June Exam 2014 Maths Paper - 1

3 Hours 150 marks

SECTION A [80 MARKS]

QUESTION 1

(a) $2x^2 - 11x + 12 = 0$

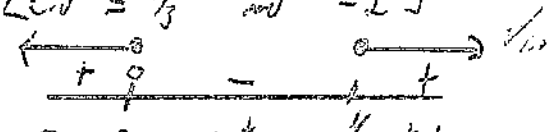
$\therefore (2x-3)(x-4) = 0$ ✓m

$\therefore x = \frac{3}{2}$ or ✓cv $x = 4$ (2)

(b) $3x^2 + 5x - 2 \geq 0$

$\therefore (3x-1)(x+2) \geq 0$ ✓c

[c.v. $= \frac{1}{3}$ and -2]



$\therefore x \in (-\infty; -2]$ ✓a or $[\frac{1}{3}; \infty)$ ✓a

OR $x \leq -2$ or $x \geq \frac{1}{3}$ (4)

(c) $9^{2x+1} = \frac{1}{27}$

$\therefore (3^2)^{2x+1} = 3^{-3}$ ✓cv

$\therefore 3^{4x+2} = 3^{-3}$ ✓a ✓a

$\therefore 4x + 2 = -3$ ✓ca

$\therefore 4x = -5$

$\therefore x = -\frac{5}{4}$ ✓a (4)

(d) $\log (x+2)^2 = 2$

$\therefore 10^2 = (x+2)^2$ ✓cv

$\therefore (x+2)^2 = 100$

$\therefore x + 2 = 10$ or $x + 2 = -10$

$\therefore x = 8$ ✓a or $x = -12$ ✓a (5)

[13]

(1)

QUESTION 2

(a) $\sum_{t=0}^{99} (3t - 1)$

(1) $(3(0)-1) + (3(1)-1) + (3(2)-1) \ldots$

$= -1 + 2 + 5 \ldots$ ✓a (1)

(2) $S_n = \frac{n}{2}(2a + (n-1)d)$ ✓cv

$n = 99 - 0 + 1 = 100$ ✓a

$a = -1$ $\therefore d = 3$

$\therefore S_n = \frac{100}{2}(2(-1) + (100-1)(3))$ ✓a

$\therefore S_n = 14750$ ✓a (4)

(b) Geometric Sequence

$T_6 = \frac{-3}{16}$ $\quad$ $T_9 = \frac{6}{256}$

$\therefore ar^5 = -\frac{3}{16}$ ✓a $\quad$ $\therefore ar^8 = \frac{6}{256}$ ✓a

$ar^8 = \frac{6}{256}$ ... (1)

$ar^5 = -\frac{3}{16}$ ... (2)

$r^3 = -\frac{1}{8}$ ✓cv (1) ÷ (2)

$\therefore r = -\frac{1}{2}$ ✓a

Sub $r = -\frac{1}{2}$ into (2)

$\therefore a(-\frac{1}{2})^5 = -\frac{3}{16}$

$\therefore a = -\frac{3}{16} \div (-\frac{1}{2})^5$

$\therefore a = 6$ ✓a

$6; (6 \times -\frac{1}{2}); (6 \times -\frac{1}{2} \times -\frac{1}{2})$ ✓cv

$= 6; -3; \frac{3}{2}$ ... ✓a (7)

[12]

Question 3

(a) $A = P(1-i)^n$

$i = 10\%$; $A = 2x$; $P = 2x$; $n = ?$

$\therefore 2x = 2x(1-0,1)^n$ ✓a

$\therefore \frac{1}{2} = (0,9)^n$ ✓c.a

$\therefore \log_{0,9}(\frac{1}{2}) = n$ ✓m

$\therefore n = 6,57\ldots$ year

$\therefore n = 6$ years 7 months ✓a (4)

