St John's College

Preliminary Examinations
July 2014
Mathematics Paper 1

Examiner: **G Evans** **Time:** **3 hrs**
Moderator: **D Grigoratos** **Marks:** **150**

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 12 pages, including an **Answer Sheet** (pages 9 and 10) and an **Information Sheet** (pages 11 and 12). Please check that your paper is complete.

2. Read the questions carefully.

3. Answer the questions on the separate paper provided, *except* **Question 4(a) and (b)** and **Question 6** which must be answered on the **Answer Sheet**. Write your name on the Answer Sheet and indicate your teacher's initials.

4. Number your answers exactly as the questions are numbered and answer the questions in the correct order.

5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.

6. Round off your answers to one decimal digit where necessary.

7. All the necessary working details must be clearly shown. Equations may not be solved solely with a calculator.

8. It is essential that you present your work neatly and logically.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Out of	22	20	11	16	5	12	14	27	14	9
Mark										
TOTAL										

SECTION A

QUESTION 1

(a) Simplify the expression:

$$\frac{2^x.9^{x+1}}{3^x.6^{x-1}}$$ (3)

(b) Solve for x, without the use of a calculator:

(1) $\log_{x+1} 4 = \frac{2}{3}$ (3)

(2) $9^x - 3^{x+1} = 54$ (6)

(c) Solve for x and y:

$$x = y - 13$$
$$\sqrt{2-x} = y - 3$$ (6)

(d) Consider the quadratic equation: $px^2 + 6x - p = 0$.

(1) Solve the equation, in terms of p, using the quadratic formula. (2)

(2) Hence, state for which value(s) of p the equation will have real roots. Give a reason for your answer. (2)

[22]

QUESTION 2

(a) Given $f(x) = 2x - 1$ solve for x: $f\left(\frac{x}{2}\right) \times \frac{f(x)}{2} \geq 0$ (5)

(b) Given:

$$g'(x) = \frac{1}{\sqrt{x}} \quad \text{and} \quad g(4) = 3$$

Find the equation of the perpendicular to the graph of g at $x = 4$. (5)

(c) (1) Determine $f'(x)$ if $f(x)=x^{\frac{3}{2}}-\frac{3}{2x}+2\sqrt{x}$ (5)

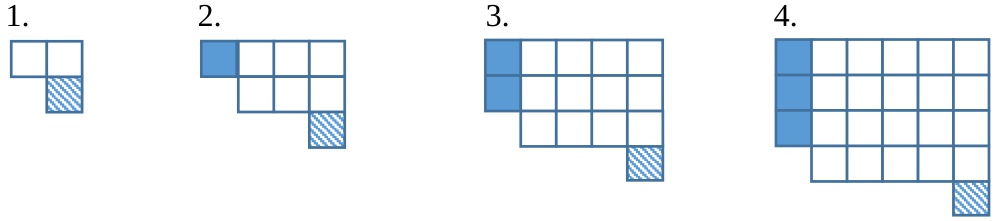
(2) Find an expression for $\frac{dy}{dx}$ if $xy+2y=x^3+2x^2$

and state any restrictions. (5)

[20]

QUESTION 3

(a) Consider the following picture pattern, which continues in the same sense indefinitely:



Calculate the number of square blocks in the 40^{th} picture. (3)

(b) The sum to infinity of a certain geometric sequence is 4 times the first term.

Calculate the common ratio. (3)

(c) In an arithmetic sequence, the n^{th} term is given as T_n and the sum of the first n terms is S_n. It is given that:

$$T_{10}-T_9=6$$

$$S_{10}-S_9=57$$

Find the value of T_1. (5)

[11]

QUESTION 4 (*Answer this question on the Answer Sheet*)

(a) Sketch the graph of $f(x)=2-\frac{12}{x+3}$ on the given set of axes on the Answer Sheet, indicating asymptotes and intercepts with the axes. (6)

(b) State the equation of the axis of symmetry which has a positive gradient and add this line to your sketch. (3)

(c) *The **world population** is the total number of living humans on Earth. As of 2013, it is estimated at 7.166 billion (7 166 000 000) by the United States Census Bureau. The world population has experienced continuous growth since the end of the Great Famine and the Black Death in 1350, when it was near 370 000 000. [Wikipedia - adapted]*

Using this information, the world population can be modelled by the formula:

$y = 370(1{,}00448)^x$

where y is in millions and x is the number of years since the year 1350.

