St John's College

Preliminary Examinations

July 2014

Mathematics Paper 2

Examiner: K Jacobs **Time: 3 hours**
Moderator: B Tobias **Marks: 150**

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 20 pages, an Information Sheet consisting of 2 pages and a Diagram Sheet consisting of 4 pages. Please check that your paper is complete.
2. Spare paper will be handed out for rough workings only.
3. Answer all the questions on this examination script. Any workings on the Diagram Sheet will not be marked. If you need additional working space, please use pages 19 – 20, and thereafter some extra folio paper.
4. Read the questions carefully.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Round off your answers to two decimal digits where necessary, unless otherwise stated.
7. All the necessary working details must be clearly shown.
8. Diagrams may not necessarily be drawn to scale.
9. It is essential that you present your work neatly and logically.

NAME: _____ **TEACHER:** _____

SECTION A			**SECTION B**		
Q	Marks		Q	Marks	
1	5		8	16	
2	17		9	3	
3	18		10	9	
4	3		11	9	
5	13		12	8	
6	8		13	9	
7	14		14	6	
A	**78**		15	12	
			B	**72**	
			TOTAL	**150**	

SECTION A

QUESTION 1

A(6; -4), B(8; 2), C(3; *a*) and D(*b*; *c*) are points on the Cartesian plane.

Determine the value of:

(a) *a* if A, B and C are collinear. (3)

(b) *b* and *c* if B is the midpoint of A and D. (2)

[5]

QUESTION 2

(a) Solve for θ correct to one decimal digit if $\tan 2\theta = \cos 150^0$ where $\theta \in [0^0; 180^0]$. (4)

(b) Prove: $\sin(180^0+\theta)\cos(90^0+\theta)+\cos(-\theta)\cos(\theta-180^0)=-\cos 2\theta$ (5)

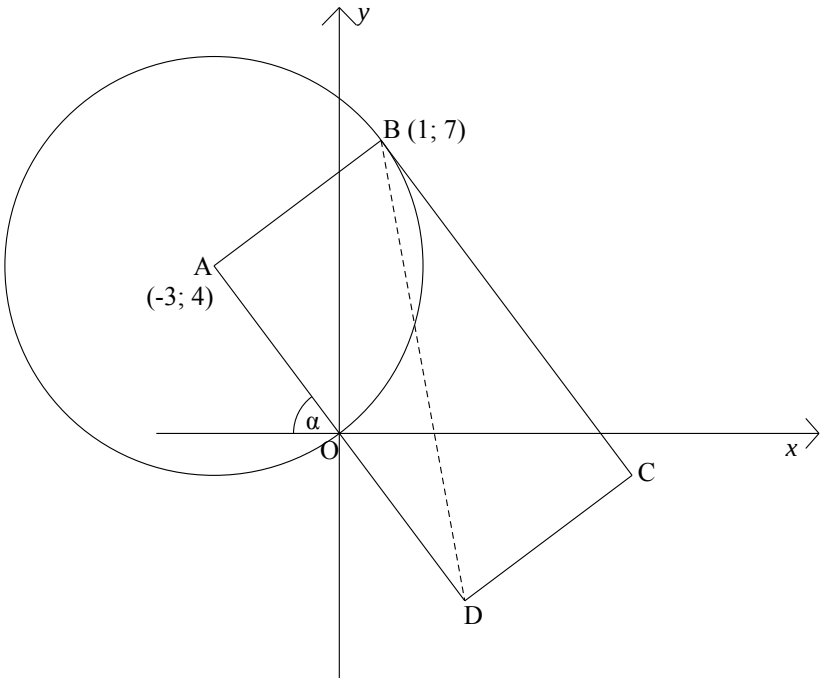
(c) (1) Prove: $\dfrac{1-\cos 2\theta}{\sin 2\theta}=\tan\theta$ (4)

(2) Hence, determine $\tan 15^0$ in simplest surd form, without the use of a calculator. (4)

[17]

QUESTION 3

ABCD is a rectangle with A(-3; 4) and B(1; 7), with O the origin.



