Parklands College of Education

September Examinations - Winter Quarter 2014

Subject	: Mathematics	Paper	: 2
Grade	: 12	Marks	: 150
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Moderator	: F. Bredell		

SOLUTIONS

QUESTION 1

Two sets of data are given here :

A : $3;8;10;11;12;x;13;15;15;17;27$

B : $3;8;10;11;12;x;y;15;15;25;28$

1.1) Determine the value of x in set A if the Mean is 13. (2)

$$\frac{3+8+10+11+12+x+13+2(15)+17+27}{11}=13$$

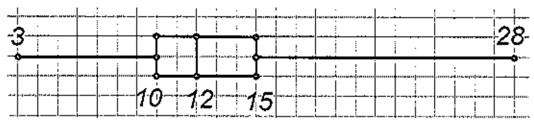
$x+131=143$

Therefore $x=12$

1.2) Determine the standard deviation for Set A. (3)

$$\text{S.d.}=\sqrt{\frac{(3-13)^2+(8-13)^2+...+(27-13)^2}{11}}=5{,}72$$

1.3) Determine the values of x and y in set B if the mean is 14 and the data is represented by the given Box-and-Whisker diagram (drawn to scale). (4)



$x=Median=12$

$$\frac{3+8+10+11+12+12+y+2(15)+25+28}{11}=14$$

$y+139=154$

$y=15$

1.4) What is the highest value that the bottom 75 % of the set B would have ? (1)

[10]

15

QUESTION 2

The table shows a selection of data consisting of pairs of values $(x;y)$.

x	1	2	3	4	5	6	7	8
y	6	4,5	5	3	4	2,5	3	1,5

2.1) Determine the equation of the line that would best fit the points if shown graphically. (4)

$y=-0{,}541x+6{,}125$

2.2) Hence, calculate the value of y if $x=4{,}5$. (2)

$y=-0{,}541(4{,}5)+6{,}125$
$=3{,}69$

2.3) Calculate the correlation coefficient. (2)

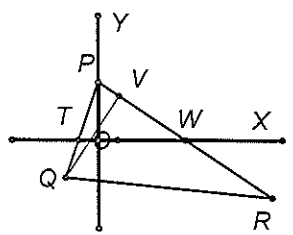
$-0{,}907$

2.4) What type of correlation does this imply ? (1)
[9]

Strong negative

QUESTION 3

In the diagram, ΔPQR, with $P=(0;6)$ and $Q=(k;r)$, is represented.
PQ intersects the X-axis at T such that $P\hat{T}O=71{,}56505°$.



3.1) If the equation of PQ is $y=mx+c$, calculate the value of m. (2)

$m=\tan 71{,}56505°=3$

3.2) If the equation of PR is $px+3y=18$, and the midpoint of PR is $W=(9;0)$, show that $p=2$. (2)

$p(9)+3(0)=18$
$p=2$

3.3) Calculate the coordinates of R. (4)

If $R=(x;y)$, then $\left(\frac{x+0}{2};\frac{y+6}{2}\right)=(9;0)$

$\frac{x+0}{2}=9;\ \frac{y+6}{2}=0$

$x=18$ and $y=-6$

3.4) If $QV\perp PR$, where $V=(t;q)$, express r in terms of k, t and q. (6)
[14]

From $2x+3y=18$: $3y=-2x+18$

So $y=-\frac{2}{3}x+6$

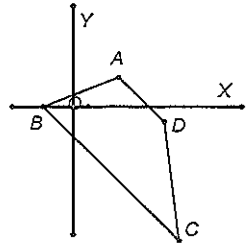
So $\frac{r-q}{k-t}=\frac{3}{2}$

$r-q=\frac{3}{2}(k-t)$

$r=\frac{3}{2}(k-t)+q$

QUESTION 4

The diagram shows $ABCD$, with $A=(3;2)$, $B=(-2;0)$, $C=(7;k)$ and $D=(6;-1)$.



