Parklands College of Education

September Examinations – Winter Quarter 2014

Subject	: Mathematics	Paper	: 1
Grade	: 12	Marks	: 150
Examiner	: F.A. du Preez	Time	: 3 hours
Moderator	: A. Vosloo ; F. Bredell ; S. Joubert		

SOLUTIONS

QUESTION 1

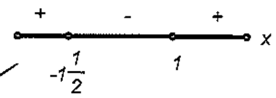
1.1) Solve for x :

1.1.1) $4x=\frac{15}{x+1}$ (4)

$4x(x+1)=15$ ✓
$4x^2+4x-15=0$ ✓
$(2x-3)(2x+5)=0$ ✓
$x=\frac{3}{2}$ or $x=-\frac{5}{2}$ ✓

1.1.2) $2x^2+x-3\geq 0$ (3)

$(2x+3)(x-1)\geq 0$ ✓



$x\leq -\frac{3}{2}$ ✓ or $x\geq 1$ ✓

1.2) Solve for x and y in the simultaneous equations $y+2x=1$ and $2x^2-y^2=x-6$ (7)

$y=1-2x$ ✓

Sub : $2x^2-(1-2x)^2=x-6$ ✓

$2x^2-(1-4x+4x^2)=x-6$
$2x^2-1+4x-4x^2=x-6$
$-2x^2+3x+5=0$ ✓
$2x^2-3x-5=0$
$(2x-5)(x+1)=0$ ✓
$x=\frac{5}{2}$ or $x=-1$ ✓

$y=1-2\left(\frac{5}{2}\right)$ or $y=1-2(-1)$
$y=-4$ ✓ or $y=3$ ✓

1.3) Simplify the following : $\dfrac{2^{n+2}-\sqrt{32y^8}}{\sqrt{4^{n+1}}-\sqrt{8y^8}}$ (5)

$$=\frac{2^n.2^2-\sqrt{16\times 2y^8}}{\sqrt{(2^2)^{n+1}}-\sqrt{4\times 2y^8}}=\frac{2^n.2^2-4y^4\sqrt{2}}{\sqrt{(2^{2n+2})}-2y^4\sqrt{2}}$$

$$=\frac{2^n.2^2-4y^4\sqrt{2}}{2^{n+1}-2y^4\sqrt{2}}=\frac{2^2(2^n-y^4\sqrt{2})}{2(2^n-y^4\sqrt{2})}=2$$

1.4) If $5^x=y$, write $\sqrt{125^x}$ in terms of y. (2)

$$=\sqrt{(5^3)^x}=(5)^{\frac{3x}{2}}=(5^x)^{\frac{3}{2}}=y^{\frac{3}{2}}$$

1.5) If $px^2+3x+4p=0$, determine the value of p for which the equation has equal roots, without solving for x. (5)

[26]

$$b^2-4ac=3^2-4p(4p)$$
$$=9-16p^2$$
$$=0$$
$$(3+4p)(3-4p)=0$$
$$p=-\frac{3}{4} \quad \text{or} \quad p=\frac{3}{4}$$

QUESTION 2

2.1) Consider the sequence $4:12:20:...:340$.

2.1.1) Determine the number of terms in this sequence. (3)

$d=8$; let $T_n=340$; $4+(n-1)(8)=340$
$(n-1)(8)=336$; $(n-1)=42$;
Therefore $n=43$

2.1.2) Hence, calculate the value of $4+12+20+...+340$. (2)(5)

$$S_{43}=\frac{43}{2}[4+340]=7\,396$$

2.2) The $3rd$ and $6th$ terms of a Geometric sequence are 24 and 192 respectively.

2.2.1) Calculate the first 3 terms of this sequence. (5)

$ar^2=24$ (1) ; $ar^5=192$ (2)

(2) / (1) : $r^3=8$; $r=2$;

From (1) : $a=\dfrac{24}{4}=6$

Therefore $6;12;24$

2.2.2) Calculate how many terms must be added to give a sum of 3 066. (4)(9)

Let $S_n=3\,066$

Then $\dfrac{6(2^n-1)}{2-1}=3\,066$;

$2^n-1=511$; $2^n=512=2^9$

Therefore $n=9$

2.3) Calculate $\displaystyle\sum_{k=1}^{5}\frac{k^2}{k+2}$. (3)

$$=\frac{1^2}{1+2}+\frac{2^2}{2+2}+\frac{3^2}{3+2}+\frac{4^2}{4+2}+\frac{5^2}{5+2}$$
$$=\frac{1}{3}+1+\frac{9}{5}+\frac{16}{6}+\frac{25}{7}$$
$$=\frac{328}{35}=9\frac{13}{35}=9.37$$