(a) Determine the gradient of AB. (2)

(b) Determine the equation of line AD, and verify that the line passes through the origin. (3)

(c) Calculate the value of the angle marked α on the diagram. (2)

(d) If the line BD is described by the equation $2y + 11x = 25$, determine the coordinates of point D. (3)

(e) If D is the point (3; -4), determine the area of rectangle ABCD. (3)

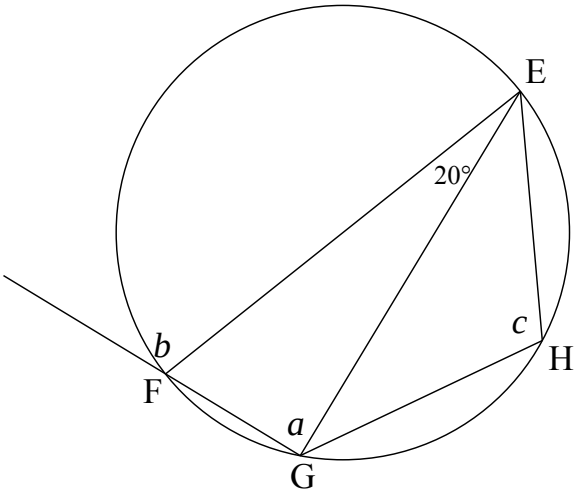
(f) Write down the equation of the circle centre A and radius AB. (3)

(g) If a second circle is drawn, centre D (3; -4) and radius DC, how many points of intersection will it have with the sketched circle, centre A? Explain. (2)

[18]

QUESTION 4

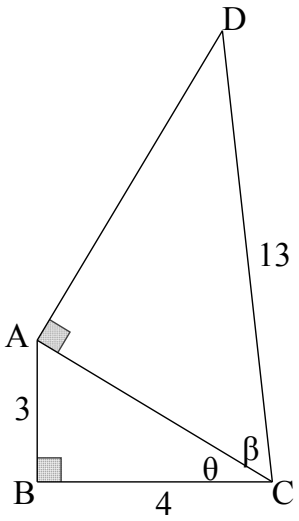
EF is a diameter of a circle with points G and H on the circumference.
If $F\hat{E}G = 20^0$, determine the value of the angles marked *a*, *b* and *c*, **stating reasons**.
[3]



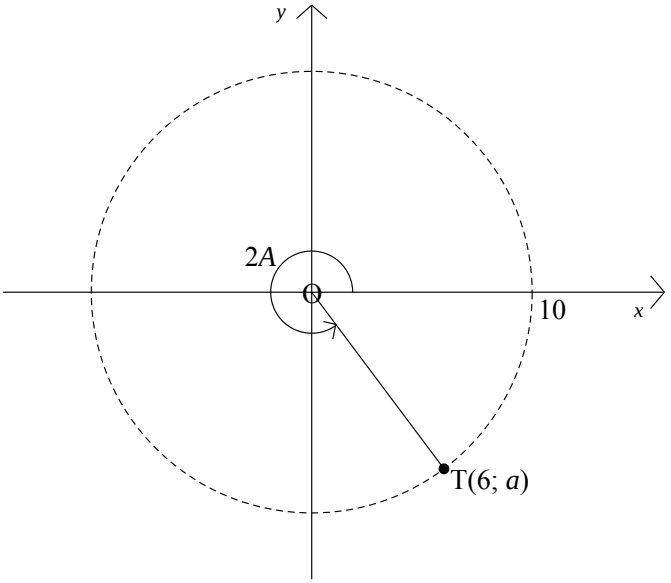
QUESTION 5

(a) ΔACD and ΔABC are sketched alongside, with $\hat{A} = 90^0$ and $\hat{B} = 90^0$ respectively. $A\hat{C}D = \beta$ and $B\hat{C}A = \theta$. AB = 3 units, BC = 4 units, and CD = 13 units.

Without the use of a calculator, determine:
$\sin(\theta + \beta)$. (4)



(b) A circle of radius 10 units is sketched below. OT is a radius that makes an angle of $2A$ with the positive x-axis, such that $270^0 < 2A < 360^0$. T is the point (6; a).



