Matric Prelim Paper 1 2014 - Memo

SECTION A

Question 1

(a)

$$\frac{2^x.3^{2x+2}}{3^x.2^{x-1}.3^{x-1}} \checkmark$$
$$= 2^{x-x+1}.3^{2x+2-x-x+1} \checkmark$$
$$= 2.3^3$$
$$= 54 \checkmark \quad [3]$$

(b)(1)

$$(x+1)^{\frac{2}{3}} = 4 \checkmark$$
$$\therefore x+1 = \pm 8 \checkmark$$
$$\therefore x = 7 \text{ or } x \neq -9 \checkmark \quad [3]$$

-1 for no $\pm$

(b)(2)

$$3^{2x} - 3.3^x - 54 = 0 \checkmark\checkmark$$
$$\text{let } k = 3^x \checkmark$$
$$\therefore k^2 - 3k - 54 = 0$$
$$\therefore (k-9)(k+6) = 0 \checkmark$$
$$\therefore 3^x = 9 \quad or \quad 3^x \neq -6 \checkmark$$
$$\therefore x = 2 \checkmark \quad [6]$$

(c)

$$\sqrt{2-x} = x+13-3 \checkmark$$
$$2-x = x^2+20x+100 \checkmark$$
$$x^2+21x+98 = 0 \checkmark$$
$$(x+14)(x+7) = 0$$
$$x = -14 \text{ or } x = -7 \checkmark \quad [6]$$
$$y \neq -1 \text{ or } y = 6 \checkmark\checkmark$$

(d)(1)

$$x = \frac{-6 \pm \sqrt{36-4p(-p)}}{2p} \checkmark$$
$$\therefore x = \frac{-6 \pm \sqrt{36+4p^2}}{2p} = \frac{-3 \pm \sqrt{9+p^2}}{p}$$
$$\checkmark \quad [2]$$

Question 1

(d)(2)

p is any real number since expression under root sign is always positive. $\checkmark\checkmark$ [2]

Question 2

(a)

$$\left(2\left(\frac{x}{2}\right)-1\right)\frac{(2x-1)}{2} \geq 0 \checkmark\checkmark$$
$$\therefore (x-1)(2x-1) \geq 0 \checkmark$$
$$\therefore x \geq 1 \quad or \quad x \leq \frac{1}{2} \checkmark\checkmark \quad [5]$$

(b)

$$m = g'(4) = \frac{1}{2} \checkmark$$
$$\therefore grad.\ perp. = -2 \checkmark$$
$$\therefore y-3 = -2(x-4) \checkmark\checkmark$$
$$\therefore y = -2x+11 \checkmark \quad [5]$$

(c)(1)

$$f(x) = x^{\frac{3}{2}} - \frac{3}{2}x^{-1} + 2x^{\frac{1}{2}} \checkmark$$
$$\therefore f'(x) = \frac{3}{2}x^{\frac{1}{2}} \checkmark + \frac{3}{2}x^{-2} \checkmark + x^{-\frac{1}{2}} \checkmark$$
$$= \frac{3\sqrt{x}}{2} + \frac{3}{2x^2} \checkmark + \frac{1}{\sqrt{x}} \quad [5]$$

(c)(2)

$$y(x+2) = x^2(x+2) \checkmark\checkmark$$
$$\therefore y = x^2; \checkmark \qquad x \neq -2 \checkmark$$
$$\therefore \frac{dy}{dx} = 2x \checkmark \quad [5]$$

