Preliminary Examinations
July 2014
Mathematics Paper 2

## SECTION A

### QUESTION 1 — 5 marks

(a) $\frac{a-2}{3-8}=\frac{2+4}{8-6}$ ✓

$\frac{a-2}{-5}=\frac{6}{2}$

$a-2=-15$

$\therefore a=-13$ ✓ (3)

(b) $\frac{b+6}{2}=8$ $\quad\frac{c-4}{2}=2$

$b=10$ ✓ $\quad c=8$ ✓ (2)

### QUESTION 2 — 17 marks

(a) $\tan 2\theta=\cos 150^0$

$\tan 2\theta=-\frac{\sqrt{3}}{2}$ ✓

$2\theta=-40,9^0+k.180^0$ ✓

$\theta=-20,4^0+k.90 \qquad k\in\mathbb{Z}$ (4)

$\therefore \theta\in\{69,6^0;159,6^0\}$ ✓

(b) $\sin(180^0+\theta)\cos(90^0+\theta)+\cos(-\theta)\cos(\theta-180^0)=-\cos 2\theta$

$LHS=(-\sin\theta)(-\sin\theta)+(\cos\theta)(-\cos\theta)$

$=\sin^2\theta-\cos^2\theta$ ✓

$=-\cos 2\theta$ (5)

$=RHS$

(c) (1) $\frac{1-\cos 2\theta}{\sin 2\theta}=\tan\theta$ ✓

$LHS=\frac{1-(1-2\sin^2\theta)}{2\sin\theta\cos\theta}$ ✓

$=\frac{2\sin^2\theta}{2\sin\theta\cos\theta}$

$=\frac{\sin\theta}{\cos\theta}$ ✓ (4)

$=\tan\theta$ ✓ $=RHS$

(2) $\tan 15^0=\frac{1-\cos 2(15^0)}{\sin 2(15^0)}$ ✓

$=\frac{1-\cos 30^0}{\sin 30^0}$

$=\frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ ✓ (4)

$=2-\sqrt{3}$ ✓

### QUESTION 3 — 18 marks

(a) $m_{AB}=\frac{7-4}{1-(-3)}=\frac{3}{4}$ ✓ ✓ (2)

(b) $m_{AD}=-\frac{4}{3}$ ✓

$y-4=-\frac{4}{3}(x+3)$ ✓ (3)

$y=-\frac{4}{3}x \quad \therefore c=0$, passes through origin ✓

(c) $\tan(180^0-\alpha)=-\frac{4}{3}$ ✓

$\tan\alpha=\frac{4}{3}$ (2)

$\alpha=53,13^0$ ✓

(d) $y=-\frac{4}{3}x$ ... 1) $\quad 2y+11x=25$ ... 2)

*Subs* 1) int *o* 2)

$2\left(-\frac{4}{3}x\right)+11x=25$ ✓

$-\frac{8}{3}x+11x=25$ (3)

$\frac{25}{3}x=25$

$x=3$ ✓

$\therefore y=-4$ ✓

$\therefore D(3;-4)$

(e) *Area* = *AB.AD* ✓

$=\sqrt{(1-(-3))^2+(7-4)^2}\ .\ \sqrt{(-3-3)^2+(4-(-4))^2}$ ✓

$=5\ .\ 10$ (3)

$=50\ units^2$ ✓

(f) $(x+3)^2+(y-4)^2=25$ ✓ ✓ ✓ (3)

*(g)* One point of intersection. ✓ (2)

AD = 2(AO) i.e. will touch at O. ✓

## QUESTION 4 3 marks

$a=90^0$ ∠ in semi circle ✓

$b=110^0$ ext. ∠ in ΔEFG ✓

$c=110^0$ ext. ∠ of cyclic quad EFGH ✓

## QUESTION 5 13 marks

(a) $\sin(\theta+\beta)$

$=\sin\theta\cos\beta+\cos\theta\sin\beta$

$=\left(\frac{3}{5}\right)\left(\frac{5}{13}\right)+\left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$ (4)

$=\frac{15}{65}+\frac{48}{65}$

$=\frac{63}{65}$ ✓

(b) (1) $6^2+a^2=10^2$ ✓ (2)

$a=-8$ ✓

(2) (*i*) $\cos 2A=\frac{6}{10}=\frac{3}{5}$ ✓ (1)