Calculate the value of k, if :

4.1) $AD \parallel BC$ (5)

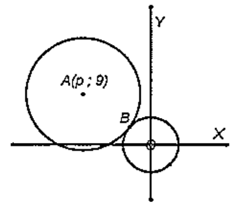
$$\frac{-1-2}{6-3}=\frac{k-0}{7-(-2)}$$
$$\frac{k}{9}=-1$$
$$k=-9$$

4.2) C is a point on the circle with B as centre and radius $k\sqrt{2}$. (5)
[10]

$$(x-(-2))^2+(y-0)^2=(k\sqrt{2})^2$$
$$(x+2)^2+y^2=2k^2$$
Sub C : $(7+2)^2+k^2=2k^2$
$$k^2=9^2$$
Therefore $k=-9$ or $k=9$

QUESTION 5

The circle with centre $A(p;9)$ and radius 10, and the circle with equation $x^2+y^2=25$, touch at point B.



5.1) Calculate the value of p. (3)

$$(p-0)^2+(9-0)^2=(15)^2$$
$$p^2=144$$
$$p=-12$$

5.2) If $A=(-12;9)$, determine the equation of AO. (3)

$$m=\frac{9-0}{-12-0}=-\frac{3}{4}$$
Therefore the equation is $y=-\frac{3}{4}x$

5.3) Hence, determine the coordinates of B. (6)

Sub $y=-\frac{3}{4}x$ into $x^2+y^2=25$
$$x^2+\left(-\frac{3}{4}x\right)^2=25$$
$$x^2+\frac{9x^2}{16}=25 \quad 16x^2+9x^2=400 \quad 25x^2=400$$
$$x^2=16 \quad x=-4$$
$$y=-\frac{3}{4}(-4)=3$$
$$B=(-4;3)$$

5.4) Determine the equation of the common tangent at B. (3)
[15]

$m_t=\frac{4}{3}$; so $y=\frac{4}{3}x+c$

$B=(-4;3)$: $3=\frac{4}{3}(-4)+c=-\frac{16}{3}+c$

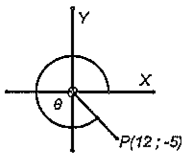
$c=\frac{25}{3}$

Therefore $y=\frac{4}{3}x+8\frac{1}{3}$

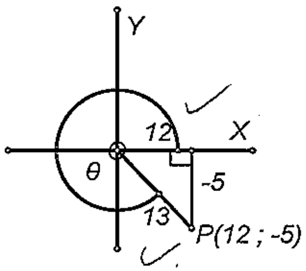
QUESTION 6

CALCULATORS MAY NOT BE USED IN THIS QUESTION.

6.1) In the diagram, $P(12;-5)$ is a point in the plane, with *reflex* $X\hat{O}P=\theta$.



6.1.1) Copy the diagram onto your page, and complete the diagram, showing all the relevant sides and values. (2)



6.1.2) Determine the value of $\frac{\cos 2\theta}{\cos(180°+\theta)}$. (3)(5)

$$\frac{\cos 2\theta}{-\cos\theta}=-\frac{2\cos^2\theta-1}{\cos\theta}$$

$$=-\frac{2\left(\frac{12}{13}\right)^2-1}{\frac{12}{13}}$$

$$=-\left(\frac{288}{169}-1\right)\times\frac{13}{12}=-\frac{119}{156}$$

6.2) If $\sin 25°=p$, determine $\sin 130°.\cos 155°$ in terms of p. (4)

$=\sin 50°(-\cos 25°)$

$=2\sin 25°\cos 25°(-\cos 25°)$

$=-2\sin 25°\cos^2 25°=-2\sin 25°(1-\sin^2 25°)$

$=-2p(1-p^2)$

6.3) The identity $\frac{\sin 5x}{\sin x}-\frac{\cos 5x}{\cos x}=4\cos 2x$ is given.

