

Gr 12 - July 2014 Memo

Question 1 [18]

d) (1) $(x^{2/3} - 4)(x^{1/3} + 2) = 0$
 $x^{2/3} = 4$ or $x^{1/3} = -2$
 $x = \pm 8$ ✓ $x = -8$ ✓

(2) $\frac{x^2 + 1}{x + 1} \leq 0$ (2)

$\therefore x + 1 < 0$

$x < -1$ ✓ (1)

(3) $2^{2x+1} - 3(2^{2x-1}) + 4^x = 12$

$2^{2x+1} - 3 \cdot 2^{2x-1} + 2^{2x} = 12$

$2^{2x}(2^1 - 3 \times 2^{-1} + 1) = 12$

$2^{2x}(1 \frac{1}{2}) = 12$

$2^{2x} = 8$ ✓

$\therefore 2x = 3$

$x = \frac{3}{2}$ ✓ (4)

(4) $x^2 - 4x - 1 = \frac{2}{x^2 - 4x - 2}$

Let $k = x^2 - 4x$

$k - 1 = \frac{2}{k - 2}$ ✓

$k^2 - 3k + 2 = 2$ ✓

$k(k - 3) = 0$

$x^2 - 4x = 0$ ✓ or

$x(x - 4) = 0$

$x = 0$ ✓ or $x = 4$

$x^2 - 4x - 3 = 0$
 $x = \frac{4 \pm \sqrt{28}}{2}$

$= 4,65$ or $-0,65$

(5)

$$b) (x^2 - 5)(2x^2 - 3x + 9) = 0$$

$$(x^2 - 5)(2x - 3)(x - 3) = 0$$

$$x = \pm\sqrt{5} \quad \text{or} \quad x = \frac{3}{2} \quad \text{or} \quad x = 3$$

$$(1) \quad x = 3 \quad (2)$$

$$(2) \quad x = \pm\sqrt{5} \\ = \pm 2, 24. \quad (2)$$

$$c) \quad \Delta = 0$$

$$12 - 3a^2 = 0$$

$$4 = a^2$$

$$\pm 2 = a \quad (2)$$

Question 2 [16]

$$d) (1) \quad 2; 21 \quad (1)$$

$$(2) \quad S_{100} = 2 \times 50 + \frac{50}{2} [2 \times 5 + 49(4)] \\ = 100 + 5150 \\ = 5250 \quad (4)$$

$$b) \quad ar^6 = \frac{3645}{64} \quad \text{--- (1)} \quad \quad ar^3 = \frac{135}{8} \quad \text{--- (2)}$$

$$\frac{ar^6}{ar^3} = \frac{\frac{3645}{64}}{\frac{135}{8}}$$

$$r^3 = \frac{27}{8}$$

$$\therefore r = \frac{3}{2}$$

$$\text{Subs into (1):} \quad a \left(\frac{3}{2}\right)^3 = \frac{135}{8} \\ a = 5 \quad (4)$$

$$c) \quad 5 + 15 + 45 + \dots \quad S_n = 605$$

$$605 = \frac{5(3^n - 1)}{3 - 1}$$

$$242 = 3^n - 1$$

$$243 = 3^n$$

$$3^5 = 3^n$$

(3)

$$\therefore n = 5$$

$$d) \quad \sum_{n=1}^{\infty} (k - \frac{3}{2})^n = (k - \frac{3}{2}) + (k - \frac{3}{2})^2 + (k - \frac{3}{2})^3 + \dots$$

$$S_{\infty} = \frac{(k - \frac{3}{2})}{1 - (k - \frac{3}{2})}$$

$$= \frac{2k - 3}{2} \div \frac{2 - 2k + 3}{2}$$

$$= \frac{2k - 3}{5 - 2k}$$

(4)

Question 3 [27]

$$a) \quad (1) \quad y = 1 \quad (1)$$

$$(2) \quad \frac{-4}{x} + 1 = -x + 4$$

$$-4 + x = -x^2 + 4x$$

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$$x = 4 \quad \text{or} \quad x = -1$$

$$y = -(-1) + 4 = 5$$

$$\therefore A(-1; 5)$$

(4)

$$(3) \quad A(-1; 5) \quad B(-5; 1)$$

$$AB = \sqrt{(-1+5)^2 + (5-1)^2} \\ = 4\sqrt{2} \quad (3)$$

$$(4) \quad h(x) = \frac{-4}{x+2} + 1 \quad (1)$$

$$(5) \quad y = (x+2) + 1 \quad \text{or} \quad y = -(x+2) + 1 \\ = x + 3 \quad (1) \quad = -x - 1$$

$$(6) \quad [-1; 0) \quad [4; \infty) \quad (2)$$

$$b) \quad (1) \quad y = a(x-2)(x-4)$$

Subs (0; 8)