(*ii*) $\sin 4A=2\sin 2A.\cos 2A$ ✓

$=2\left(\frac{-8}{10}\right)\left(\frac{3}{5}\right)$ ✓ (3)

$=-\frac{24}{25}$ ✓

(*iii*) $2\cos^2 A-1=\cos 2A$

$\cos^2 A=\frac{\left(\frac{3}{5}\right)+1}{2}=\frac{4}{5}$ ✓ (3)

$\therefore \cos A=\pm\frac{2\sqrt{5}}{5}$ ✓ ✓

## QUESTION 6 8 marks

*(a)* H = -0,88W + 33,95 ✓ ✓ ✓ straight line (3) *(accept in terms of x and y)*

*(b)* H = -0,88 (9,75) + 33,95 = 25,37 *cm* ✓ ✓ (2)

*(c)* *r* = -0,99 ✓ Strong correlation ✓ ∴ very reliable (3)

QUESTION 7 14 marks

*(a)* 30 boys (1)

*(b)* 60 – 30 = 30 boys (1)

*(c)* 165 *cm* (1)

*(d)* 170 – 160 = 10 *cm* (2)

*(e)* 40% of 120 = 48

120 – 48 = 72

$\therefore k = 166{,}5$ *cm* *(accept 167 cm)* (2)

*(f)* 30; 30; 19 (2)

*(g)* mean = 165 *cm* (2)

*(h)* (1) Mean will decrease (1)

(2) Standard deviation will increase (1)

(3) Skewed to the left (1)

**SECTION B**

QUESTION 8 16 marks

*(a)* $x^2 - 8x + y^2 - 4y = -7$

$(x-4)^2 + (y-2)^2 = -7 + 16 + 4$ (4)

$(x-4)^2 + (y-2)^2 = 13$

$P(4; 2)$

$PQ = \sqrt{13}$

*(b)* $\hat{Q}_1 = 90^0$ (tan $\perp$ radius)

$\hat{S}_1 = 90^0$ (tan $\perp$ radius) (3)

$\therefore \hat{Q}_1 + \hat{S}_1 = 180^0$

$\therefore$ PQRS is a cyclic quad (opp $\angle$s of quad are supplementary)

*(c)* $M_{PR} = \left(\frac{4+11}{2}; \frac{2+6}{2}\right) = \left(\frac{15}{2}; 4\right)$

$PR = \sqrt{(4-11)^2 + (2-6)^2}$

$= \sqrt{65}$ (5)

$\therefore r = \frac{\sqrt{65}}{2}$

$\therefore circle: \left(x - \frac{15}{2}\right)^2 + (y-4)^2 = \frac{65}{4}$

*(d)* $RS^2 = PR^2 - PS^2$ *pythag*

$RS^2 = 65 - 13$ (4)

$RS^2 = 52$

$RS = 2\sqrt{13}$ accept $\sqrt{52}$

QUESTION 9 3 marks

$p = 8$ *or* $p = 9$ workings

## QUESTION 10 — 9 marks

(a) $a^2 = b^2 + c^2 - 2bc\cos A$

$\therefore \cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$ ✓ (1)

(b) (1) In $\Delta ABC$, $\cos B\hat{A}C = \dfrac{6^2 + 7^2 - 8^2}{2(6)(7)}$ ✓

$\cos B\hat{A}C = \dfrac{1}{4}$ ✓

$\therefore B\hat{A}C = 75,52^0$ ✓ (4)

$\therefore B\hat{O}C = 2(75,52^0) \approx 151^0$ ✓ ∠at centre = 2 x ∠ at circumf

(2) $\hat{C}_1 = 14,48^0$ ✓ int. ∠s of isosc Δ

In $\Delta OBC$:

$\dfrac{OB}{\sin 14,48^0} = \dfrac{8}{\sin 151^0}$ ✓ (4)

$\therefore OB = 4,13$ ✓

$\therefore$ diameter $= 8,26$ units ✓

## QUESTION 11 — 9 marks

(a) $a = 3;\ b = -3$ ✓ ✓ (2)

(b) (1) $f(x) = 3\cos 2x - 3$

$= 3(1 - 2\sin^2 x) - 3$ ✓ (2)

$= -6\sin^2 x$ ✓

(2) $-6\sin^2 x = \sin x$

$-\sin x(6\sin x + 1) = 0$

$\sin x = 0$ ✓ *or* $\sin x = -\dfrac{1}{6}$ ✓ (5)