6.3.1) Prove this identity. (6)

$$LHS=\frac{\sin 5x\cos x-\cos 5x\sin x}{\sin x\cos x}$$

$$=\frac{\sin(5x-x)}{\sin x\cos x}$$

$$=\frac{\sin 4x}{\sin x\cos x}=\frac{2\sin 4x}{2\sin x\cos x}$$

$$=\frac{2(2\sin 2x\cos 2x)}{\sin 2x}$$

$$=4\cos 2x=RHS$$

6.3.2) Determine the smallest positive value of x for which this identity is undefined. (1)(7)
[16]

$x=90°$

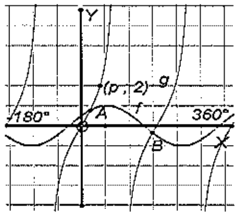
QUESTION 7

Determine the general solution of x if $\sin x\cos x = 3\cos^2 x$. [7]

$\sin x\cos x - 3\cos^2 x = 0$
$\cos x(\sin x - 3\cos x) = 0$
$\cos x = 0$ or $\sin x - 3\cos x = 0$
$\cos x = 0$ or $\sin x = 3\cos x$
$\cos x = 0$ or $\frac{\sin x}{\cos x} = 3$
$\cos x = 0$ or $\tan x = 3$
$x = \pm 90° + 360°n$ or $x = 71.57° + 180°n$, $n \in Z$

QUESTION 8

In the diagram, the trigonometric functions $f(x) = \sin(x+30°)$ and $g(x) = 2\tan x$ (partially) are represented, for $x \in [-180°; 360°]$.



8.1) Write down the value of p. (1)

$p = 45°$

8.2) Write down the Period of g. (1)

$180°$

8.3) Show that the x-coordinates of A and B can be determined from the equation $\sqrt{3}\sin x\cos x + \cos^2 x = 4\sin x$ (do not solve for x). (4)

At A and B : $\sin(x+30°) = 2\tan x$

$\sin x\cos 30° + \cos x\sin 30° = \frac{2\sin x}{\cos x}$

$\sin x\frac{\sqrt{3}}{2} + \cos x\frac{1}{2} = \frac{2\sin x}{\cos x}$

$\times 2\cos x : \sqrt{3}\sin x\cos x + \cos^2 x = 4\sin x$

8.4) Determine the new equation of f if the graph is shifted by $60°$ to the right. (1)

$y = \sin(x - 30°)$

8.5) Write down the equations of each of the asymptotes which are not drawn. (1)
[8]

$x = \pm 90°$; $x = 270°$

QUESTION 9

9.1)

9.1.1) Complete the following : For any ΔABC, $\sin A = \frac{\cdots}{b}$ (1)

$\sin A = \frac{a.\sin B}{b}$

9.1.2) Hence, if $\hat{B} = 2\hat{A}$, show that $\tan A = \frac{2a\sin A}{b}$. (3)(4)

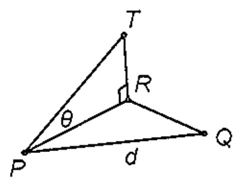
$\tan A = \frac{\sin A}{\cos A}$

$= \frac{\frac{a.\sin B}{b}}{\cos A} = \frac{a.\sin 2A}{b.\cos A}$

$= \frac{a.2\sin A\cos A}{b.\cos A}$

$= \frac{2a\sin A}{b}$

9.2) In the diagram, the observation points P, Q and R lie in the same horizontal plane. A balloon is situated at T, which is directly above R. The angle of elevation of T from P is θ. $P\hat{Q}R = 90° - \alpha$ and $QR = \frac{1}{2}PQ$, where $PQ = d$ units.



