Memo Gr 12 P1

Section A.

Question 1

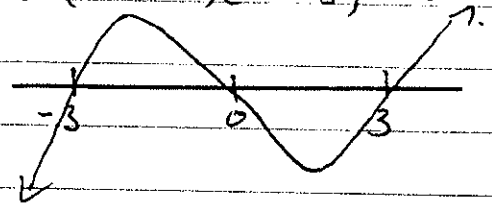
a (i) i) $x = 5^-$

(ii) $x = \sqrt{3}$ or $x = -\frac{3}{2}$ or $x = 5^-$

(2) $x^3 - 9x < 0$

$x(x^2 - 9) < 0$

$x(x-3)(x+3) < 0$



$\therefore \quad x < -3 \quad \text{or} \quad 0 < x < 3$

(3) $16^x \times \frac{1}{4^{x-1}} = \frac{1}{64}$

$4^{2x} \times 4^{-x+1} = 4^{-3}$

$x + 1 = -3$

$x = -4$

(4) $x^2 + 2x - 9 = 0$

$x^2 + 2x + 1 = 9 + 1$

$(x+1)^2 = 10$

$x + 1 = \pm\sqrt{10}$

$x = -1 \pm \sqrt{10}$

(b) $f(x) = 2^x \quad (a; 6)$

$6 = 2^a$

$\log 6 = \log 2^a$

$\log 6 = a \log 2$

$a = 2,58$

(c) $\left(1 - \frac{2}{\sqrt{x}}\right)\left(1 + \frac{2}{\sqrt{x}}\right) = 3$

$1 - \frac{4}{x} = 3$

$x - 4 = 3x$

$-2x = 4$

$x = -2$

$\therefore \sqrt{-2}$ no real roots.

(d) $\log_2 x + \log_2 y = 4$

$\log_2 xy = 4$

$xy = 16$ (2)

$x + y = 10$

$x = 10 - y$ (1)

Subst (1) into (2)

$(10 - y)y = 16$

$10y - y^2 = 16$

$0 = y^2 - 10y + 16$

$0 = (y - 8)(y - 2)$

$y = 8$ ✓ or $y = 2$ ✓

$x = 2$ ✓ $x = 8$ ✓

$(2; 8)$ $(8; 2)$

Question 2

(a) $f(x) = 3x^2 - 5$

$f(x+h) = 3(x+h)^2 - 5$

$= 3x^2 + 6xh + 3h^2 - 5$

$f'(x) = \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 5 - (3x^2 - 5)}{h}$ ✓

$= \lim_{h \to 0} \frac{6xh + 3h^2}{h}$

$= \lim_{h \to 0} \frac{h(6x + 3h)}{h}$

$= 6x$

b) $y = \frac{6}{\sqrt[3]{x^6}} - \frac{1}{6x^6}$

$= \frac{6}{x^2} - \frac{1}{6x^6}$

$= 6x^{-2} - \frac{1}{6}x^{-6}$

$= -12x^{-3} + x^{-7}$

$= -\frac{12}{x^3} + \frac{1}{x^7}$

c) i. $f(x) = ax^3 + q$

$f'(x) = 3ax^2$

(ii) $12 = 3a(2)^2$ and $3 = (2)^3 + q$

$1 = a$ $-5 = q$

$\therefore f(x) = x^3 - 5$

(iii) m tang $= 12$

$\perp m = -\frac{1}{12}$

$\therefore y - 3 = -\frac{1}{12}(x - 2)$

$y = -\frac{1}{12}x + \frac{19}{6}$

d) $y = \sqrt[3]{\frac{x+7}{2}}$

$x = \sqrt[3]{\frac{y+7}{2}}$

$x^3 = \frac{y+7}{2}$

$2x^3 = y + 7$

$2x^3 - 7 = y$

$\therefore f'(x) = 6x^2$

Question 3

(a) 180° ; 205 ; 230

$a = 180 \qquad d = 25 \qquad n = 15$

$T_n = 180 + (15-1)25$

$= R530$

(b) $S_n = R5\,880 \qquad d = 25 \qquad a = 180$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$5880 = \frac{n}{2}[2(180) + (n-1)(25)]$

$11\,760 = n(360 + 25n - 25)$

$11\,760 = n(335 + 25n)$

$0 = 25n^2 + 335n - 11\,760$

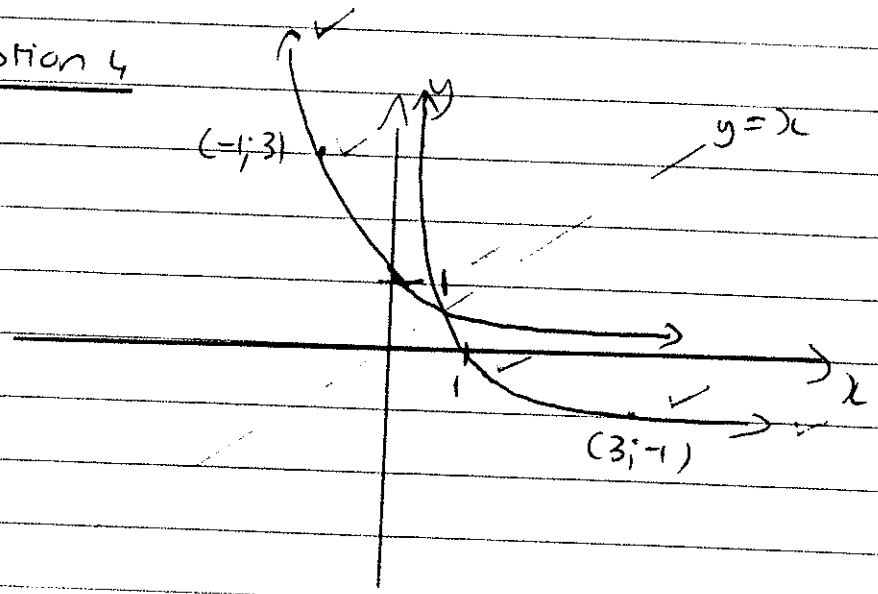
$0 = 5n^2 + 67n - 2352$

$n = \frac{-(67) \pm \sqrt{(67)^2 - 4(5)(-2352)}}{2(5)}$

$n = 16$ or $n \neq -\frac{147}{5}$

Question 4

a)



