SECTION A

QUESTION 1

a) Calculate the value of t if $T\ (2;1)$ is the midpoint of the line segment joining $S(7;t)$ and $R(-3;-2)$ (2)

$\frac{t-2}{2} = 1$ ✓

$t = 4$ ✓

b) Calculate the value of a if $(-18;17)$; $(-3;7)$ and $(a;a)$ are collinear (5)

$\frac{17-7}{-18+3}$ ✓ $= \frac{7-a}{-3-a}$ ✓

$10(-3-a) = -105+15a$ ✓

$-25a = -75$ ✓

$a = 3$ ✓

c) P(−3; 2) and Q(5; 8) are two points in the Cartesian plane. Determine the equation of the line perpendicular to PQ, which passes through P. Leave your answer in the form $ax+by+c=0$. (5)

$m_{PQ} = \frac{6}{8} = \frac{3}{4}$ ✓

$m_{\perp} = -\frac{4}{3}$ ✓

$y = -\frac{4}{3}x + c$ sub $(-3;2)$

$2 = -\frac{4}{3}(-3) + c$

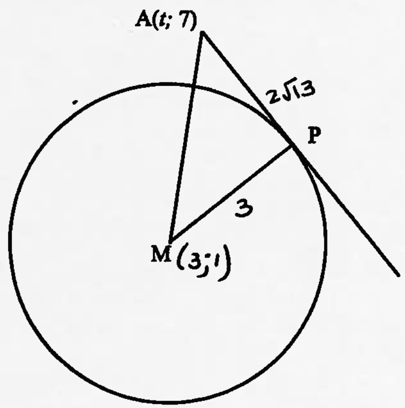
$2 = 4 + c$ ✓

$-2 = c$ ✓

$y = -\frac{4}{3}x - 2$

$3y + 4x + \cancel{6} = 0$ ✓

d) The circle drawn below has equation $(x-3)^2 + (y-1)^2 = 9$



1) Write down the circle centre and the length of the radius

$(3;1)$ ✓ radius $= 3$ ✓

2) If the length of the tangent from a point A (t; 7) to a point P on t units, calculate the value of t, where $t > 0$.

$(t-3)^2 + (7-1)^2$ ✓ $= 3^2 + (2\sqrt{13})^2$ ✓ Pyth

$t^2 - 6t + 9 + 36 = 9 + 52$ ✓

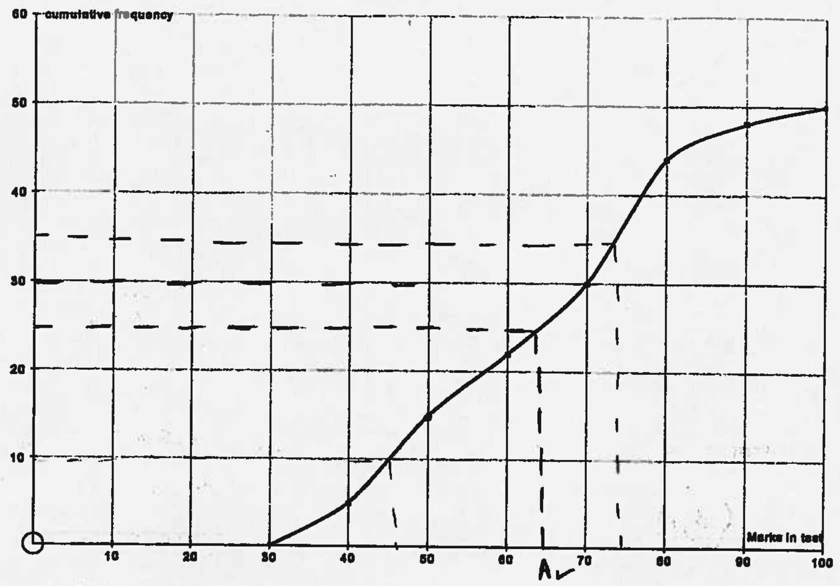
$t^2 - 6t - 16 = 0$

$(t-8)(t+2) = 0$ ✓

$t = 8$ ✓ or $t = -2$ ✓ (N/A)

QUESTION 2

A class of 50 pupils wrote a Physical Science examination. The marks obtained by the pupils were tabulated and an ogive was drawn as shown below.