(b) $A = P(1+i)^n$ ✓a

$A = 500\,000$; $n = 5$; $P = 200\,000$

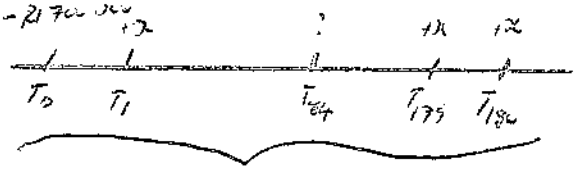
$\therefore 500\,000 = 200\,000(1+i)^5$ ✓c.a

$\therefore \frac{5}{2} = (1+i)^5$

$\therefore \sqrt[5]{\frac{5}{2}} - 1 = i$ ✓c.a

$\therefore i = 20\,11\%$ ✓a (4)

(c)(i) Loan = 2 000 000 − 300 000 = 1 700 000 ✓a



$i^{(12)} = \frac{0,09}{12}$ (mark)

$P = \frac{x[1-(1+i)^{-n}]}{i}$ ✓m

(ii) $\therefore 1\,700\,000 = \frac{x[1-(1+\frac{0,09}{12})^{-180}]}{\frac{0,09}{12}}$ ✓a ✓a

$\therefore x = R17\,242,53$ ✓a (5)

(c) (2) $\underset{T_0}{-R1\,700\,000}$ $\underset{T_1}{+17\,242,53}$ $\underset{T_{84}}{+17\,242,53}$

Balance Outstanding = Borrowed with interest − Paid back with interest ✓m

$B0 = 1\,700\,000(1+\frac{0,09}{12})^{84} - \frac{x[(1+i)^n-1]}{i}$ ✓a

$= 1\,700\,000(1+\frac{0,09}{12})^{84} - \left(\frac{17\,242,53[(1+\frac{0,09}{12})^{84}-1]}{\frac{0,09}{12}}\right)$ ✓a

$= R1\,176\,948,53$ ✓a (5)

(3) Interest = Paid back − borrowed back

$= (R17\,242,53 \times 180) - R1\,700\,000$ ✓a ✓a

$= R1\,403\,655,40$ ✓a (3)

[21]

Question 4

(a) (1) $x = 3$ ✓a (1)

(2) $y = a(x-x_1)(x-x_2)$

$\therefore y = a(x-1)(x-5)$ ✓a

Sub in $(0; 10)$

$\therefore 10 = a(0-1)(0-5)$ ✓m

$\therefore 10 = 5a$

$\therefore 2 = a$ ✓a

$\therefore y = 2(x-1)(x-5)$

$\therefore y = 2(x^2 - 6x + 5)$

$\therefore y = 2x^2 - 12x + 10$

$\therefore f(x) = 2x^2 - 12x + 10$ ✓a (4)

(3) Dom: $x \in R$ ✓a

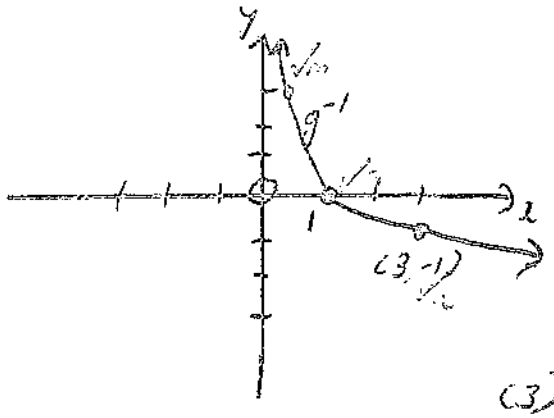
$x_{TP} = 3$

$\therefore y_{TP} = f(3) = 2(3)^2 - 12(3) + 10$

$\therefore y_{TP} = -8$ ✓a

$\therefore$ Range: $y \in [-8; \infty)$ (4)

(b) (1)



(3)

(2) $g^{-1}(x) = \log_{1/3}(x)$ (2)

(3) $y = x$ (1)

(4) $g: y = (\frac{1}{3})^x$

↓ x-axis

$-y = (\frac{1}{3})^x$ ✓

↓ y-axis

$-y = (\frac{1}{3})^{-x}$

$\therefore y = -(\frac{1}{3})^{-x}$ ✓ (2)

$\therefore y = -(3)^x$

[10]

