} \times \cot 1^{o}$

$= \tan 45^{o}$

$= 1$

Or do the same using sin and cos reciprocals

[3 marks]