



MICHAELHOUSE

Mathematics Department

**A BLOCK EXAMINATION
CORE MATHEMATICS PAPER 1
SEPTEMBER 2014**

Examiner: Mr S B Coxon
Time: 3 hours

Moderator: Mr P Stevens
Marks: 150

PLEASE READ THE INSTRUCTIONS CAREFULLY

1. This question paper consists of 10 page(s) plus an Information Sheet. Please check that your paper is complete.
2. Question 7 (b) must be answered on the set of axes provided, attached to the front cover of your answer book.
3. Read the questions carefully.
4. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
5. All the necessary working details must be clearly shown; giving an answer only will not necessarily give you full marks.
6. It is in your own interest to write legibly and to present your work neatly.
7. Round off your answers to **1 decimal digit** where necessary.

NAME: _____

MATHS TEACHER: _____

SECTION A
QUESTION 1

(a) Given: $x(x + 3) = m.$

Solve for x if:

(1) $m = 0$ (2)

(2) $m = x + 3$ (4)

(b) Solve for x :

(1) $8^{2\log x} = 64$ (3)

(2) $(x - 3)^2 < 9$ (4)

[13 marks]

QUESTION 2

The roots of a quadratic equation $g(x) = 0$ are given by:

$$x = \frac{-20 \pm \sqrt{400 - 48h}}{24}.$$

(a) Determine the roots if $h = 1.$ (2)

(b) For which value(s) of h will the roots of $g(x)$ be real? (2)

(c) Write down the largest possible integral value of h for which the roots will be rational. (2)

(d) (1) For which value of h will one of the roots be 0? (2)

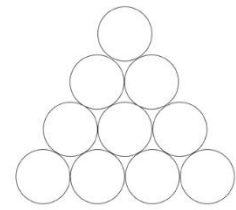
(2) Now write down the value of the other root. (2)

[10 marks]

QUESTION 3

- (a) Given the following arithmetic sequence: $x + 1 ; 3x - 2 ; 5x - 5 ; \dots$
- (1) Find the common difference in terms of x . (2)
- (2) Determine, in terms of x , the sixth term. (2)
- (3) If the sum of the first six terms is 105, find x . (4)
- (b) Fifty five round pipes are stacked in the same way as shown in the figure.

Determine the number of pipes that must be placed in the bottom layer in order to have one pipe in the top layer.



(5)

[13 marks]

QUESTION 4

- (a) Given $f(x) = \frac{1}{2}x^2 - 9$, determine $f'(x)$ from first principles. (4)
- (b) Find, leaving your answer with positive exponents:
- (1) $f'(x)$ given $f(x) = \frac{x^2-4}{2-x}$. (3)
- (2) $D_x(\sqrt{x} + 2kx - k)$. (3)
- (c) Given $f(x) = x^3 - 20x + 30$.

Find the equation of the tangent to $f(x)$ at the point $(3 ; y)$. (5)

[15 marks]