Prove that $TR = \frac{d\sqrt{5 - 4\sin\alpha}}{2}.\tan\theta$. (7)

[11]

In ΔTRP : $\frac{TR}{PR} = \tan\theta$

Then $TR = PR.\tan\theta$ (1)

In ΔPRQ : $PR^2 = d^2 + \left(\frac{d}{2}\right)^2 - 2d\left(\frac{d}{2}\right)\cos(90° - \alpha)$

$= d^2 + \frac{d^2}{4} - d^2 \sin\alpha = \frac{5d^2 - 4d^2\sin\alpha}{4}$

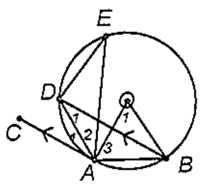
$= \frac{d^2(5 - 4\sin\alpha)}{4}$

Then $PR = \frac{d\sqrt{5 - 4\sin\alpha}}{2}$

From (1) : $TR = \frac{d\sqrt{5-4\sin\alpha}}{2}.\tan\theta$

QUESTION 10

10.1) In the circle with centre O, $O\hat{A}B = 70°$ and AC is a tangent to the circle at A, such that $AC \parallel BD$.



Calculate the value of each of the following angles, giving reasons :

10.1.1) $\hat{O}_1$ (3)

$O\hat{B}A = O\hat{A}B = 70°$ ($OA = OB$; radii)
$\hat{O}_1 = 180° - 2(70°)$ (angle sum in $\Delta OAB = 180°$)
$\hat{O}_1 = 40°$

10.1.2) $\hat{D}_1$ (2)

$\hat{D}_1 = \frac{1}{2}(40°) = 20°$ (central angle = 2 x inscribed angle)

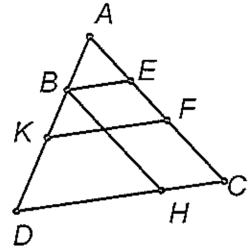
10.1.3) $\hat{A}_1$ (2)

$\hat{A}_1 = \hat{D}_1 = 20°$ (alt. angles ; $DB \parallel CA$)

10.1.4) $\hat{E}$ (2)(9)

$\hat{E} = \hat{A}_1 = 20°$ (AC tangent ; AD chord)

10.2) In the diagram, $\frac{DH}{HC} = \frac{7}{2}$, $AE : EF = 2:3$, $BE \parallel KF \parallel DC$ and $BH \parallel AC$.



Calculate the value of $\frac{AF}{FC}$, giving reasons. (7)

[16]

$\frac{AB}{BD} = \frac{CH}{HD} = \frac{2}{7}$ ($BH \parallel AC$)

and $\frac{AE}{EC} = \frac{AB}{BD} = \frac{2}{7}$ ($BE \parallel DC$)

If $EF = 3y$, then $AE = 2y$, $EC = 7y$ and

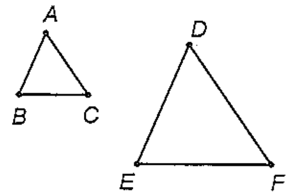
so $FC = 7y - 3y = 4y$

Then $\frac{AF}{FC} = \frac{2y + 3y}{4y}$

$= \frac{5}{4}$

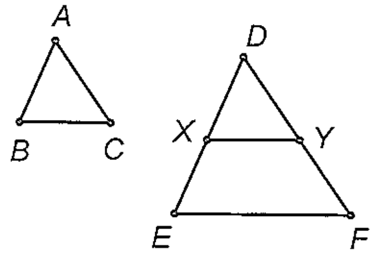
QUESTION 11

11.1) Triangles ABC and DEF are given, such that $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$. Prove the theorem which states that $\frac{AB}{DE} = \frac{AC}{DF}$.



(8)

Draw $DX = AB$, and $DY = AC$. Draw XY.



In ΔDXY and ΔABC :

(1) $DX = AB$ (constr.)

(2) $\hat{A} = \hat{D}$ (given)

(3) $DY = AC$ (constr.)