$$8 = a(0-2)(0-4)$$

$$1 = a$$

$$\therefore y = 1(x-2)(x-4) \\ = x^2 - 6x + 8$$

$$\therefore a = 1 \quad b = -6 \quad c = 8 \quad (4)$$

$$(2) \quad \text{Subs } (1; 3) \text{ into: } y = k^x \\ 3 = k^1 \quad (1)$$

$$(3) \quad x = 3 \quad \text{subs into} \quad y = x^2 - 6x + 8 \\ = (3)^2 - 6(3) + 8 \\ = -1$$

$$\therefore y \geq -1 \quad (2)$$

(4) (1) $f^{-1}(x)$ — one to many / vertical line, no function (1)

$$(2) \quad x \geq 3 \quad \text{or} \quad x \leq 3 \quad (1)$$

$$c) (1) (i) A(1; 0) \checkmark \quad (1)$$

$$(ii) B(27; -3) \checkmark$$

$$y = \log_{1/3} x \\ (1) \quad -3 = \log_{1/3} x$$

$$\therefore x = \left(\frac{1}{3}\right)^{-3} \\ = 27$$

$$(2) (0; 27) \checkmark \quad (2)$$

$$(3) f^{-1}(x) = \left(\frac{1}{3}\right)^x \checkmark \quad (1)$$

$$(4) f(x) = -\log_{1/3} x \\ = \log_3 x \checkmark \quad (1)$$

Question 4 [10]

$$a) f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - (x^2 - 3x)}{h} \\ = \lim_{h \rightarrow 0} \frac{2xh + h^2 - 3h}{h} \\ = \lim_{h \rightarrow 0} 2x + h - 3 \\ = 2x - 3 \quad (4)$$

$$b) (1) y = 2\sqrt{x} - \frac{4}{9}x^3 + \pi \\ = 2x^{1/2} - \frac{4}{9}x^3 + \pi$$

$$\frac{dy}{dx} = x^{-1/2} - \frac{4}{3}x^2 \quad (2)$$

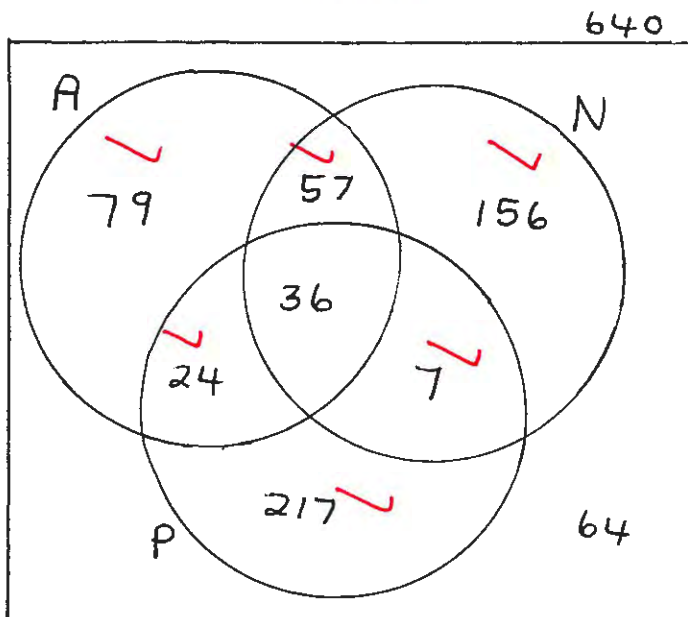
$$(2) \quad 8x^3 - 2xy + y - 1 = 0$$

$$\begin{aligned} & y - 2xy \\ \checkmark & y(1 - 2x) \\ & y \end{aligned}$$

$$\begin{aligned} & = 1 - 8x^3 \\ & = 1 - 8x^3 \\ & = \frac{(1 - 2x)(1 + 2x + 4x^2)}{(1 - 2x)} \\ & = 1 + 2x + 4x^2 \end{aligned}$$

$$\frac{dy}{dx} = 2 + 8x \quad \checkmark \quad y \quad (4)$$

Question 5 [6]



$$\begin{aligned} 136 - x + x + 213 - x + 36 + 7 + 24 + 217 + 64 &= 640 \\ -x + 697 &= 640 \\ 57 &= x \end{aligned}$$

Question 6 [8]

$$\begin{aligned} \text{a)} \quad & \frac{a^{3/2} + 5a^{1/2} + 6a^{-1/2}}{a^2 - 4} \\ &= \frac{a^{-1/2} (a^2 + 5a + 6)}{(a+z)(a-z)} \\ &= \frac{a^{-1/2} (a+3)(a+z)}{(a+z)(a-z)} \\ &= \frac{a^{-1/2} (a+3)}{(a-z)} \end{aligned}$$