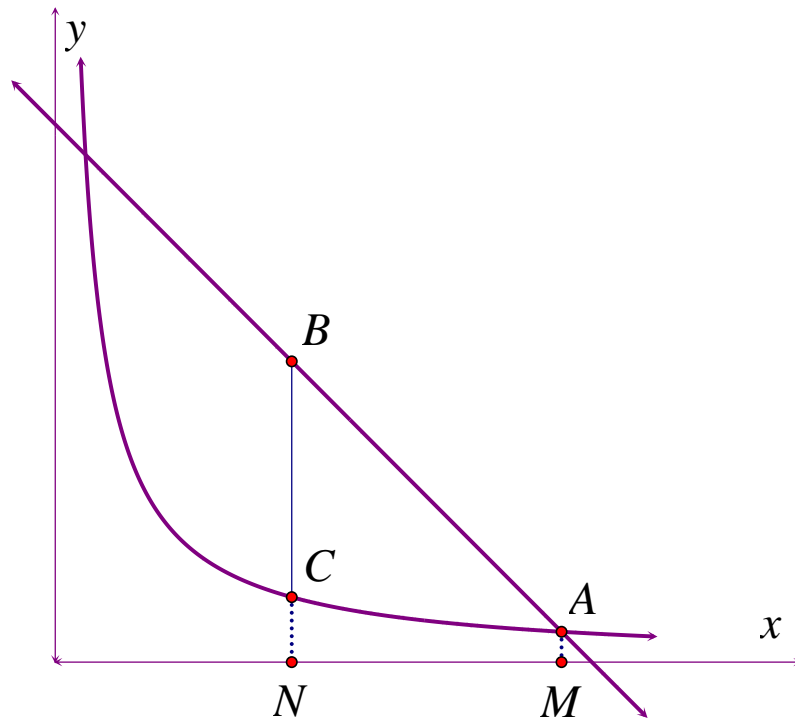
QUESTION 5

- (a) A farmer has just bought a new tractor for R 800 000. He has decided to replace this tractor in 5 years' time when it's trade-in value will be R 200 000. The replacement cost of a new tractor is expected to increase by 8% per annum.
- (1) The farmer wants to pay cash for the new replacement tractor in 5 years time, after trading in his present tractor for R 200 000. How much extra cash will he need to pay for his new tractor in 5 years time? (3)
- (2) One month after purchasing his present tractor, the farmer deposits x rands into an account to build up a sinking fund for the new tractor. The interest rate for this account was 12% per annum compounded monthly. The farmer continues to deposit the same amount at the end of each month for a total of 60 months. At the end of 60 months he has exactly the amount that is needed to purchase a new tractor after he trades in his present tractor. Calculate the amount he deposits each month. (4)
- (b) Jon has been a valued client and has not claimed from his Life Insurance to date. On his 70th birthday, the insurance company pay him a lump sum of R 1 250 000. Jon decides to invest this money and to draw a monthly salary from this. The bank offers him a favourable interest rate of 5,6% per annum, compounded monthly. Jon decides to withdraw R 15 000 monthly, starting one month after investing the lump sum.
- (1) Calculate how many months he will be able to withdraw R 15 000 per month (to the closest number of months). (4)
- (2) After 84 months, Jon withdraws the remaining amount of money to take his wife on a world cruise. Calculate the amount of money he will have (outstanding balance), immediately after the 84th withdrawal. (3)

[14 marks]

SECTION B
QUESTION 6

The figure shows the graphs of $xy = 12; x > 0$ and $x + y = 13$.



- (a) Find the co-ordinate of point A. (4)
- (b) Determine the possible lengths of CN if $BC = 6$ units. (5)
- (c) Find the value of x , for which BC will have a maximum length, $x \in [1; M]$. (4)

[13 marks]

QUESTION 7

Given the equation $2^x = x^2$, which has three real roots.

- (a) By means of inspection, determine two of these roots which are integer values. (2)
- (b) Sketch the graphs of $f(x) = 2^x$ and $g(x) = x^2$ on the set of axes provided. (3)
- (c) Use your graph to determine the approximate value of the third root. Show on your graph using the letter A where this reading was taken. (2)
- (d) For what values of x will $g(x) > f(x)$; $x \geq 0$? (2)
- (e) Determine the equations of $y = f^{-1}(x)$ and $y = g^{-1}(x)$, in the form $y = \dots$. (3)
- (f) Is $g^{-1}(x)$ a function? Explain. (2)
- (g) Solve for x : $g(x) = g^{-1}(x)$. (2)
- (h) Draw rough sketches of the following functions on separate axes. Show how each graph is related to the function $g(x)$:
- (1) $g(-x)$
- (2) $g(-x + 1)$
- (3) $g(-x + 1) + 1$ (3)

[19 marks]

QUESTION 8

(a) The first term of a geometric series with n terms is x and the sum of the series is $\frac{x(x^{10}-1)}{x^2-1}$.