Study the graph and answer the following:

a) What was the median mark? Show where you take your reading with "A" (2)

± 64-65 ✓

b) If 70% of the pupils passed the test, what was the passing mark? Show your calculations (2)

0,7 × 50 = 35 ✓ read from 15

50 pupils ✓

c) If not more than 20% failed what was the passing mark? (1)

0,2 × 50 = 10 → 45 ✓

d) How many pupils got between 50 and 70 marks for the examination? (1)

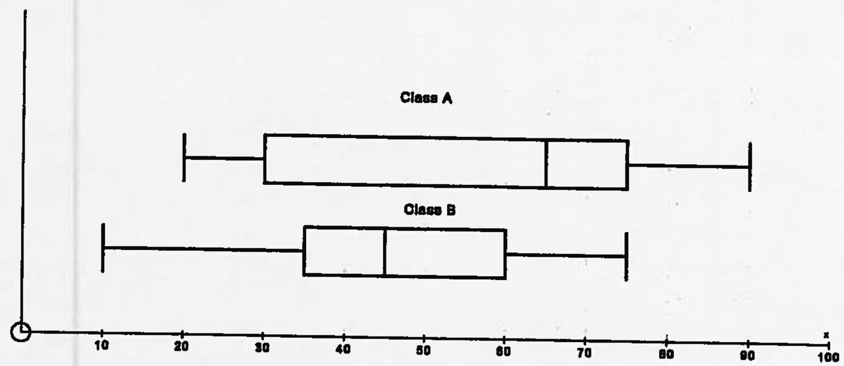
30−15 = 15 ✓

[6]

QUESTION 3

The parallel box plots show the English results of class A and class B in the April examinations.

Class A had a median mark of 65%.



a) Which class would have a higher mean mark? (1)

Class A ✓

b) Which class had a greater interquartile range? (1)

Class A ✓

c) What percentage of class B scored less than 60%? (1)

75% ✓

d) If all the learners in class A were given an extra 5% what would happen to the standard deviation of class A? (1)

Stays the same ✓

[4]

QUESTION 4

The data below shows the energy levels, in kilocalories per 100 g, of 10 different snack foods.

1	4	2	7	8
440	520	480	560	615
550	620	680	545	490
6	9	10	5	3

a) Calculate the median energy level of the snack foods. (2)

$\frac{545 + 550}{2} = 547{,}5$ ✓✓

b) Calculate the mean energy level of the snack food.

$\frac{5500}{10} = 550$ ✓✓

(2)

c) Calculate the standard deviation.

$\sigma = 69{,}03 = 69{,}0$ ✓

(1)

d) The energy levels, in calories per 100g of 10 different breakfast cereals had a mean of 545,7 kilocalories and a standard deviation of 28 calories. Which of the two types of food show greater variations in energy levels? What do you conclude?

Snack foods. ✓
Breakfast cereals better to eat. Energy levels more consistent / reliable ✓

(2)
[7]

QUESTION 5

a) Determine $\sin x$ if $\cos(x - 10°) = 0{,}666$ and $-90 \le x \le 0°$. (4)

$\cos(x - 10°) = 0{,}666$

$x - 10° = \pm 48{,}24° + k\,360°$ ✓

$x \cancel{-10°} = -38{,}24°$ ✓

$\sin(-38{,}24°)$ ✓ $= -0{,}62$

$= -0{,}6$ ✓

b) If $\sin 40° = a$, express the following in terms of a:

(1) $\sin 140°$ (1)

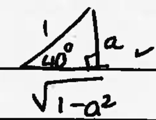
$= \sin 40°$

$= a$ ✓

(2) $\cos 140°$ (3)

$-\cos 40$ ✓

$= -\sqrt{1-a^2}$ ✓



(3) $\sin 100°$ (3)

$= \sin 2(40°)$ ✓

$= 2\sin 40° \cos 40°$ ✓

$= 2a\sqrt{1-a^2}$ ✓

c) **Simplify without using a calculator:**

1) $-1-\sin(90^{o}-x)\cos(x-180^{o})$ (3)