(1) Determine the value of a. (2)

(2) Determine without the use of a calculator:

(i) $\cos 2A$ (1)

(ii) $\sin 4A$ (3)

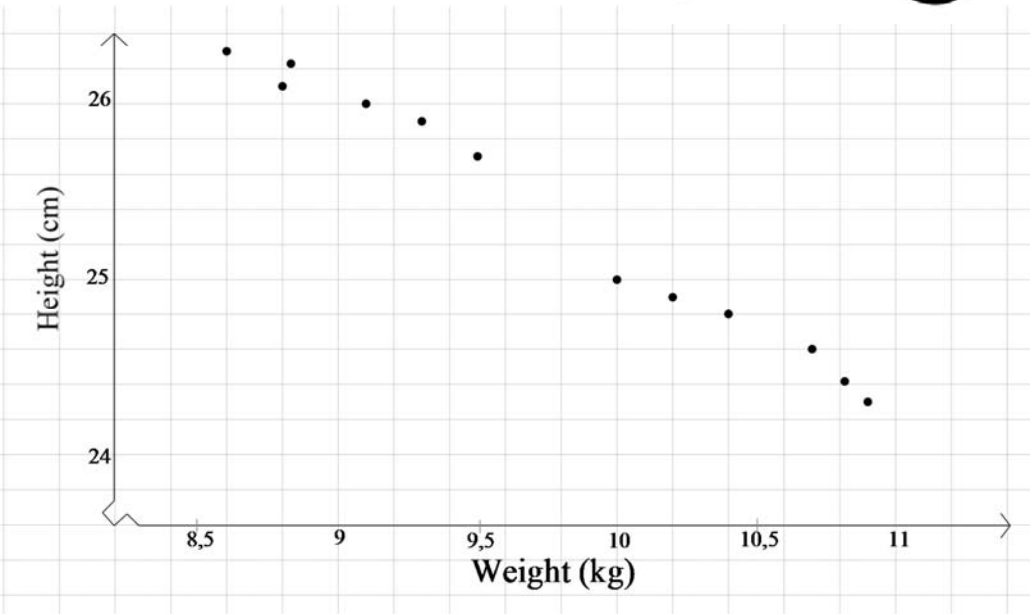
(iii) $\cos A$ (3)

[13]

QUESTION 6

In BMX dirt-bike racing, jumping high or "getting air" depends on many factors: the rider's skill, the angle of the jump, and the weight of the bike. Below is a set of data recording the maximum height a rider can achieve for various bike weights, which is then plotted on a graph.

Weight (kg)	Height (cm)
8,6	26,3
8,8	26,1
9,1	26
9,3	25,9
9,5	25,7
10	25
10,2	24,9
10,4	24,8
10,7	24,6
10,9	24,3



(a) Calculate the line of best fit that models the data. (3)

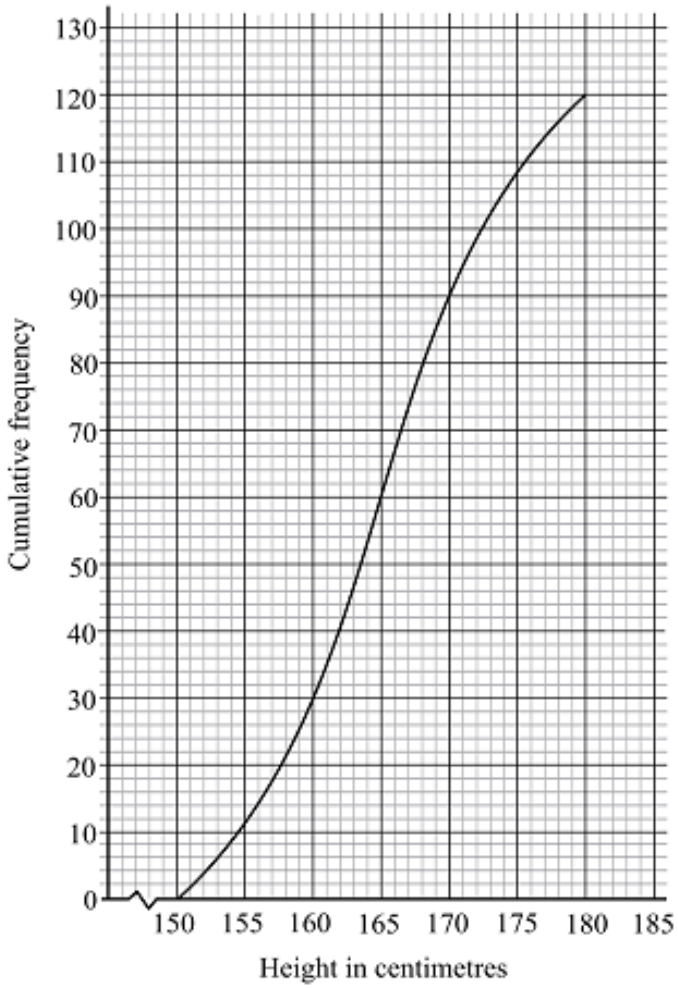
(b) Using the equation of the line of best fit, estimate what height can be achieved if the weight of the bike is 9,75 kg? (2)