Question 3

(a)

$$T_n = n^2 + 2n \quad ✓✓$$

$$\therefore T_{40} = 1680 \quad ✓ \qquad [3]$$

(b)

$$4a = \frac{a}{1-r} \quad ✓$$

$$\therefore 1-r = \frac{1}{4} \quad ✓$$

$$\therefore a = \frac{3}{4} \quad ✓ \qquad [3]$$

(c)

$$T_{10} - T_9 = 6 \quad \therefore d = 6 \; ✓$$

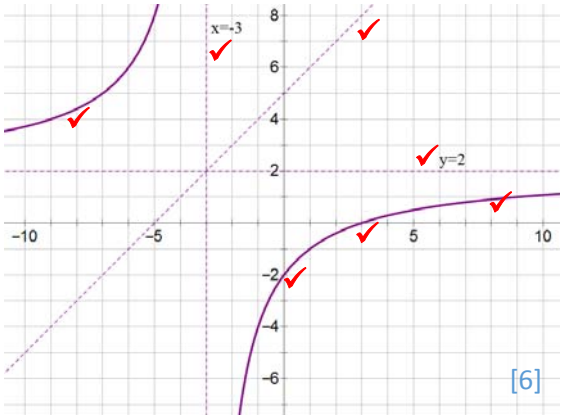
$$S_{10} - S_9 = 57 \quad \therefore T_{10} = 57 \; ✓$$

$$\therefore a + 9(6) = 57 \; ✓✓$$

$$\therefore a = 3 = T_1 \; ✓ \qquad [5]$$

Question 4

(a)



[6]

(b)

y = x + 5 ✓✓ [3]

(c)(1)

$$1800 - 1350 = 450 \quad ✓$$

$$\therefore y = 370(1,00448)^{450} \; ✓$$

$$= 2\,766 \; million \quad ✓ \qquad [3]$$

Question 4

(c)(2)

$$5000 = 370(1,00448)^n \quad ✓$$

$$\therefore n = 582,48 \; ✓✓$$

$$\therefore year = 1350 + 582 = 1932 \; ✓$$

$$\therefore 1930's \qquad [4]$$

Question 5

(a)

$$3x^2 + 2x = 1 \quad ✓$$

$$\therefore 3x^2 + 2x - 1 = 0$$

$$(3x-1)(x+1) = 0$$

$$\therefore x = \frac{1}{3}; \; ✓ \quad x \neq -1 \; ✓ \qquad [3]$$

Note they may not work back from the given answer.

(b)

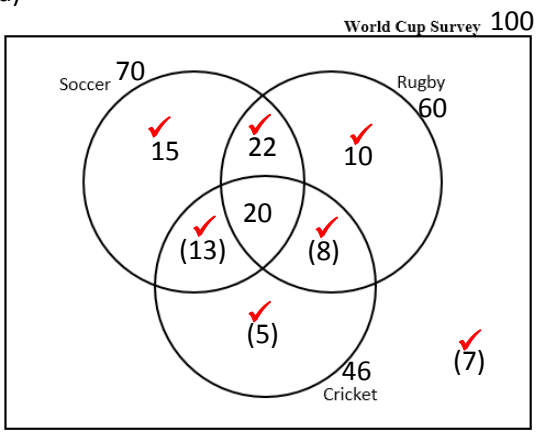
$$2\left(\frac{1}{3}\right) \times 60 \quad ✓$$

$$= 40 \quad ✓ \qquad [2]$$

SECTION B

Question 6

(a)



[7]

(b) $\frac{7}{100} = 0,07$ ✓ ✓ [2]

(c)

$$p(S) \times p(R) \; ✓$$

$$= 0,7 \times 0,6 = 0,42 \; ✓$$

$$P(S \text{ and } R) = 0,42 \; ✓$$

$$\therefore \text{independent} \qquad [3]$$

Question 7

(a)

$$F = P(1+in)$$
$$= 150\,000(1+0{,}12\times 5)$$
$$= R240\,000$$ [3]

(b)

$$0{,}12 = \left(1+\frac{r}{12}\right)^{12} - 1$$
$$r = 11{,}39\%$$ [3]

(c)

$$150\,000 = \frac{x\left[1-\left(1+\frac{0{,}1139}{12}\right)^{-60}\right]}{\frac{0{,}1139}{12}}$$
$$= R3290{,}61$$ [4]