$-1-\cos x \checkmark . \cos(180^{\circ}+x)$

$= -1-\cos x.(-\cos x) \checkmark$

$= -1+\cos^2 x$

$= \underline{-\sin^2 x} \checkmark$

2) $\sin 15^{o}\cos 15^{o}$ (3)

$= \dfrac{2\sin 15^{\circ}\cos 15^{o}}{2} \checkmark$

$= \dfrac{\sin 2(15^{o})}{2} = \dfrac{\sin 30^{o}}{2} \checkmark = \dfrac{1}{4} \checkmark$

3) $(1-\sqrt{2}\cos 15^{o})(1+\sqrt{2}\cos 15^{o})$ (4)

$1-2\cos^2 15^{o} \checkmark$

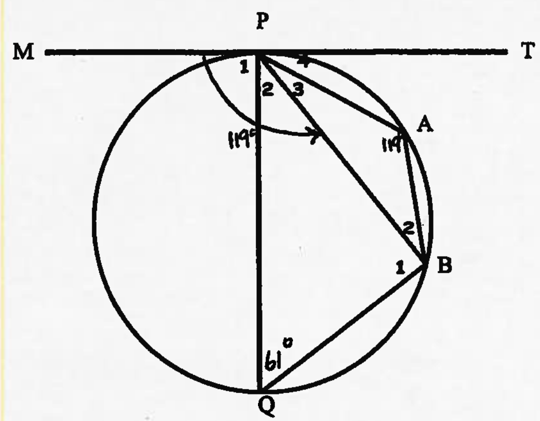
$= -\cos 2(15^{o}) \checkmark$

$= -\cos 30^{o} \checkmark$

$= -\dfrac{\sqrt{3}}{2} \checkmark$

[21]

QUESTION 6



PQ is a diameter. $M\hat{P}B = 119^{o}$ and MT is a tangent.
Find the magnitude of:

a) $\hat{Q}$ (3)

$\hat{A} = 119^{o} \checkmark$ tan chord theorem ✓

$\hat{Q} = 61^{o} \checkmark$ opp ∠s. cyclic quad.

b) $\hat{P}_2$ (3)

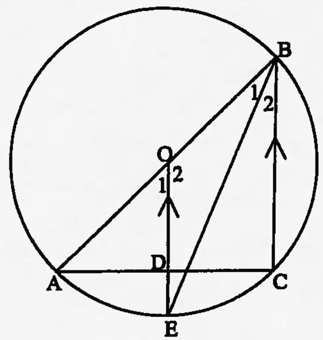
$\hat{P}_1 = 90^{o} \checkmark$ rad ⊥ tang ✓

$\therefore \hat{P}_2 = 29^{o} \checkmark$ $M\hat{P}B = 119^{o}$ given

[6]

QUESTION 7

AOB is the diameter of the circle AECB, with O the centre. OE || BC and OE meets AC at D. B and E are joined. Prove that:



a) AD = DC. (4)

AO = OB radii
DO || CB given
$\therefore$ AD = DC line mtpt || to 2nd line
OR $\hat{C} = 90°$ ∠ in semi circle
$\hat{D} = 90°$ corr ∠s DO || CB
$\therefore$ AC = DC line centre ⊙ ⊥ to chord

b) EB bisects $A\hat{B}C$. (3)

$\hat{B}_1 = \hat{E}$ radii
$= \hat{B}_2$ alt ∠s OE || BC
$\therefore$ EB bisects $A\hat{B}C$
OR
$\hat{O}_1 = 2\hat{B}_1$ ∠ at centre
$\hat{O}_1 = \hat{B}_1 + \hat{B}_2$ corr ∠s OD || BC
$\therefore \hat{B}_1 = \hat{B}_2$

c) If $O\hat{E}B = x$, express $B\hat{A}C$ in terms of x, giving reasons for your answer. (2)

$\hat{B}_2 = x$ alt ∠s OE || BC
$\therefore A\hat{B}C = 2x$
$\therefore \hat{A} = 90° - 2x$ ∠s of Δ

[9]