Question 5

(a) (b) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ ✓m

$= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$ ✓c

$= \lim_{h \to 0} \frac{x - (x+h)}{\frac{x(x+h)}{h}}$ ✓m

$= \lim_{h \to 0} \left[\left(\frac{-h}{x(x+h)} \right) \times \left(\frac{1}{h} \right) \right]$

$= \lim_{h \to 0} \frac{-1}{x^2 + xh}$ ✓a

$= \frac{-1}{x^2}$ ✓a (5)

(b)(i) $y = \frac{x^2 - 5x + 6}{x - 2}$

$\therefore y = \frac{(x-3)(x-2)}{x-2}$

$\therefore y = x - 3$ ✓a

$\therefore \frac{dy}{dx} = 1$ ✓a (2)

(ii) $y = \frac{3}{\sqrt[3]{x^2}} + 2\sqrt{x}$

$\therefore y = 3x^{-\frac{2}{3}} + 2x^{\frac{1}{2}}$ ✓ ✓

$\frac{dy}{dx} = -2x^{-\frac{5}{3}} + x^{-\frac{1}{2}}$ ✓ ✓ ✓ (4)

(c) At $x = 3$:

$y = (3)^2 = 9$ (3; 9) ✓a

$m = \frac{dy}{dx} = 2x$

$\therefore$ Gradient at $x = 3$

$m = 2(3)$

$\therefore m = 6$ ✓a

$y = mx + c$

$\therefore y = 6x + c$

Sub in (3; 9)

$\therefore 9 = 6(3) + c$ ✓a

$\therefore -9 = c$

$\therefore y = 6x - 9$ ✓a (4)

[15]

SECTION A : [29 Marks]

SECTION B (70 MARKS)

QUESTION 6

(a) $A = \sqrt{\frac{x-2}{x+3}}$; $B = \sqrt{(x-5)(x+4)}$

(1) A undefined when $x = -3$ (1)

(2) Real: $(x-5)(x+4) \geq 0$

$\therefore x \in (-\infty; -4] \cup [5; \infty)$

$(x \leq -4 \text{ or } x \geq 5)$ (3)

(3) $x = 6$ (1)

(4) $x = -3$ (1)

(b) $x \in \mathbb{Q} : f(x) = 3 \cdot x \in \mathbb{Q}' : f(x) = \frac{5}{4}$

$2f(f(\sqrt{2}))$

$= 2f(\frac{5}{4})$

$= 2(3)$

$= 6$ (2)

[8]

(5)

QUESTION 7

(a) (1) 5; 11; A; 35; 53; 75

6; A-11; 35-A; 18

A-11-6; 35-A-(A-11)

quadratic → constant 2nd difference.

$\therefore A - 11 - 6 = 35 - A - (A - 11)$

$\therefore A - 17 = 35 - 2A + 11$

$\therefore 3A = 63$

$\therefore A = 21$ (2)

(2) Sequence: 5; 11; 21; 35; 53 ...

a; 6; 10; 14; 18

d; 4; 4; 4

2nd diff

$T_n = a + d(n-1) + \frac{2\text{nd diff}}{2}(n-1)(n-2)$

$\therefore T_n = 5 + 6(n-1) + \frac{4}{2}(n^2 - 3n + 2)$

$\therefore T_n = 5 + 6n - 6 + 2n^2 - 6n + 4$

$\therefore T_n = 2n^2 + 3$ (4)

(b) $S_n = \frac{2a + (n-1)d}{2} n^2$

(1) $S_3 = \frac{2(3) + 5(3)^2}{2}$

$\therefore S_3 = 36$ (1)

(2) $S_2 = \frac{2(2) + 5(2)^2}{2} = 14$

$T_3 = S_3 - S_2$

$= 36 - 19$

$= 17$ (3)

(b) (3) $S_1 = \dfrac{9(1)+5(1)^2}{2}$

$\therefore S_1 = 7$ ✓a

$T_2 = S_2 - S_1$

$\therefore T_2 = 19 - 7$

$\therefore T_2 = 12$ ✓a

$S_1 = 7 = T_1$

$\therefore$ Sequence: 7; 12; 17

A. Sequence: $a = 7$, $d = 5$ ✓ ✓a

$\therefore T_n = 5n + 2$ (4)