2.4) Consider the convergent Geometric Series $1+\left(\frac{x-1}{2x+3}\right)+\left(\frac{x-1}{2x+3}\right)^2+\ldots$

Calculate the value of x if the sum to infinity is equal to $1\frac{1}{2}$. (4)

[21]

$\frac{1}{1-\frac{x-1}{2x+3}}=\frac{3}{2}$; $\frac{1}{\frac{2x+3-(x-1)}{2x+3}}=\frac{3}{2}$

$\frac{1}{\frac{2x+3-x+1}{2x+3}}=\frac{3}{2}$; $\frac{1}{\frac{x+4}{2x+3}}=\frac{3}{2}$

$\frac{2x+3}{x+4}=\frac{3}{2}$; $2(2x+3)=3(x+4)$

$4x+6=3x+12$;

Therefore $x=6$

QUESTION 3

The quadratic sequence $1;2x-1;5x-2;25;8x+17;\ldots$ is given.

3.1) Calculate the value of x. (4)

d_1 : $2x-2;3x-1;27-5x;8x-8;\ldots$
d_2 : $x+1;28-8x;13x-35;\ldots$

Therefore $x+1=28-8x$
Therefore $9x=27$
Therefore $x=3$

3.2) Calculate the $n-th$ term in the form $T_n=an^2+bn+c$. (5)

$1;5;13;25;41;\ldots$
d_1 : $4;8;12;16;\ldots$
d_2 : $4;4;4;\ldots$

$2a=4$ $a=2$
$3(2)+b=4$; $b=-2$
$2-2+c=1$; $c=1$

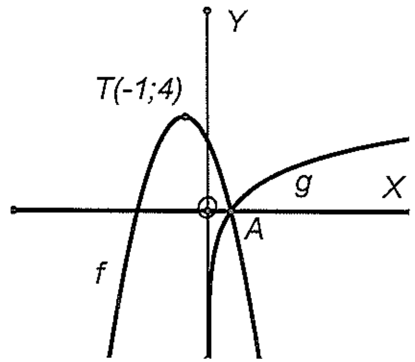
$T_n=2n^2-2n+1$

[9]

QUESTION 4

In the diagram, $f(x)=ax^2+bx+c$ and $g(x)=\log_2 x$ are represented.

The turning point of f is $T(-1;4)$. The graph of g intersects the X-axis at A.



4.1) Write down the coordinates of A. (1)

$A=(1;0)$

4.2) Hence, determine the equation of f. (5)

$y=a(x+1)^2+4$
$(1;0)$: $0=a(1+1)^2+4=4a+4$
$-4a=4$; $a=-1$
$y=-(x^2+2x+1)+4=-x^2-2x-1+4$
Therefore $y=-x^2-2x+3$

4.3) Determine the equation of g^{-1} in the form $y=\ldots$ (2)

$y=2^x$

4.4) If the point T is also the intersection point of the asymptotes of the function $h(x)=\frac{k}{x-p}+q$, write down the values of p and q. (2)

$p=-1$
$q=4$

4.5) If the graph of h also passes through the point A, calculate the value of k. (3)

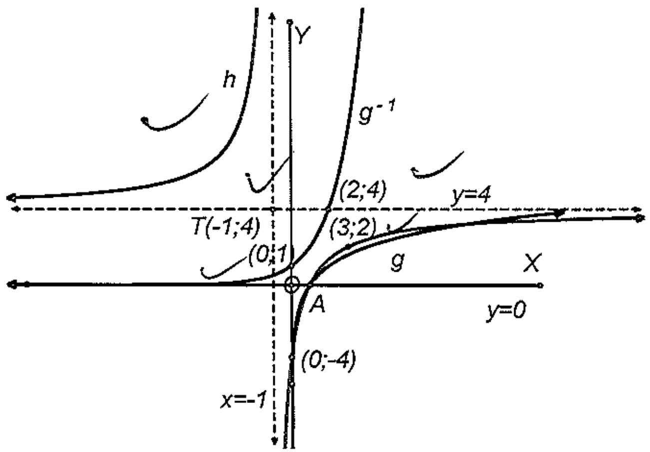
$(1;0)$: $0=\frac{k}{1+1}+4$

$\frac{k}{1+1}=-4$

$k=2(-4)=-8$

4.6) Using the set of axis on the DIAGRAM sheet, draw neat sketch graphs of g^{-1} and h, showing all the necessary calculations, and indicating the intercepts with the axes, asymptotes and one more point on each graph. (6)

Y-intercept for h : $y=-\frac{8}{0+1}+4=-4$; so $(0;-4)$



4.7) Solve for x if $g(x)<-1$. (3)