(c) With reference to the correlation coefficient, comment on the reliability of the answer in (b). (3)

[8]

QUESTION 7

The cumulative frequency curve below shows the heights of 120 boys.



(a) How many boys were less than 160 cm? (1)

(b) Find the number of boys who are between 160 and 165 cm. (1)

(c) Find the median height of the boys. (1)

(d) Determine the interquartile range. (2)

(e) Given that 40% of the boys are taller than k cm, find the value of k. (2)

(f) Complete the following frequency table. (2)

***h* (cm)**	$150 \le h < 155$	$155 \le h < 160$	$160 \le h < 165$	$165 \le h < 170$	$170 \le h < 175$	$175 \le h < 180$
Frequency	11	19				11

(g) Hence, calculate the mean height of the boys. (2)

(h) 10 more students are measured and their heights were found to be less than 150 cm. Without calculations, describe how the following might change for the new data set:

(1) the mean (1)

(2) the standard deviation (1)

(3) the skewness. (1)

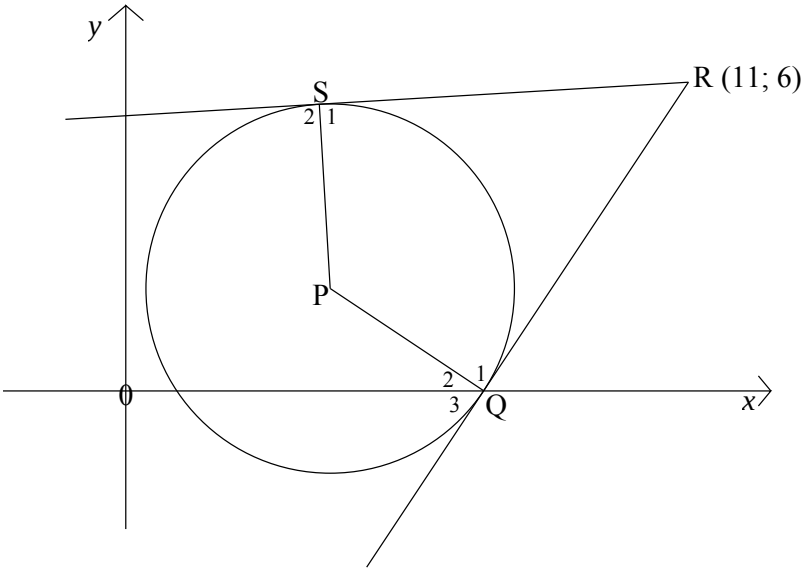
[14]

78 Marks

SECTION B

QUESTION 8

A circle centre P and radius PQ is sketched with Q on the *x*-axis. QR and SR are tangents to the circle at Q and S respectively, intersecting at point R (11; 6).



(a) If the circle is defined by $x^2 - 8x + y^2 - 4y = -7$, determine the coordinates of the centre P, and the length of radius PQ, leaving your answers in surd form. (4)

(b) Prove that quadrilateral PQRS is cyclic, **stating reasons**. (3)

(c) Hence determine the equation of the circle that passes through the points P, Q, R and S. (5)

(d) Find the length of RS, leaving your answer in surd form. (4)

[16]

QUESTION 9

The table below shows the marks gained in a test by a group of students.