$x = 0^0 + k.180^0$ *or* $x = -9,59^0 + k.360^0$ ✓

*or* $x = 189,59^0 + k.360^0$ ✓ $k \in \mathbb{Z}$

$\therefore x = 0^0$ *or* $180^0$ ✓

## QUESTION 12 — 8 marks

(a)

$\hat{A}_3 = \hat{B} = x$ tan-chord ✓ (2)

$\hat{B} = \hat{C}_2 = x$ tan-chord ✓

*OR*

$AE = CE$ tangents from the same point

$\therefore \hat{C}_2 = \hat{A}_3 = x$ equal ∠s opp equal sides

(b)

$\hat{A}_2 + \hat{A}_3 = 90^0$ tan ⊥ radius ✓ (2)

$\hat{B} = x$ proven in (a)

$\therefore \hat{D} = 90^0 - x$ int. ∠s of Δ ✓

(c)

$\hat{C}_1 = 90^0$ ∠ in semi-circle ✓

$\hat{C}_2 = x$ proven in (a)

$\therefore \hat{C}_3 = 90^0 - x$ ∠s on str line ✓ (4)

$\therefore \hat{C}_3 = \hat{D} = 90^0 - x$

$\therefore CE = ED$ equal sides opp. equal ∠s ✓

But $CE = EA$ tangents from same pt

$\therefore$ CE = ED = EA ✓

$\therefore AE = ED$

## QUESTION 13 — 9 marks

(a)

In $\Delta AEC$ and $\Delta DEB$

$\hat{C} = \hat{B}$ $\angle$s in same segment ✓

$\hat{A} = \hat{D}$ $\angle$s in same segment ✓

$A\hat{E}C = D\hat{E}B$ vert. opp $\angle$s ✓

$\therefore \Delta AEC \;///\; \Delta DEB$ AAA ✓

$\therefore \frac{CE}{EB} = \frac{AE}{ED}$ ✓ (5)

$\therefore CE.ED = BE.EA$

(b)

$QT = TS = 3$ line from centre perp chord ✓

Let $PT = 4x$

$\therefore TR = x$ given

$PT.TR = QT.TS$ proven in (a)

$(4x)(x) = (3)(3)$ ✓

$x^2 = \frac{9}{4}$

$x = \frac{3}{2}$ ✓ (4)

$\therefore$ diameter $= 5x = 5\left(\frac{3}{2}\right) = 7,5$ units ✓

## QUESTION 14 — 6 marks

(a) $\frac{\text{area } \Delta SUW}{\text{area } \Delta RSW} = \frac{\frac{1}{2}UW.h}{\frac{1}{2}WR.h} = \frac{\frac{1}{2}(3x)h}{\frac{1}{2}(5x)h} = \frac{3}{5}$ ✓ (2)

(b) $\frac{TQ}{QR} = \frac{2}{5}$ ✓ (2)

(c) $\frac{TV}{SW} = \frac{1}{3}$ ✓ (2)

## QUESTION 15 — 12 marks

(a) $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(7)^2(24)$ ✓ (2)

$= 392\pi \ cm^3$ ✓

(b) $h = \frac{24}{3} = 8 \ cm$ ✓

$\frac{r}{R} = \frac{h}{H}$ (3)

$\therefore r = \frac{Rh}{H} = \frac{(7)(8)}{24}$ ✓

$= \frac{7}{3} \ cm$ ✓

(c) $V_{air} = V_{cone} - V_{water}$

$= 392\pi - \frac{1}{3}\pi\left(\frac{7}{3}\right)^2(8)$ ✓

$= 392\pi - \frac{392}{27}\pi$ ✓ (3)

$= \frac{10192}{27}\pi = 377,48\pi \ cm^3$ ✓

(d) $\frac{24}{7} = \frac{h_{air}}{r}$

$\therefore h = \frac{24r}{7}$ ✓

$V_{air} = 377,48\pi = \frac{1}{3}\pi r^2 h$ (4)

$377,48\pi = \frac{1}{3}\pi r^2\left(\frac{24r}{7}\right)$ ✓

$r^3 = 330,30$

$r = 6,91$ ✓

$\therefore h = 23,70$

$h_{water} = 24 - 23,70 = 0,3cm$ ✓