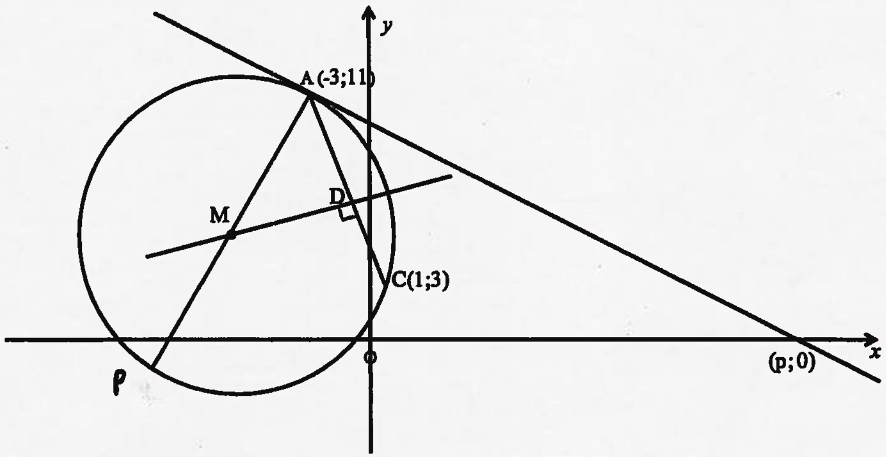
TOTAL :73 marks

SECTION B

QUESTION 8

A(-3; 11) and C(1;3) and P are points on the circle with centre M. MD ⊥ AC

AP, the diameter, has equation $y = 3x + 20$



a) Determine the equation of DM, the perpendicular bisector of AC. (5)

$D\left(\frac{-3+1}{2}; \frac{11+3}{2}\right)$

$= (-1; 7)$

$m_{AC} = \frac{11-3}{-3-1} = -2$

$\therefore m_{MD} = \frac{1}{2}$

$\therefore 7 = \frac{1}{2}(-1) + c$

$7\frac{1}{2} = c$

$y = \frac{1}{2}x + 7\frac{1}{2}$

b) Show that the equation of the circle is $(x+5)^2 + (y-5)^2 = 40$. (6)

$3x + 20 = \frac{1}{2}x + 7\frac{1}{2}$ ✓

$\frac{5}{2}x = \frac{-25}{2}$ ✓

$x = -5$ ✓

$y = 3(-5) + 20$

$= 5$ ✓ $\therefore M(-5; 5)$

$AM = \sqrt{(-3+5)^2 + (11-5)^2}$ ✓

$= \sqrt{4+36}$

$AM^2 = 40$ ✓

$\therefore (x+5)^2 + (y-5)^2 = 40$

c) Determine, correct to one decimal place, the size of $M\hat{A}D$. (3)

$\tan\theta = 3$ $\quad$ $\tan\alpha = -2$

$\theta = 71{,}6^\circ$ ✓ $\quad$ $\alpha = -63{,}4^\circ + 180^\circ$

$\alpha = 116{,}6^\circ$ ✓

$\therefore M\hat{A}D = 45^\circ$ ✓

d) Find the numerical value of p, if the tangent to the circle at A meets the x axis at the point (p; 0) (3)

$(x+5)^2 + (y-5)^2 = 40$ $\quad$ OR $\quad$ $(-3+5)(x+5) + (11-5)(y-5) = 40$ ✓

$m_{tan} = -\frac{1}{3}$ ✓ $\quad$ $2x + 10 + 6y - 30 = 40$

$\frac{11-0}{-3-p} = \frac{-1}{3}$ ✓ $\quad$ $2x + 6y = 60$

$y = 0$ $\quad$ $2x = 60$ ✓

$3 + p = 33$ $\quad$ $x = 30$

$p = 30$ ✓ $\quad$ $p = 30$ ✓

[17]

QUESTION 9

a) If $\cos 35^\circ = k$, prove that $\sin^2 70^\circ = 4k^2 - 4k^4$ (5)

$\sin^2 2(35^\circ)$ ✓

$= (2\sin 35^\circ \cos 35^\circ)^2$ ✓

$= 4\sin^2 35^\circ \cos^2 35$ ✓

$= 4(1-\cos^2 35^\circ)\cos^2 35^\circ$ ✓

$= 4\cos^2 35^\circ - 4\cos^4 35^\circ$ ✓

$= \underline{4k^2 - 4k^4}$

b) For which values of x in the interval $0^o \le x \le 270^o$ is the following identity undefined?