(c) ... $T_n = 3n - 2$ ... Biggest ... $3 - 2$ $= 9999 - 2 = 9997$ ✓ (3) [17]

QUESTION 8

$f(x) = -2/x$

(a)(1) $f(x) = -2/x$

a right ; b up

$h(x) = \dfrac{-2}{x-a} + b$ ✓a ✓a (2)

(6)

(2)

A/o/s pass through (a;b) and gradient $= -1$ ✓a

$y = mx + c$

$\therefore y = -x + c$

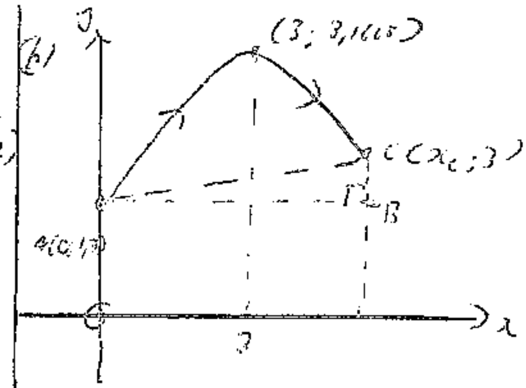
Sub in (a;b)

$\therefore b = -a + c$

$\therefore b + a = c$

$\therefore y = -x + a + b$ ✓a (3)

(b)



(1) $y = a(x-p)^2 + q$

$\therefore y = a(x-3)^2 + 3{,}1625$ ✓a

$\therefore 1{,}7 = a(0-3)^2 + 3{,}1625$ ✓a

$\therefore -1{,}4625 = 9a$ $\therefore a = -0{,}1625$

$\therefore y = -0{,}1625(x-3)^2 + 3{,}1625$ ✓a

At C: $3 = -0{,}1625(x_c - 3)^2 + 3{,}1625$

$\therefore -0{,}1625 = -0{,}1625(x_c - 3)^2$

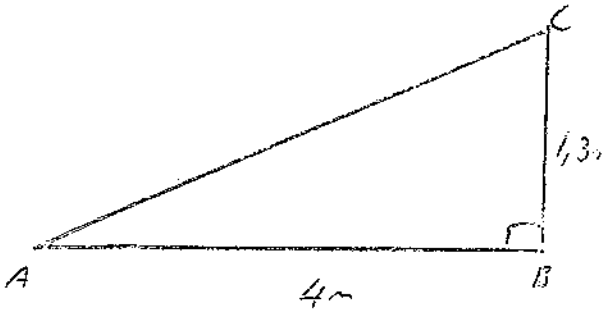
$\therefore (x_c - 3)^2 = 1 \Rightarrow x_c - 3 = 1$ or $x_c - 3 = -1$

$\therefore x_c = 4$ or $x_c = 2$

$\therefore x_c = 4$ $\therefore$ AB $= 4$ units (6)

(8b)

(2)



$AC^2 = AB^2 + BC^2$ ✓m

$\therefore AC^2 = 4^2 + 1,3^2$

$\therefore AC = 4,2m$ ✓a (2)

OR

A(0;1,7) B(4;3)

$\therefore AC = \sqrt{(0-4)^2 + (1,7-3)^2}$ ✓m

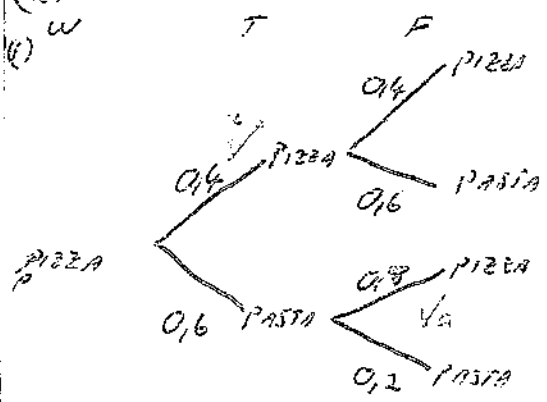
$\therefore AC = \sqrt{17,69}$

$\therefore AC = 4,2m$ ✓a (2)