(4)

$$\begin{aligned} \text{b)} \quad & x^2 - 5x + 7 = 0 \\ \Delta &= b^2 - 4ac \\ &= (-5)^2 - 4(1)(7) \\ &= -3 \end{aligned}$$

Roots non-real.

$$\begin{aligned} kx^2 + kx + 2 &= 0 \\ \Delta &= b^2 - 4ac \\ &= k^2 - 4k(2) \\ &= k^2 - 8k \end{aligned}$$

$$\begin{aligned} \therefore k^2 - 8k &< 0 \\ k(k - 8) &< 0 \end{aligned}$$

(4)

$$0 < k < 8.$$

Question 7 [14]

$$\begin{aligned} a) \quad A &= P(1 - i)^n \\ 75\,000 &= 250\,000(1 - 0,18)^n \end{aligned}$$

$$\frac{75\,000}{250\,000} = (0,82)^n$$

$$n = \log_{0,82} \frac{75}{250}$$

$$n = 6,0668\dots$$

\therefore In the 7th year (5)

$$\begin{aligned} b) \quad A &= P(1 + i)^n \\ &= 250\,000(1 + 0,08)^5 \\ &= R\,367\,332,02 \end{aligned} \quad (2)$$

$$c) \quad 367\,332,02 = \frac{x \left[\left(1 + \frac{0,095}{12}\right)^{60} - 1 \right]}{\frac{0,095}{12}}$$

$$x = R\,4\,806,61 \quad (4)$$

$$\begin{aligned} d) \quad 1 + i &= \left(1 + \frac{i}{m}\right)^m \\ 1 + i &= \left(1 + \frac{0,095}{12}\right)^{12} \\ i &= \left(1 + \frac{0,095}{12}\right)^{12} - 1 \end{aligned}$$

$$i = 0,09924\dots$$

\therefore 9,92% p.a. (3)

Question 8 [19]

$$\begin{aligned} \text{a) } m &= 3x^2 + 2ax + b \\ &= 3(4)^2 + 2a(4) + b \\ 0 &= 48 + 8a + b \end{aligned} \quad \text{--- (1)}$$

$$\begin{aligned} \text{Subs (1; 0): } 0 &= 1 + a + b + 12 \\ -13 &= a + b \end{aligned} \quad \text{--- (2)}$$

$$\begin{aligned} \therefore -48 - 8a &= -13 - a \\ -7a &= 35 \\ a &= -5 \end{aligned}$$

Subs into (2):

$$\begin{aligned} -13 &= -5 + b \\ -8 &= b \end{aligned}$$

(7)

$$y = x^3 - 5x^2 - 8x + 12.$$

$$\begin{aligned} \text{b) } 3x^2 - 10x - 8 &= 0 \\ (3x + 2)(x - 4) &= 0 \\ x = -\frac{2}{3} \quad \text{or} \quad x &= 4 \\ y = 14\frac{22}{27} \end{aligned}$$

$$Q \left(-\frac{2}{3}; 14\frac{22}{27} \right)$$

(4)

$$\begin{aligned} \text{c) } x^3 - 5x^2 - 8x + 12 &= 0 \\ (x - 1)(x^2 - 4x - 12) &= 0 \\ (x - 1)(x - 6)(x + 2) &= 0 \\ x = 1 \quad \text{or} \quad x = 6 \quad \text{or} \quad x &= -2 \end{aligned}$$

$$\therefore P(-2; 0) \quad U(6; 0)$$

(2)

$$\begin{aligned}
 d) \quad m_{\tan} &= 3x^2 - 10x - 8 \\
 &= 3(-2)^2 - 10(-2) - 8 \\
 &= 24 \checkmark
 \end{aligned}$$

$$\begin{aligned}
 y - 0 &= 24(x + 2) \checkmark \\
 y &= 24x + 48. \quad (3)
 \end{aligned}$$

$$\begin{aligned}
 e) \quad &(-\infty; -2] \checkmark \\
 &[-\frac{2}{3}; 1] \checkmark \\
 &[4; 6] \quad (3)
 \end{aligned}$$

Question 9 [8]

A: 51 ; 49 ; 47 ; ...

B: 25 ; 24 ; 23 ; ...

$$\begin{aligned}
 a) \quad S_n &= \frac{n}{2} [2a + (n-1)d] \checkmark \\
 S_t &= \frac{t}{2} [2 \times 51 + (t-1)(-2)] \checkmark \\
 &= \frac{t}{2} [102 - 2t + 2] \\
 &= 52t - t^2 \checkmark \quad (3)
 \end{aligned}$$

$$\begin{aligned}
 b) \quad S_n &= \frac{n}{2} [2a + (n-1)d] \checkmark \\
 S_t &= \frac{t}{2} [2 \times 25 + (t-1)(-1)] \checkmark \\
 &= \frac{t}{2} [50 - t + 1] \\
 &= \frac{51t}{2} - \frac{t^2}{2} \checkmark
 \end{aligned}$$

$$52t - t^2 + \frac{51t}{2} - \frac{t^2}{2} = 1000 \checkmark$$

$$104t - 2t^2 + 51t - t^2 = 2000 \checkmark$$

$$0 = 3t^2 - 155t + 2000 \checkmark$$

$$0 = (3t - 80)(t - 25) \checkmark$$

$$t = \frac{80}{3}$$

$$= 26,6$$

NA.