(1) Estimate the world population, to the nearest million, in the year 1800. (3)

(2) In which decade did the population first exceed 5 000 million? (4)

[16]

QUESTION 5

The probability that Lionel Messi scores in a game of soccer is $2x$ whilst the probability that he does not score is $3x^2$.

(a) Determine the value of x, showing that its value is $\frac{1}{3}$. (3)

(b) In how many of the next 60 games is Messi likely to score? (2)

[5]

74 Marks

SECTION B

QUESTION 6 (*Answer this question on the Answer Sheet*)

A survey was conducted on a group of 100 students to see who would watch the following World Cups: Soccer, Rugby and Cricket. The results are summarised as follows:

Soccer: 70	Rugby: 60	Cricket: 46
Only Soccer: 15	Only Rugby: 10	All three: 20
Soccer and Rugby: 42		

(a) Represent the given information using the Venn Diagram on the answer sheet. (7)

(b) What is the probability that a randomly selected student will not watch any of the World Cups? (2)

(c) Are the events "Soccer" and "Rugby" independent? Give a mathematical motivation for your answer. (3)

[12]

QUESTION 7

In order to buy their first cars, Connor and Liam both take out loans for R150 000 with an interest rate of 12% per annum over a period of 5 years with monthly instalments starting in one month's time. However, Connor's loan is with simple interest and Liam's loan is with monthly compounding.

(a) Determine the total value of Connor's loan, including interest. (3)

(b) Calculate the percentage nominal interest rate applicable to Liam's loan, giving the answer ***correct to 2 decimal places***. (3)

(c) Calculate Liam's monthly repayments. (4)

(d) Who pays more interest - Connor or Liam? Explain why this is so. (4)

[14]

QUESTION 8

(a) For the graph defined by $f(x) = ax^2 + bx + c$ the following information is given:

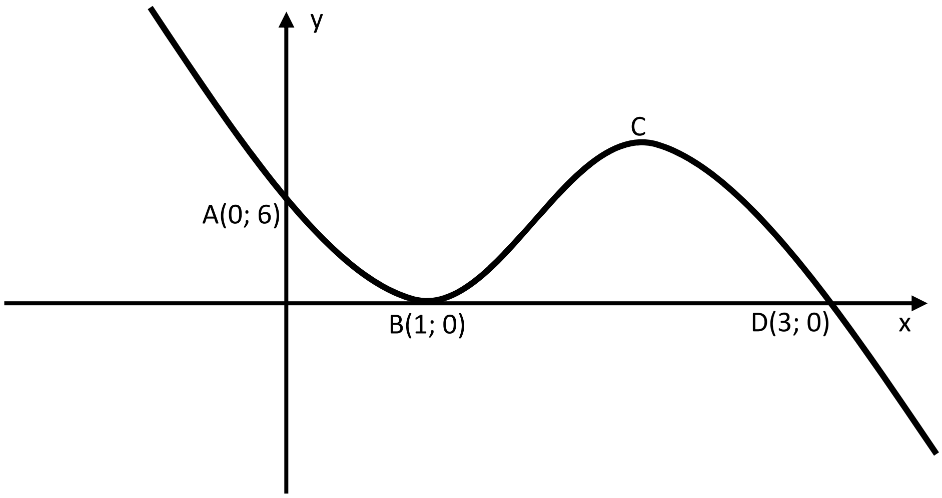
$a > 0$, $b > 0$ $b^2 - 4ac = 0$

(1) Make a neat rough sketch of $f(x)$, showing its correct orientation and position with respect to the axes. (3)

(2) Write down the coordinates of the turning point, using the parameters a, b and c, where necessary. (2)

(b) The graph of a cubic function $g(x)$ is shown below (*not to scale*).

A is the y-intercept, B and D are x-intercepts and B and C are stationary points.



(1) Find the equation of the graph, proving that $g(x) = -2x^3 + 10x^2 - 14x + 6$, (*N.B. you may not assume this result in your working*) (6)

(2) Find the value(s) of x for which:

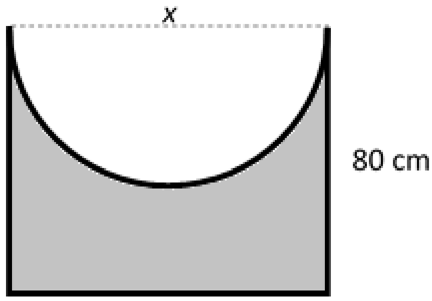
(i) $g(x)$ is increasing (5)

(ii) $g'(x)$ is increasing (3)

(3) For which values of k, will $g(x) = k$ have exactly three real solutions? (2)

(c) A semi-circle of diameter x is cut from a rectangle with a length of 80 cm.