$$\frac{\cos x - \cos 2x + 2}{3\sin x - \sin 2x} = \frac{1+\cos x}{\sin x}$$ (6)

$3\sin x = \sin 2x$ or $\sin x = 0$ ✓

$3\sin x = 2\sin x \cos x$ ✓ $x = 0^\circ + k360^\circ$

$\frac{3}{2} = \cos x$ ✓ $x = 180^\circ + k360^\circ$

No soln $= x$ ✓ $x = k180^\circ$ ✓

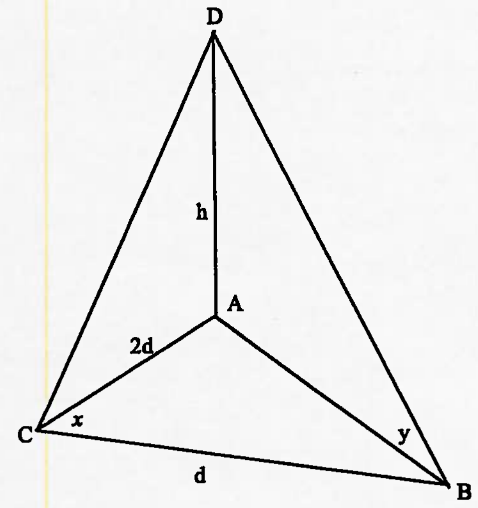
$0^\circ; 180^\circ$ ✓ $k \in \mathbb{Z}$

[11]

QUESTION 10

In the given diagram A,B, and C are the vertices of a triangular garden with AC = 2d and CB = d. A tree standing at A, is observed from B, and the angle of elevation of D, the top of the tree is measured as y.

$A\hat{C}B = x$ and $AD = h$



a) Show that $AB = d\sqrt{5 - 4\cos x}$ (3)

$AB^2 = d^2 + (2d)^2 - 2d.2d\cos x$

$= d^2 + 4d^2 - 4d^2\cos x$

$= d^2(5 - 4\cos x)$

$AB = d\sqrt{5 - 4\cos x}$

b) Hence, determine the value of h in terms of x and y. (3)

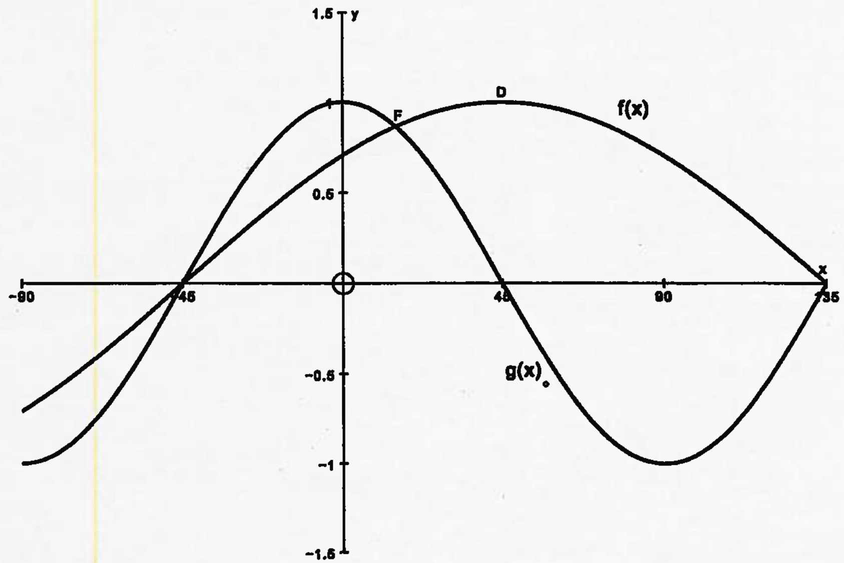
$\tan y = \frac{h}{AB}$

$h = d\sqrt{5-4\cos x} \cdot \tan y$

[6]

QUESTION 11

This sketch shows the graphs of $f(x) = \sin(x + 45^o)$ and $g(x)$ both for $x \in [-90^o; 135^o]$:



a) Write down the equation of $g(x)$. (2)

$g(x) = \cos 2x$

b) For which values of x is $g(x) > f(x)$ in the given interval, given that the x coordinate of F is 15^o? (2)