Therefore $\Delta DXY \equiv \Delta ABC$ (SAS)

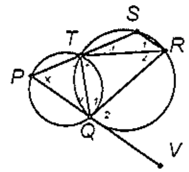
Then $D\hat{X}Y = \hat{B}$
$= \hat{E}$ (given)

Then $XY \parallel EF$ (corr. angles are equal)

Therefore $\dfrac{DX}{DE} = \dfrac{DY}{DF}$

Therefore $\dfrac{AB}{DE} = \dfrac{AC}{DF}$ ($DX = AB$ and $DY = AC$)

11.2) PQV and QR are tangents to the respective circles on the right and left hand sides. $\hat{P} = x$ and $P\hat{Q}T = y$.



11.2.1) Write down, giving reasons, one more angle equal to x and one more angle equal to y. (3)

$\hat{Q}_1 = \hat{P} = x$ (QR tangent; QT chord)
$\hat{R}_2 = T\hat{Q}P = y$ (QP tangent; QT chord)

11.2.2) Write $\hat{S}$ and $R\hat{Q}V$ in terms of x and / or y, giving reasons. (4)

$\hat{S} = 180° - \hat{Q}_1$ (opp. angles of cyclic quad $TQRS$)
$= 180° - x$
$R\hat{Q}V = 180° - (x + y)$ (angles on straight line PQV)

11.2.3) Prove that $PQ \parallel SR$. (2)

$\hat{P} + \hat{S} = x + 180° - x = 180°$

Therefore $PQ \parallel SR$ (co-int. angles are supplementary)

11.2.4) Prove that $\Delta PQT /// \Delta QRT$. (3)

In $\Delta PQT, \Delta QRT$:
(1) $\hat{P} = \hat{Q}_1$ (QR tangent; QT chord)
(2) $T\hat{Q}P = \hat{R}_2$ (QP tangent; QT chord)
(3) $\hat{T}_1 = \hat{T}_2$ (sum of angles in a triangle is $180°$)

Therefore $\Delta PQT /// \Delta QRT$ (AAA)

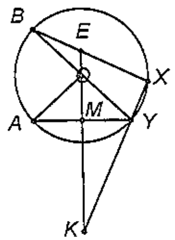
11.2.5) If SV is drawn such that $SV = SP$ and SV intersects QR at W, prove that QT is a tangent to circle WQV. (3)(15)
[23]

$\hat{Q}_1 = \hat{P}$ (proven)
$= \hat{V}$ ($SV = SP$)

Therefore QT is a tangent to circle WQV (converse of Tangent theorem)

QUESTION 12

O is the centre of the circle and M is the midpoint of chord AY.
BEX, BOY, $EOMK$ and XYK are straight lines.



12.1) Prove that $EMYX$ is a cyclic quadrilateral. (4)

$O\hat{M}A = 90°$ ($AM = MY$)
$\hat{X} = 90°$ (BY diameter)

Therefore $O\hat{M}A = \hat{X}$
Therefore $EMYX$ is a cyclic quadrilateral (ext. angle = int. opp. angle)

12.2) Prove that $AM = \dfrac{XE.MK}{XK}$ (6)

In $\Delta KMY . \Delta KXE$:
(1) $\hat{K}$ common angle
(2) $K\hat{M}Y = \hat{X} = 90°$ (proven)
(3) $K\hat{Y}M = X\hat{E}M$ (sum of angles in a triangle is 180°)

Therefore $\Delta KMY /// \Delta KXE$ (AAA)

Therefore $\dfrac{KM}{KX} = \dfrac{MY}{XE} = \dfrac{KY}{KE}$

Therefore $\dfrac{KM}{KX} = \dfrac{AM}{XE} = \dfrac{KY}{KE}$ ($AM = MY$)

Therefore $AM = \dfrac{XE.MK}{XK}$

12.3) If $\hat{B} = \hat{K}$, name another cyclic quadrilateral (apart from $BAYX$). (1)
[11]

$EBKY$