Calculate the value of x which gives the maximum possible (shaded) area. Give your answer in terms of π.

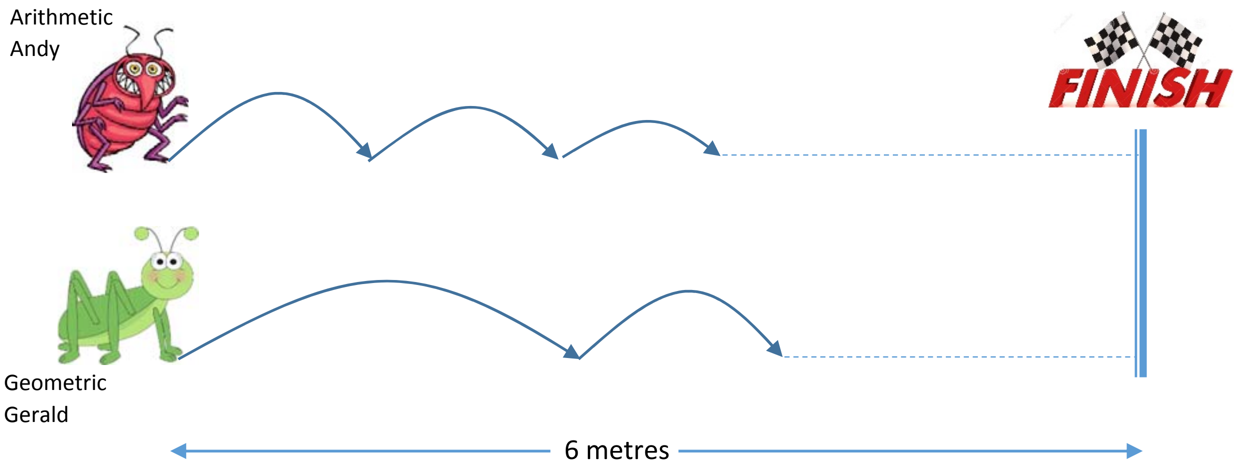


(6)

[27]

QUESTION 9

(a) Andy is an arithmetic bug and Gerald is a geometric bug and they are having a marathon bug-race over a distance of 6 metres. Andy hops in the sequence: 50 cm, 48 cm, 46 cm, 44 cm, 42 cm ....etc. Gerald starts with a hop of 100 cm and each of his subsequent hops is 85% of the length of the previous hop. Assume both bugs take the same time to make one hop.



(1) How many hops does Andy take to cross the finishing line? (4)

(2) Which bug crosses the finishing line first? (4)

(b) The following result is given: $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$

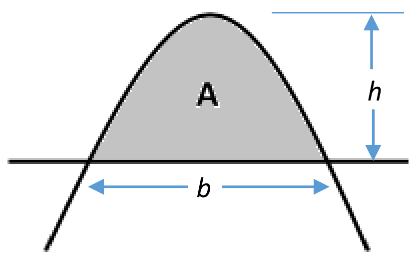
(1) Calculate the sum of the square numbers: 1 + 4 + 9 + ... + 10 000 (3)

(2) Determine the value of: $\sum_{r=3}^{40} r^2$ (3)

[14]

QUESTION 10

Consider the area, A, contained between a parabola and a horizontal line.

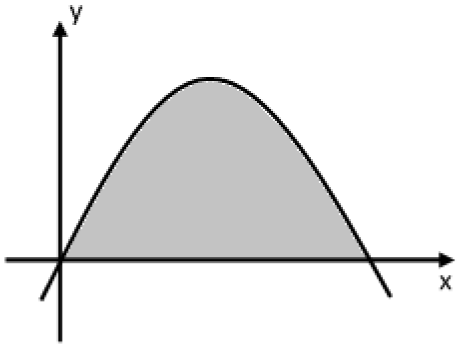


Let b be the horizontal base of the area and h the height, i.e. h is the vertical distance from the turning point to the base.