Mark	1	2	3	4	5
Number of students	5	10	p	6	2

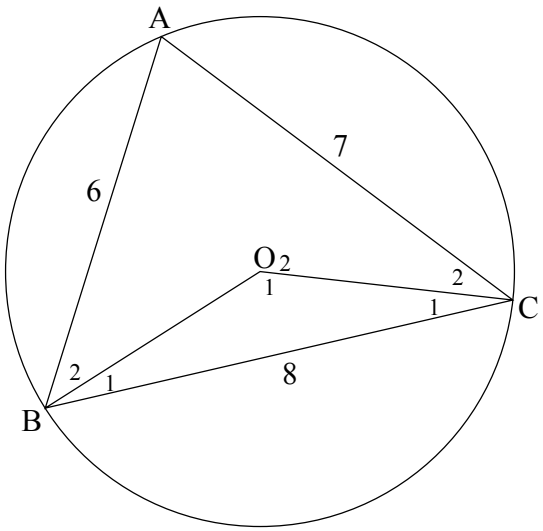
The median is 3 and the mode is 2. Find two possible values of p.

[3]

QUESTION 10 **REASONS MUST BE STATED**

(a) In any ΔABC write down $\cos A$ in terms of the sides a, b, and c. (1)

(b) In the diagram, O is the centre of the circle, AB = 6 units, AC = 7 units, and BC = 8 units.



(1) Show by calculation that obtuse $B\hat{O}C = 151^0$ to the nearest degree. (4)

(2) Hence, calculate the diameter of the circle, correct to two decimal places. (4)

[9]

QUESTION 11

The funtion f is defined by $f(x) = a\cos 2x + b$, for $0^0 \le x \le 180^0$, with $f(0^0) = 0$ and $f(90^0) = -6$.

(a) Determine the values of *a* and *b*. (2)

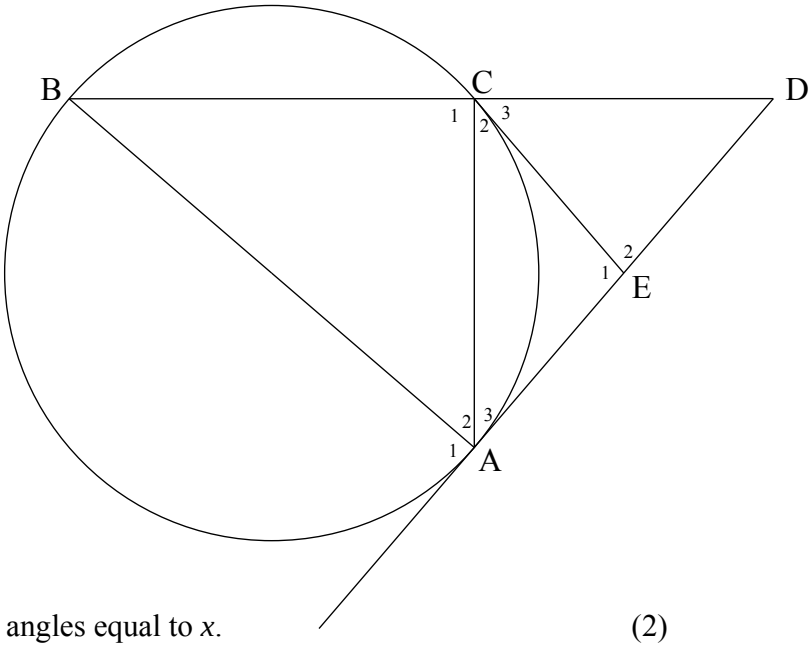
(b) (1) Show that $f(x)$ can be written as $f(x) = -6\sin^2 x$. (2)

(2) Hence calculate to one decimal digit the *x*-coordinates of the points where $f(x)$ intersects $g(x) = \sin x$. (5)

[9]

QUESTION 12 **REASONS MUST BE STATED**

AD and EC are tangents to the circle at points A and C respectively.
AB is a diameter of the circle.



(a) If $\hat{A}_3 = x$, determine two other angles equal to x. (2)

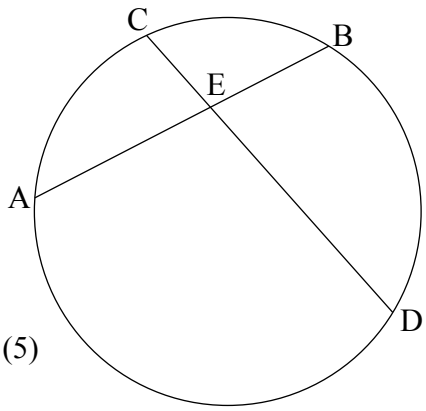
(b) Determine $\hat{D}$ in terms of x. (2)