(1) Determine the value of n . (2)

(2) Show that $T_n = x^{2n-1}$. (3)

(3) If $x = 3$, find n if $T_n = \frac{1}{3}$. (3)

(b) (1) For which values of k does the series below converge? (3)

$$\sum_{n=1}^{\infty} \left(k - \frac{3}{2}\right)^n$$

(2) If the series does converge, prove that: (3)

$$\sum_{n=1}^{\infty} \left(k - \frac{3}{2}\right)^n = \frac{2k-3}{5-2k}$$

(3) Hence, or otherwise, evaluate: (3)

$$\sum_{n=1}^{\infty} \left(k - \frac{3}{2}\right)^n \text{ if } k = 1$$

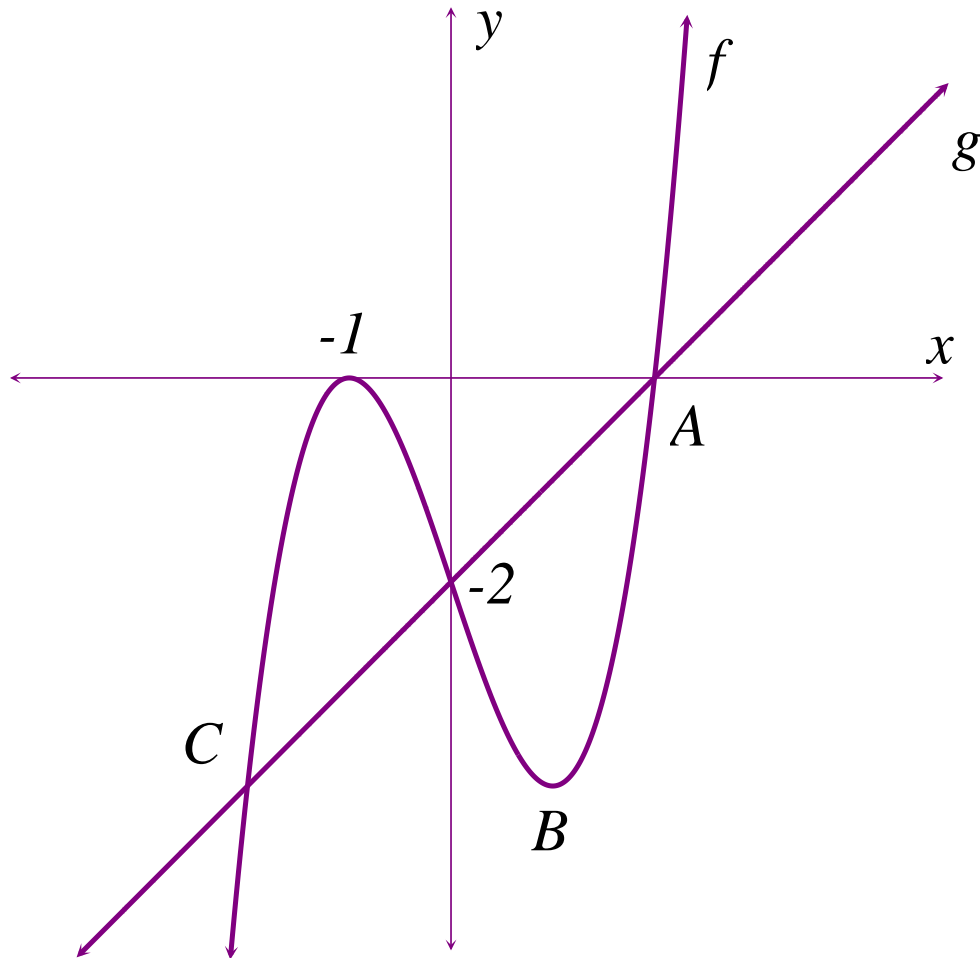
and if $k = -1$.