$\log_2 x<-1$

$x<2^{-1}$

Therefore $0<x<\frac{1}{2}$

4.8) How much and how must the graph of g^{-1} be shifted so that the equation of the new graph is $y=\frac{2^x}{4}$? (2)

[24]

$y=2^{x-2}$;

Two units to the right

QUESTION 5

5.1) Determine after how many years will an investment of $R\,20\,000$ be worth $R\,42\,000$ if interest is calculated at $14\,\%$ p.a., interest compounded half-yearly. (4)

$A=P\left(1+\frac{r}{200}\right)^{2n}$; $\left(1+\frac{r}{200}\right)^{2n}=\frac{A}{P}$

$2n=\log_{\left(1+\frac{14}{200}\right)}\left(\frac{42\,000}{20\,000}\right)=10{,}97$

$n=5{,}48$

5.2) The value of a car depeciates to $30\,\%$ of its original value after 7 years. Calculate the rate, in percentage p.a., of depreciation. (4)

$A=P\left(1-\frac{r}{100}\right)^n$; $\left(1-\frac{r}{100}\right)^n=\frac{A}{P}$

$\left(1-\frac{r}{100}\right)=\sqrt[n]{\frac{A}{P}}$; $\left(1-\frac{r}{100}\right)=\sqrt[7]{0{,}3}$

$-\frac{r}{100}=\sqrt[7]{0{,}3}-1$; $r=-100\left(\sqrt[7]{0{,}3}-1\right)=15{,}8\,\%$

OR $A=P\left(1-\frac{rn}{100}\right)$; $-\frac{rn}{100}=\frac{A}{P}-1$

$r=\frac{100}{n}(1-\frac{A}{P})$

$r=\frac{100}{7}(1-0{,}3)=10\,\%$

5.3) Monthly payments on a loan of $R\,260\,000$ begin one month after the loan has been granted, and the rate of interest is 17.5% p.a., interest compounded monthly.

5.3.1) Calculate the value of each payment if the loan must be paid back over a period of 10 years. (4)

Let the monthly repayment be x rands

Then $\dfrac{x\left(1-\left(1+\frac{17.5}{1200}\right)^{-12(10)}\right)}{\frac{17.5}{1200}}=260\,000$

Therefore $x=\dfrac{260\,000\times\frac{17.5}{1200}}{1-\left(1+\frac{17.5}{1200}\right)^{-120}}=R\,4\,601.45$

5.3.2) Calculate the total amount which has been paid back after 7 full years. (3)(7)

[15]

$A=84\times R\,4\,601.45$

$A=R\,386\,521.80$ (or $A=R\,386\,521.61$)

QUESTION 6

6.1) If $f(x)=7-2x^2$, determine $f'(x)$ from first principles. (5)

$f(x+h)=7-2(x+h)^2$

$=7-2(x^2+2xh+h^2)$

$=7-2x^2-4xh-2h^2$

$f(x+h)-f(x)=-4xh-2h^2$

$\dfrac{f(x+h)-f(x)}{h}=\dfrac{h(-4x-2h)}{h}$

$=(-4x-2h)$

$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$f'(x)=\lim\limits_{h\to 0}(-4x-2h)$

$=-4x$

6.2) Determine $\dfrac{dy}{dx}$ if $y=\sqrt{(3x)}-\sqrt{3}x$ (3)

$y=\sqrt{3}x^{\frac{1}{2}}-\sqrt{3}x$

$\dfrac{dy}{dx}=\sqrt{3}\left(\dfrac{1}{2}\right)x^{-\frac{1}{2}}-\sqrt{3}$ / $\dfrac{dy}{dx}=\dfrac{\sqrt{3}}{2x^{\frac{1}{2}}}-\sqrt{3}=\dfrac{\sqrt{3}}{2\sqrt{x}}-\sqrt{3}$

6.3) Determine at which value of x will the equation of the tangent to $f(x)=3x^2-4x$ be $y=8x-12$. (3)

[11]

$f'(x)=6x-4=8$

$6x=12$

$x=2$

QUESTION 7

Given : $f(x)=x^3-14x^2+49x-36$.

7.1) Calculate the coordinates of the X-intercepts of f. (6)

$f(1)=(1)^3-14(1)^2+49(1)-36$

$=1-14+49-36=0$

Therefore $(x-1)$ is a factor of $f(x)$.