(c) Prove that AE = ED. (4)

[8]

QUESTION 13 **REASONS MUST BE STATED**

A theorem you may never have seen before states:
If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

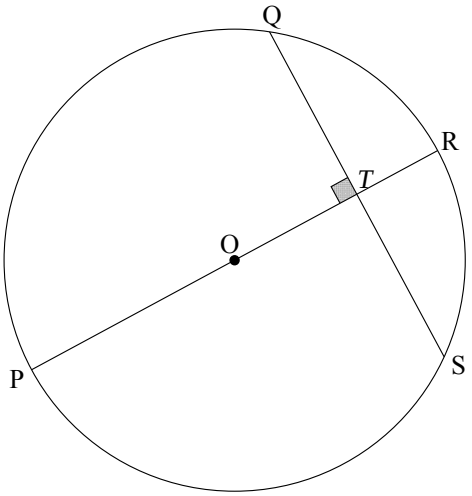


(a) **Given:** Chords AB and CD intersect at point E.

By constructing AC and BD, prove CE . ED = BE . EA (5)

(b) P, Q, R and S are points on a circle, centre O, such that PR is perpendicular to QS.
If QS = 6 units, and PT : TR = 4 : 1, determine the length of the diameter of the circle.

(4)

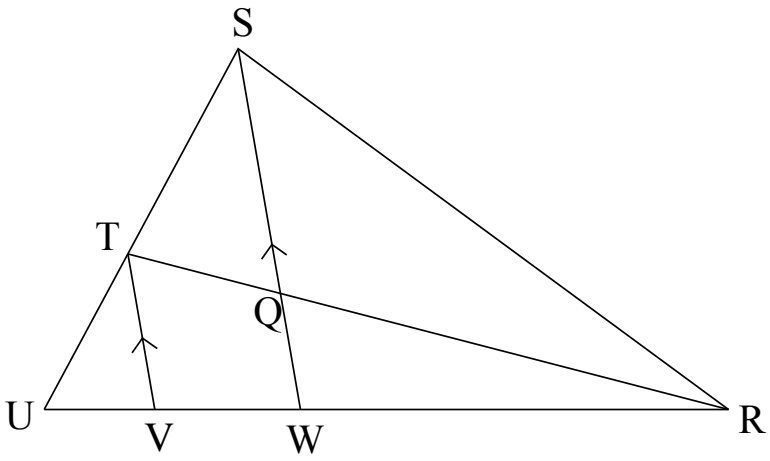


[9]

QUESTION 14

Triangle RSU is sketched with T on side SU, and V and W on side UR, such that $TV \parallel SW$. TR intersects SW in Q.

$UW = \frac{3}{8}UR$ and $2TU = ST$



Determine the following ratios:

(a) $\frac{\text{area of } \Delta SUW}{\text{area of } \Delta RSW}$ (2)

(b) $\frac{TQ}{QR}$ (no reasons required) (2)

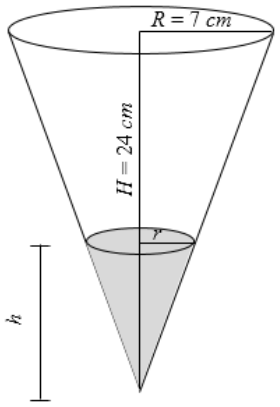
(c) $\frac{TV}{SW}$ (no reasons required) (2)

[6]

QUESTION 15

A sealed vertical inverted cone (i.e. vertex down) has a radius, R, of 7 *cm* and height, H, of 24 *cm*. Water is filled to one third of the cone's height.

$V_{cone} = \frac{1}{3}(\text{base area})(\text{height})$

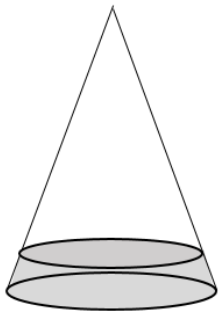


(a) Determine the volume of the cone, leaving your answer in terms of π. (2)

(b) Determine the height of the water, h, and the radius, r, of the water surface. (3)

(c) Determine the volume of the air in the cone, leaving your answer in terms of π. (3)

(d) The cone is now turned upside down. Find the height of the water. (4)



[12]

72 Marks

Additional working space