$-45^o < x < 15^o$

c) For which values of x is $f'(x).g'(x) > 0$ for $x \in [0^o; 135^o]$? (2)

$45^o < x < 90^o$

[6]

QUESTION 12

Solve for x correct to 1 decimal digit if $-\sin(2x-10^\circ)=\cos(x+50^\circ)$; $x \in (-180^\circ;180^\circ)$

$\sin(-2x+10^\circ) = \cos(x+50^\circ)$

$\sin(-2x+10^\circ) = \sin[40^\circ - x] + k360^\circ$

$-2x+10^\circ = 40^\circ - x + k360^\circ;\ k \in Z$

$-x = 30^\circ + k360^\circ$

$x = -30^\circ + k360^\circ$

OR $-2x+10^\circ = 180^\circ - (40^\circ - x) + k360^\circ$

$-2x+10^\circ = 140^\circ + x + k360^\circ$

$-3x = 130^\circ + k360^\circ$

$x = -43{,}3^\circ + k120^\circ$

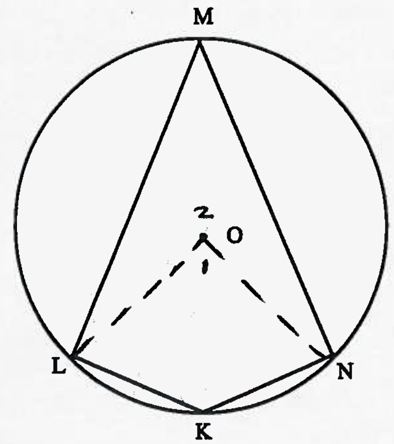
$x \in \{-30^\circ; -163{,}3^\circ; -43{,}3^\circ; 76{,}7^\circ\}$

[7]

QUESTION 13

REASONS NEED TO BE GIVEN IN THIS QUESTION.

a) In the diagram K, L, M and N are points on the circle centre O. Use the diagram to prove the theorem that states that $\hat{M}+\hat{K}=180^\circ$ (5)



Draw LO and ON ✓

$\hat{O}_1 = 2\hat{M}$ ✓ ∠ at centre 2× ∠ at circum ✓

$\hat{O}_2 = 2\hat{K}$ ∠ at centre

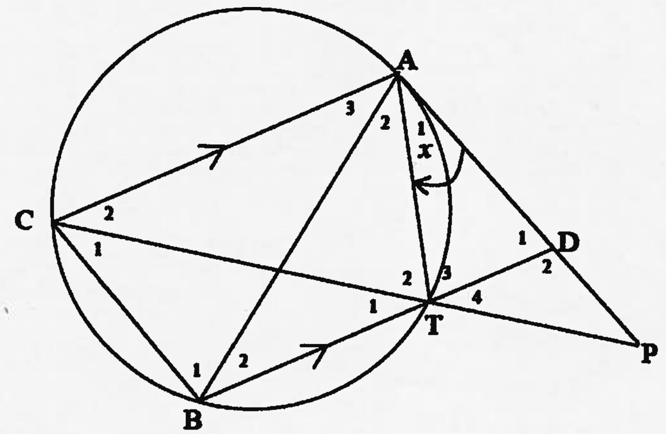
but $\hat{O}_1+\hat{O}_2 = 2\hat{M}+2\hat{K}$

$\therefore \hat{O}_1+\hat{O}_2 = 2(\hat{M}+\hat{K}) = 360^\circ$ ✓ ∠s around a point

Now $\hat{M}+\hat{K} = 180^\circ$ ✓

b) In the diagram DA is a tangent to the circle ACBT at A. CT and AD are produced to meet at P. BT is produced to cut PA at D. AC, CB, AB and AT are joined. AC//BD.