The area of A is given by the formula: $A = \frac{2}{3}bh$.

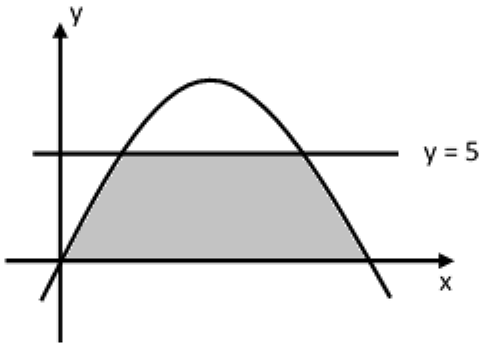
In both cases below the equation of the curve is $y = 6x - x^2$. You are required to find the shaded area.

(a)



(4)

(b)



(5)

[9]

76 Marks

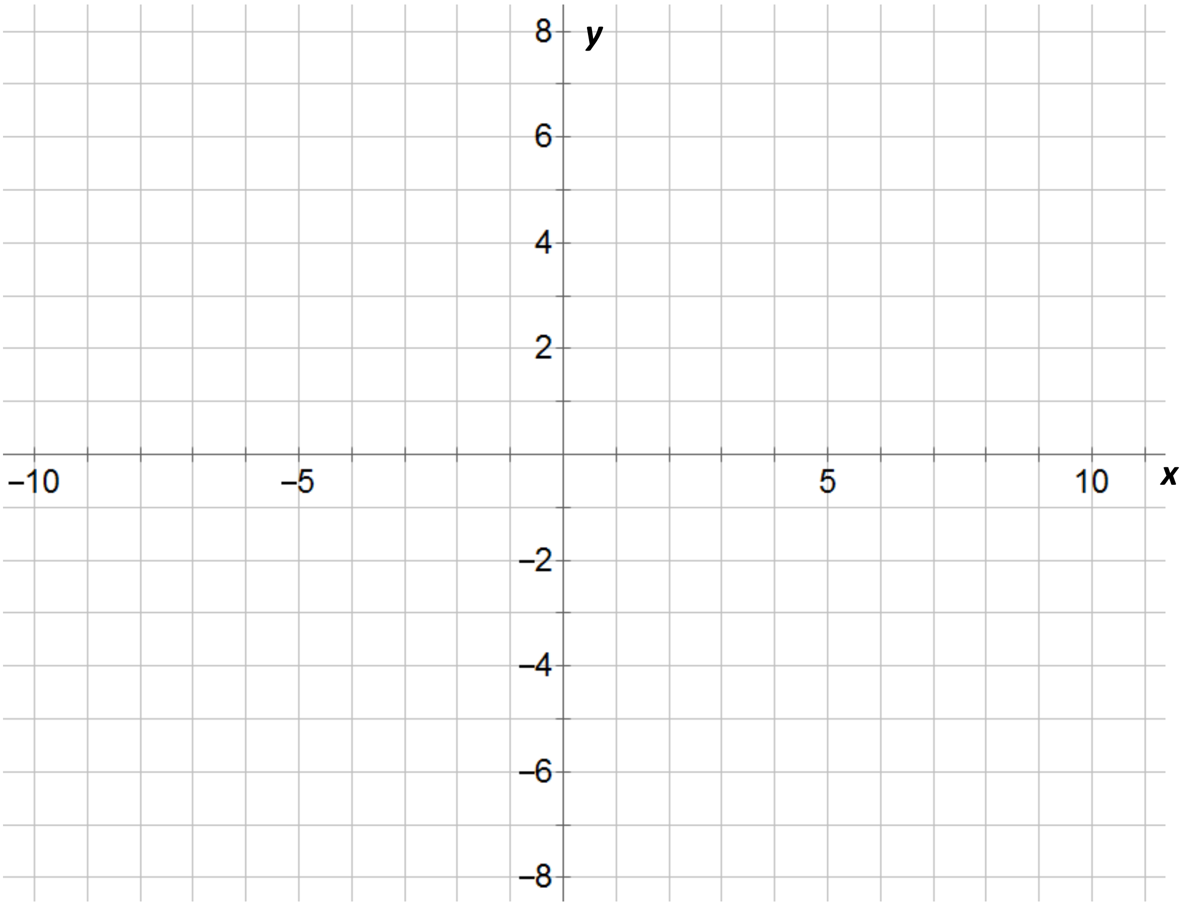
TOTAL: 150 marks

ANSWER SHEET Name: ______________________

Teacher:

KJ GE WLY DG BT SM JJ

Question 4(a)



[6]

Question 4(b)

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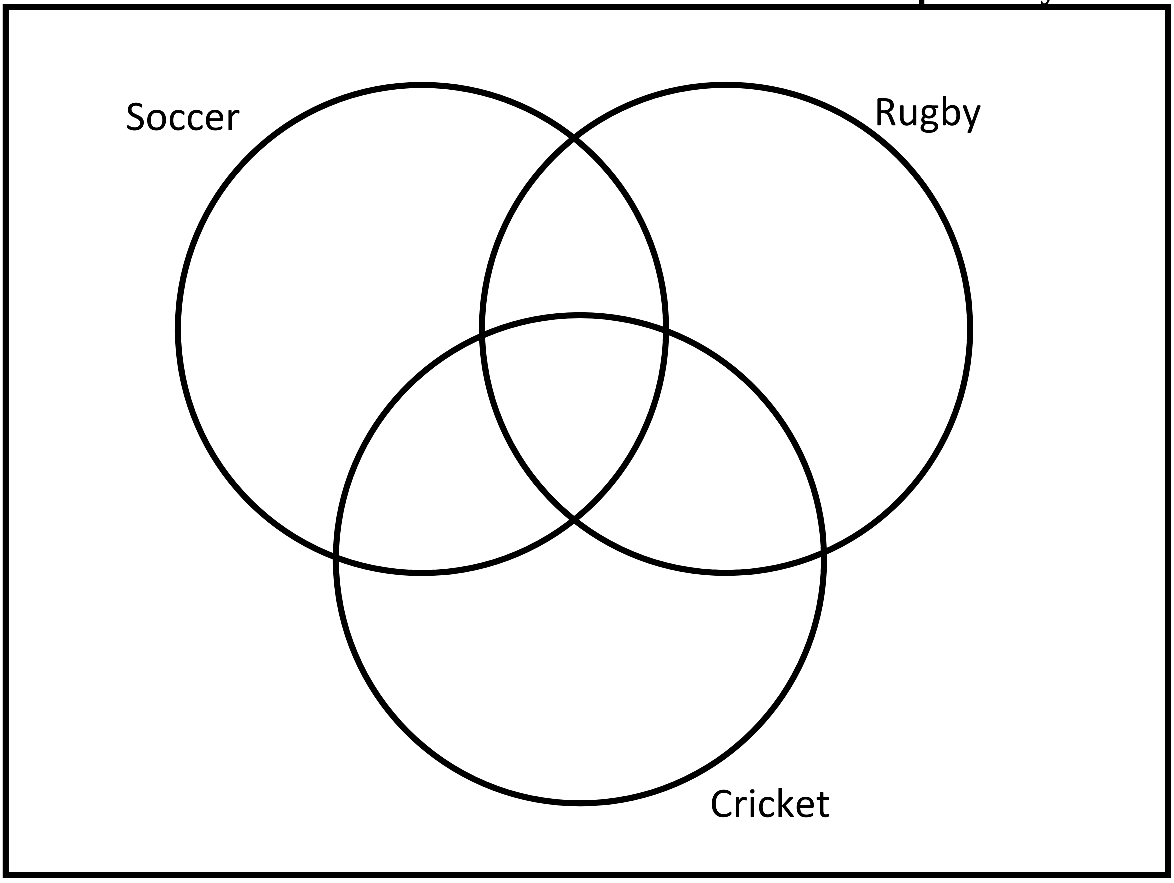
[3]

Question 6(a)

World Cup Survey



[7]

Question 6(b)

[2]

Question 6(c)

[3]

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \; ; \; r \neq 1$$

$$S_\infty = \frac{a}{1-r} \; ; \; -1 < r < 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 + i)^n$$

$$A = P(1 - i)^n$$

$$F = x\left[\frac{(1+i)^n - 1}{i}\right]$$

$$P = x\left[\frac{1 - (1+i)^{-n}}{i}\right]$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2} \; ; \; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$

$\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$

$\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\overline{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum(x - \overline{x})(y - \overline{y})}{\sum(x - \overline{x})^2}$$