[17 marks]

QUESTION 9

The graph represents the functions f and g with

$$f(x) = px^3 + qx - 2 \text{ and } g(x) = x - 2.$$

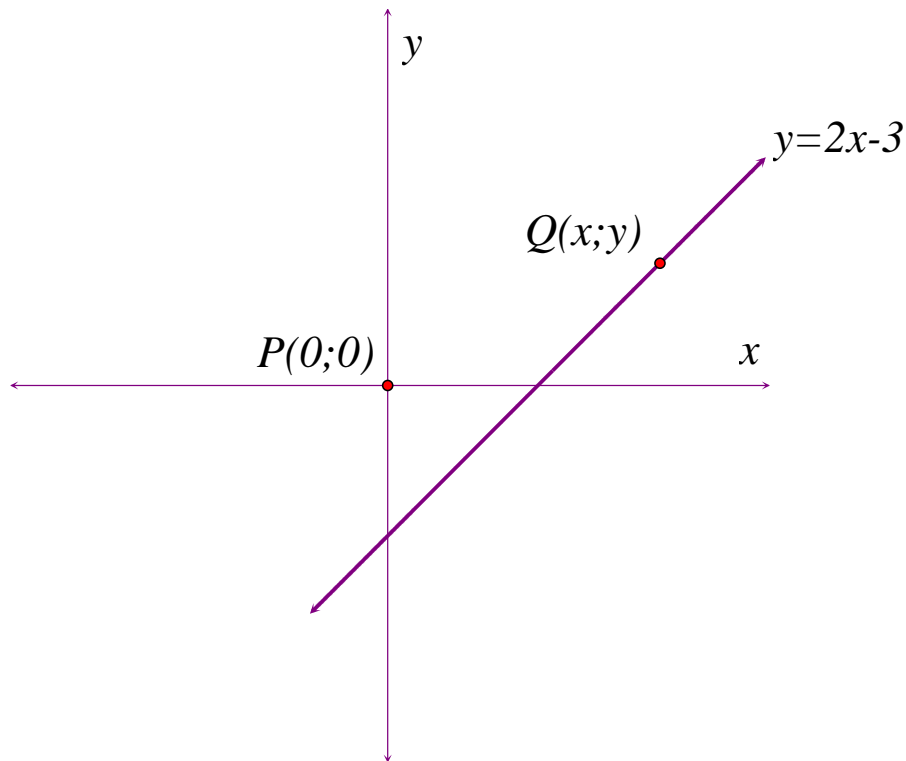


- (a) Determine the co-ordinates of point A. (1)
- (b) Show by calculation, that $p = 1$ and $q = -3$. (4)
- (c) Show that BC is parallel to the x - axis. (6)
- (d) For which values of x is $x \cdot f'(x) \leq 0$? (3)

[14 marks]

QUESTION 10

In the figure point $P(0; 0)$ is the origin and $Q(x; y)$ is any point on the straight line $y = 2x - 3$.



- (a) Show that $(PQ)^2 = 5x^2 - 12x + 9$. (3)
- (b) Now, calculate the shortest distance from the origin to the straight line. (4)
- (c) Find the equation of the line which is the shortest distance from the origin to the given line $y = 2x - 3$. (3)

[10 marks]

QUESTION 11

The Bailey-Borwein-Plouffe Formula (BBP) is a formula used for calculating π discovered by Simon Plouffe in 1995.

$$\pi = \sum_{n=0}^{\infty} \frac{1}{16^n} \left[\frac{4}{8n+1} - \frac{2}{8n+4} - \frac{1}{8n+5} - \frac{1}{8n+6} \right]$$

- (a) Using your calculator, write out π to 8 decimal places. (1)
- (b) Now calculate π (to 8 decimal places) using the formula below: (4)

$$\pi = \sum_{n=0}^2 \frac{1}{16^n} \left[\frac{4}{8n+1} - \frac{2}{8n+4} - \frac{1}{8n+5} - \frac{1}{8n+6} \right]$$

- (c) How many decimals of π are accurate, using the answer obtained in (b) above? (1)

[6 marks]

QUESTION 12

The notation $[x]$ means the largest integer less than or equal to x :

For example: $[3,7] = 3$ and $[5] = 5$.

- (a) Show that $x = 35$ is a solution to the equation: $x - [\sqrt{x}]^2 = 10$. (2)
- (b) Find one other value of x which satisfies the above equation. (1)
- (c) Sketch the graph of $y = [x]$; $0 \leq x \leq 4$. (3)

[6 marks]