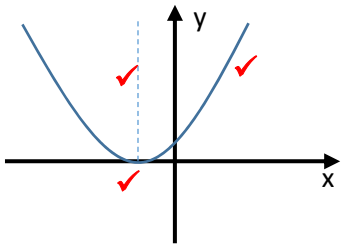
(d) Liam:
$$60\times 3290{,}61 - 150\,000$$
$$= R47\,436{,}82$$

Connor:
$$240\,000 - 150000$$
$$= R90\,000$$

Connor pays more since, with SI, interest is calculated on the initial loan rather than on a decreasing balance (CI). [4]

Question 8

(a)(1)



[3]

(a)(2) $\left(-\frac{b}{2a};0\right)$ [2]

Question 8

(b)(1)

$$y = A(x-1)^2(x-3)$$
$$\therefore 6 = A(-1)^2(-3)$$
$$\therefore A = -2$$
$$\therefore y = -2\left(x^2-2x+1\right)(x-3)$$
$$\therefore y = -2x^3+10x^2-14x+6$$ [6]

Note they may not work back from the given answer.

(b)(2)(i)

$$g'(x) = -6x^2+20x-14$$
$$0 = 3x^2-10x+7$$
$$0 = (3x-7)(x-1)$$
$$\therefore x = \frac{7}{3} \quad or \quad x = 1$$
$$\therefore 1 < x < \frac{7}{3}$$ [5]

(b)(2)(ii)

$$g''(x) = -12x+20$$
$$\therefore x = \frac{5}{3} \text{ (inflection)}$$
$$\therefore -\infty < x < \frac{5}{3}$$ [3]

(b)(3) $x = \frac{7}{3} \quad \therefore y = \frac{64}{27}$

$$\therefore 0 < k < \frac{64}{27}$$ [2]

(c)

$$A = 80x - \frac{\pi\left(\frac{x}{2}\right)^2}{2}$$
$$\therefore A = 80x - \frac{\pi x^2}{8}$$
$$\therefore A' = 80 - \frac{\pi x}{4}$$
$$\therefore x = \frac{320}{\pi}\ cm$$ [6]

Question 9

(a)(1)

$$600 = \frac{n}{2}\left[2(50) + (n-1)(-2)\right]$$

$$\therefore 600 = 50n - n^2 + n$$

$$\therefore n^2 - 51n + 600 = 0$$

$$\therefore n = 19; \quad n \neq 33 \quad (rounding\ up)$$

[4]

(a)(2)

$$600 = \frac{100\left(1 - 0{,}85^n\right)}{0{,}15}$$

$$\therefore n = 15 \quad (rounding\ up)$$

Gerald crosses first. [4]

(b)(1)

$$\sum_{r=1}^{100} r^2 = \frac{100(101)(201)}{6}$$

$$= 338350$$

[3]

(b)(2)

$$\sum_{r=3}^{40} r^2 = \sum_{r=1}^{40} r^2 - T_1 - T_2$$

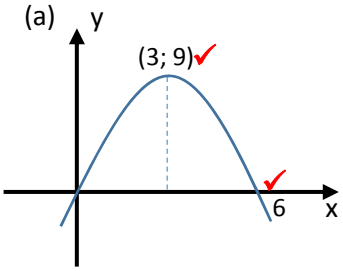
$$= \frac{40(41)(81)}{6} - 1 - 4$$

$$= 22135$$

[3]

Question 10

(a)



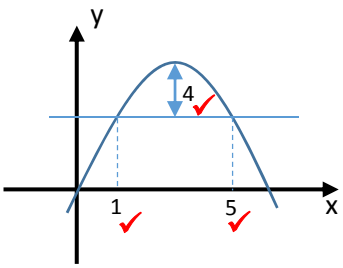
$$A = \frac{2}{3} \times 6 \times 9$$

$$= 36\,units^2$$

[4]

Question 10

(b)



Top area =

$$\frac{2}{3} \times 4 \times 4$$

$$= \frac{32}{3}\ units^2$$

Shaded area =

$$36 - \frac{32}{3}$$

$$= \frac{76}{3}\ units^2$$

[5]

TOTAL: [150]