[13]

QUESTION 9

(a)



(2)

(b) P(Pizza Friday)

$= (0,4 \times 0,6) + (0,6 \times 0,2)$ ✓a ✓a

$= 0,36$ ✓a

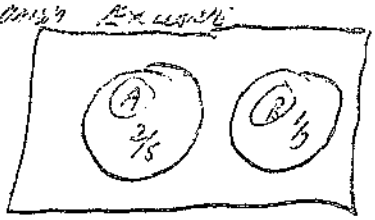
(3)

(7)

(6) $P(A) = \frac{2}{5}$, $P(B) = \frac{1}{3}$

(a) Mutually Exclusive

(a) (i)



$P(A \cup B)' = 1 - \frac{2}{5} - \frac{1}{3}$ ✓m ✓a

$= \frac{4}{15}$ ✓a (0,27)

(3)

(ii)

(2) Independent ∴

$P(A) \times P(B) = P(A \cap B)$ ✓m

$\therefore P(A \cap B) = \frac{2}{5} \times \frac{1}{3}$

$= \frac{2}{15}$ ✓a

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ✓m

$\therefore P(A \cup B) = \frac{2}{5} + \frac{1}{3} - \frac{2}{15}$

$= \frac{3}{5} = 0,6 = 60\%$ ✓a

(4)

[12]

QUESTION 10

$$f(x) = \frac{x^3+qx-px^2-pq}{x-p}$$

(a)(1) $f(p)$ is undefined ✓✓ (2)

(2) $f(x) = \frac{x(x^2+q)-p(x^2+q)}{x-p}$

$f(x) = \frac{(x^2+q)(x-p)}{x-p}$ ✓ fac

$\therefore f(x) = x^2+q$ ✓

$\therefore f'(x) = 2x$ ✓

[accept correct answer: $f'(p)$ does not exist]

$\therefore f'(p) = 2p$ ✓ (4)

(3) $\lim_{x\to p} f(x)$

$= \lim_{x\to p}(x^2+q)$ ✓

$= p^2+q$ ✓ (2)

(b) (1) neg ✓ (2) pos ✓

(3) pos ✓ (4) neg ✓

(5) neg ✓ (6) zero ✓

(7) 0 ✓ (8) pos ✓ (4)

(c) ⊥ to $2y = -4x+6$

⊥ to $y = -2x+3$.

$\therefore m_{tan} = \frac{1}{2}$ ✓

$\therefore f'(x) = \frac{1}{2}$.

$f(x) = 2x^3-4x-6$

$\therefore f'(x) = 6x^2-4$ ✓

$f'(x) = \frac{1}{2}$

$\therefore 6x^2-4 = \frac{1}{2}$ ✓

$\therefore 6x^2 = \frac{9}{2}$

$\therefore x^2 = \frac{3}{4}$

$\therefore x = \frac{\pm\sqrt{3}}{2}$ ✓

$\therefore x = 0{,}9$ ✓ or $x = -0{,}9$ (5)

[17]

(9)

$4x^2 + px + 9 = 0$ $(a=4;\ b=p;\ c=9)$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore x = \frac{-p \pm \sqrt{p^2 - 4(4)(9)}}{2(4)}$$ ✓ ✓

$$\therefore x = \frac{-p \pm \sqrt{p^2 + 144}}{8}$$

Diff roots $= 5$

$$\therefore \frac{-p + \sqrt{p^2+144}}{8} - \left(\frac{-p - \sqrt{p^2+144}}{8}\right) = 5$$ ✓ ✓

$$\therefore \frac{-p + \sqrt{p^2+144} + p + \sqrt{p^2+144}}{8} = 5$$

$$\therefore \frac{2\sqrt{p^2+144}}{8} = 5$$

$\therefore \sqrt{p^2+144} = 20$ $\therefore p^2 + 144 = 400$

$\therefore p^2 = 256$ ✓

$\therefore p = \pm 16$ ✓ ✓ [5]

SECTION B: 72 MARKS

TOTAL: 150 MARKS