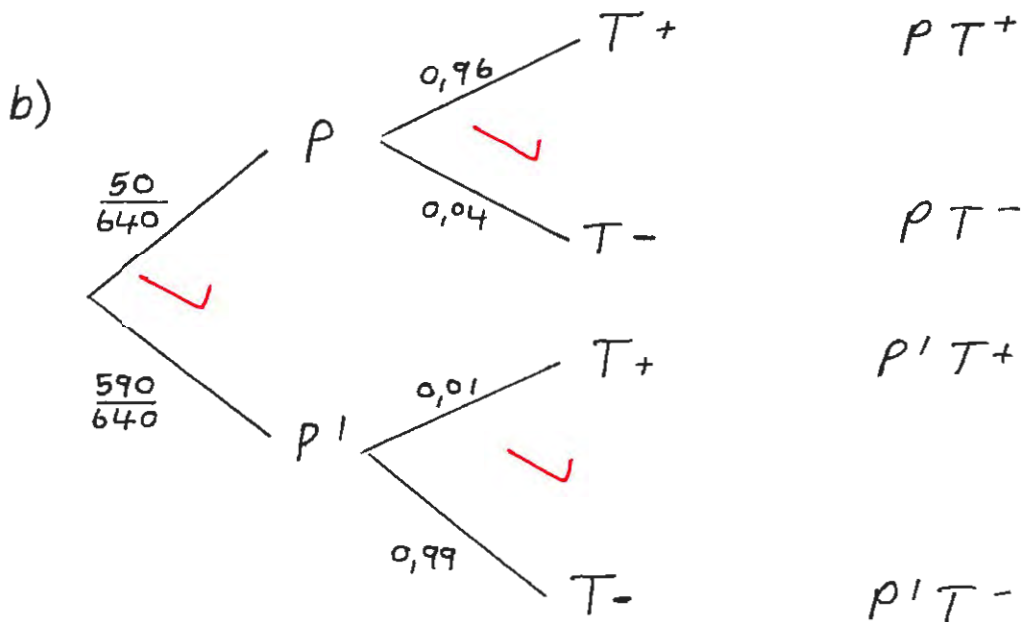
$$\text{or } t = 25 \text{ s} \checkmark$$

(5)

Question 10 [10]

a) (1) $\frac{6!}{2!} = 360$ (2)

(2) $\frac{2! \cdot 5!}{360} = \frac{2}{3}$ (3)



$\left(\frac{50}{640} \times 0,96\right) + \left(\frac{590}{640} \times 0,01\right)$
 $= 8,42\%$ (5)

Question 11 [7]

a) $h = 20(t^2 - \frac{1}{3}t^3)$
 $= 20t^2 - \frac{20}{3}t^3$

$\frac{dh}{dt} = 40t - 20t^2$
 $= 40(1,5) - 20(1,5)^2$
 $= 15 \text{ m/s}$

(2)

$$b) \quad 40 - 40t = 0$$

$$40 = 40t$$

$$1 = t \quad (2)$$

$$c) \quad 40t - 20t^2 = 0$$

$$20t(2 - t) = 0$$

$$t = 0 \quad \text{or} \quad t = 2$$

NA

$$\therefore h = 20 \left(2^2 - \frac{1}{3} (2)^3 \right)$$

$$= 26 \frac{2}{3} \text{ m.} \quad (3)$$

Question 12 [7]

a) Subs $x = a$ into $y = \frac{1}{3}x$
 $= \frac{1}{3}a$

$$\therefore D(a; \frac{1}{3}a)$$

$$m = -1x^{-2}$$

$$= \frac{-1}{a^2}$$

$$y - \frac{1}{3}a = -\frac{1}{a^2}(x - a)$$

$$a^2y - a = -1(x - a)$$

$$a^2y - a = -x + a$$

$$a^2y + x = 2a. \quad (4)$$

b) $a^2y + 0 = 2a$

$$y = \frac{2a}{a^2}$$

$$= \frac{2}{a}$$

$$B(0; \frac{2}{a})$$

$$a^2(0) + x = 2a$$

$$x = 2a$$

$$C(2a; 0)$$

$$\text{Area} = \frac{1}{2}(2a)\left(\frac{2}{a}\right)$$

$$= 2. \quad (3)$$