1	1	-14	49	-36
		1	-13	36
	1	-13	36	0

$f(x)=(x-1)(x^2-13x+36)$

$f(x)=(x-1)(x-4)(x-9)$

$=0$

if $x=1$; 4 or 9

Therefore $(1;0)$; $(4;0)$; $(9;0)$

7.2) Calculate the coordinates of the turning (stationary) points of f. (6)

$f'(x)=3x^2-28x+49=0$

$(3x-7)(x-7)=0$

$x=\frac{7}{3}$ or 7

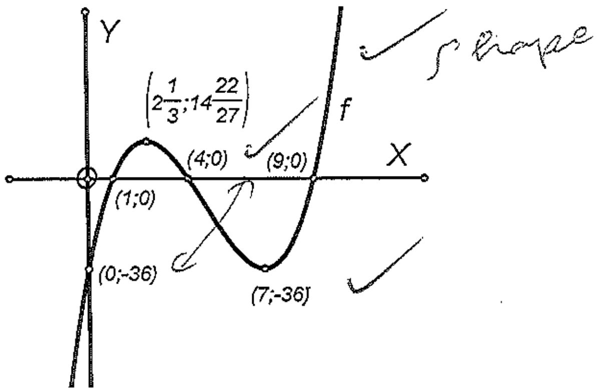
$y=f\left(\frac{7}{3}\right)$ or $f(7)$

$=\left(\frac{7}{3}\right)^3-14\left(\frac{7}{3}\right)^2+49\left(\frac{7}{3}\right)-36$ or $=(7)^3-14(7)^2+49(7)-36$

$=\frac{400}{27}$ or -36

TP's : $\left(2\frac{1}{3};14\frac{22}{27}\right)$; $(7;-36)$

7.3) Using the DIAGRAM SHEET, sketch the curve of f, showing all the intercepts with the axes and turning points. (3)



7.4) For which values of x is $f'(x)\geq 0$? (2)

[17]

$x\leq 2\frac{1}{3}$ or $x\geq 7$

QUESTION 8

8.1) The x-coordinate of a turning point of $f(x)=ax^3+6x^2$ is -2. Determine the value of a. (4)

$f'(x)=3ax^2+12x$

$f'(-2)=3a(-2)^2+12(-2)=0$

$12a=24$

$a=2$

8.2) The Volume of a closed right cylinder with radius of the circular base $r\ cm$, and height $h\ cm$, is $6\,750\pi\ cm^3$. Determine the value of r if the total external surface area has a minimum value.

$(V=\pi r^2h\ ;\ A=2\pi r^2+2\pi rh)$ (6)

[10]

$A=2\pi r^2+2\pi rh$ (1)

$h=\frac{V}{\pi r^2}=\frac{6\,750\pi}{\pi r^2}$ (2)

Sub (2) into (1) : $A=2\pi r^2+2\pi r\left(\frac{6\,750\pi}{\pi r^2}\right)$

$A(r)=2\pi r^2+\left(\frac{13\,500\pi}{r}\right)$

$A(r)=2\pi r^2+13\,500\pi r^{-1}$

$A'(r)=4\pi r-13\,500\pi r^{-2}=0$

$4\pi r=\frac{13\,500\pi}{r^2}$

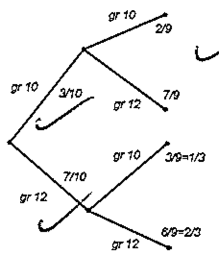
$r^3=3\,375$

$r=15$

QUESTION 9

9.1) *Ten* Sample Mathematics text books are given to a school, *three* for Grade 10, and *seven* for Grade 12, but these are sealed in brown paper without titles on the paper. A book is selected and not replaced, and a book is selected again.

9.1.1) Draw a tree diagram to illustrate the information. Indicate all the outcomes. (4)



9.1.2) Calculate the probability that both books, selected at random, will be for the same grade. (3)(7)

$$=\frac{3}{10}\times\frac{2}{9}+\frac{7}{10}\times\frac{6}{9}=\frac{48}{90}=\frac{8}{15}$$

9.2) The word I N D E F I N I T E is given.

If the letters are not repeated, calculate the total number of arrangements of these letters that can be made. (3)

[10]

Number $=\frac{10!}{3!2!2!}=151\,200$

QUESTION 10

$f(x)=ax^2+bx+c$ and $g(x)=\frac{k}{x}+q$ are given. The roots (solutions) of $f(x)=0$ are real and unequal. The tangents to $f(x)$ and $g(x)$ at $x=-1$ are parallel.

Showing all the working out, prove that $k^2+4a^2>4ac+4ak$. [7]

$b^2-4ac>0$; so $b^2>4ac$

And $f'(x)=2ax+b$

$$g'(x)=-\frac{k}{x^2}$$

Therefore $2a(-1)+b=-\frac{k}{(-1)^2}$

$$-2a+b=-k$$
$$b=2a-k$$

Therefore $b^2=(2a-k)^2=4a^2-4ak+k^2$

Therefore $4a^2-4ak+k^2>4ac$

Therefore $k^2+4a^2>4ac+4ak$