Let $\hat{A}_1 = x$



1) Prove that ΔABC/// ΔADT (6)

In Δ ABC and Δ ADT
$\hat{A}_3 = \hat{B}_2$ alt ∠s AC∥BD ✓
$\hat{B}_2 = \hat{A}_1$ tan chord ✓
∴ $\hat{A}_3 = \hat{A}_1$ ✓
$\hat{C}_1 + \hat{C}_2 = \hat{T}_3$ ✓ ext ∠ cyclic quad ✓
∴ $\hat{B}_1 = \hat{D}_1$ 3rd ∠ of Δ
∴ Δ ABC ||| Δ ADT ∠∠∠ ✓

2) Prove that PT is a tangent to the circle ADT at T. (3)

$\hat{A}_1 = \hat{A}_3$ proved
$\hat{A}_3 = \hat{T}_1$ ∠s same seg ✓
$\hat{T}_1 = \hat{T}_4$ vertic opp ✓
∴ $\hat{A}_1 = \hat{T}_4$
∴ PT tangent (converse tan chord) ✓

3) Prove that ΔAPT/// ΔTPD (3)

In Δ APT and Δ TPD
$\hat{P} = \hat{P}$ common ✓
$\hat{A}_1 = \hat{T}_4$ proved ✓
∴ $A\hat{T}P = \hat{D}_2$ ∠s of Δ
∴ Δ APT ||| Δ TPD AAA ✓

4) If $AD = \frac{2}{3}AP$, show that $AP^2 = 3PT^2$ (4)

$\frac{AP}{TP} = \frac{PT}{PD} = \frac{AT}{TD}$ ✓ (Δ APT ||| Δ TPD equiang Δs)

$\frac{AP}{TP} = \frac{\cancel{AT}}{\cancel{PD}}$ but $AD = \frac{2}{3}AP$ ∴ $DP = \frac{1}{3}AP$ ✓

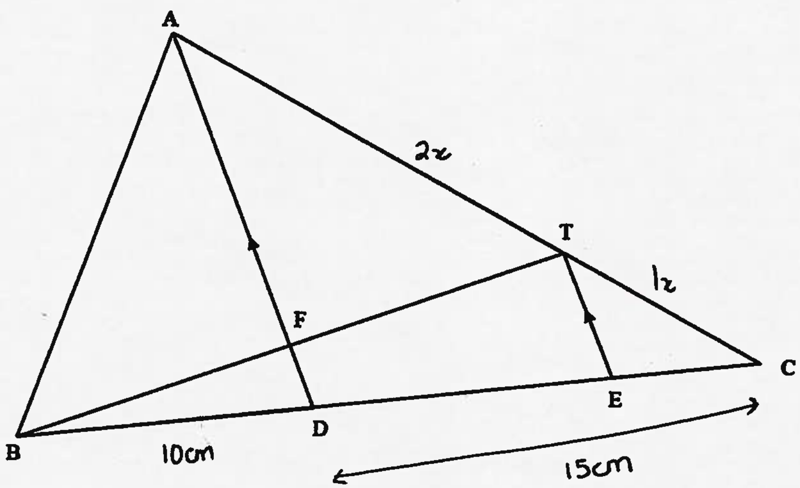
$PT^2 = AP.PD$ ✓
$= AP.\frac{1}{3}AP$
$= \frac{1}{3}AP^2$
$3PT^2 = AP^2$ ✓

[21]

QUESTION 14

In the figure below, ΔABC has D and E on BC. BD = 10cm and DC = 15cm.

AT : TC = 2 : 1 and AD// TE



a) Write down the numerical value of $\frac{CE}{ED}$ (1)

$\frac{5}{10} = \frac{1}{2}$ ✓

b) Show that D is the midpoint of BE. (2)

$\frac{EC}{CD} = \frac{1}{3} = \frac{5}{15}$ (line ∥ one side of Δ) ✓

$\therefore DE = 10$ ✓

$\therefore$ D is midpoint

c) If FD = 2,5 cm calculate TE. (2)

$FD = \frac{1}{2}TE$ ✓ (line ∥ 2nd side)

$TE = 5cm$ ✓

d) Calculate the value of $\frac{\text{Area } \Delta ADC}{\text{Area } \Delta ABD}$ (1)

$= \frac{\frac{1}{2} DC \times h}{\frac{1}{2} BD \times h} = \frac{15}{10}$

$= \frac{3}{2}$

e) Calculate the value of $\frac{\text{Area } \Delta TEC}{\text{Area } \Delta ABC}$ (3)

$= \frac{\frac{1}{2} TC \times EC \sin C}{\frac{1}{2} AC \times BC \sin C}$ ✓

$= \frac{x \times y}{3x \times 5y}$ ✓

$= \frac{1}{15}$ ✓

[9]

TOTAL : 77