(b) $y = \log_{\frac{1}{3}} x$

(c) $7 = \left(\frac{1}{3}\right)^{-1,77}$

$\log 7 = -1,77 \log \frac{1}{3}$

$\therefore \log_{\frac{1}{3}} 7 = -1,77$

$2\log_{\frac{1}{3}} 7 = -3,54$

$\log_{\frac{1}{3}} 49 = -3,54$

Questions

a(i) 100 000

(ii) 25 000

(iii) reducing balance

(4) $25\,000 = 100\,000(1-i)^4$

$\frac{1}{4} = (1-i)^4$

$0,707\ldots = 1-i$

$0,2928 = i$

$r = 29\%$

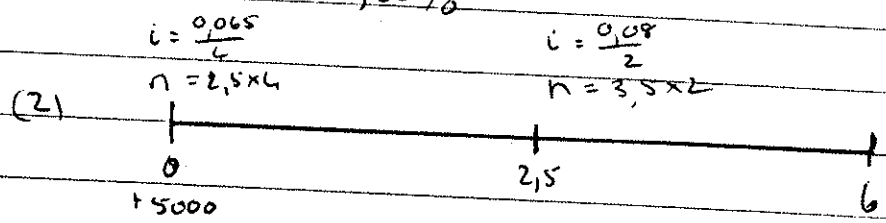
b(i) $1+ie = \left(1+\frac{i^{(m)}}{m}\right)^m$

$1+ie = \left(1+\frac{0,065}{4}\right)^4$

$ie = 0,0666$

$r = 6,66\%$

(2)



$A = 5000\left(1+\frac{0,065}{4}\right)^{10}\left(1+\frac{0,08}{2}\right)^{7}$

$= R\,7\,730,52$

(c) $500\,000 = 250\,000\left(1+\frac{0,058}{12}\right)^{12n}$

$\log 2 = 12n \log\left(\frac{6029}{6000}\right)$

$12n = 143,75\ldots$

$n = 12$ yrs

Section B:

Question 6

(a) $g(1) = -(1)^3 + 14(1)^2 - 69(1) + 36$

$= 0$

(b) $0 = x^3 - 14x^2 + 69x - 36$.

	1	−14	+69	−36
1		1	−13	36
	1	−13	36	

$(x-1)(x^2 - 13x + 36) = 0$

$(x-1)(x-9)(x-4) = 0$

$x = 1$ or $x = 9$ or $x = 4$.

(c) $g'(x) = -3x^2 + 28x - 69$

$0 = 3x^2 - 28x + 69$

$0 = (3x - 7)(x - 7)$

$x = \frac{7}{3}$ or $x = 7$

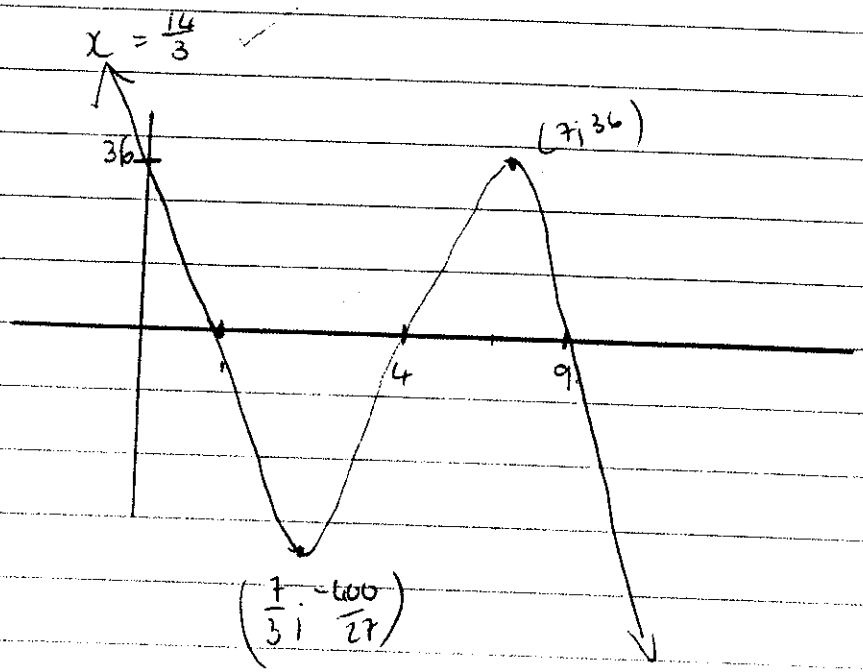
$y = \frac{-400}{27}$ $y = 36$

$(\frac{7}{3}; -\frac{400}{27})$ $(7; 36)$

(d) $-6x + 28 = 0$

$x = \frac{14}{3}$ ✓

(e)



(f) 36 down; 14,8 up

$\therefore \quad 0 < k < 50,8.$

(g) $g'(x) = -3x^2 + 28x - 49$

$g'(2) = -3(2)^2 + 28(2) - 49$

$= -5$

$g(2) = -(2)^3 + 14(2)^2 - 49(2) + 8$

$= -14$

$(2, -14)$

$y + 14 = -5(x - 2)$

$y = -5x - 4.$

Question 7.

(a) $60 - 2x$ ✓

(b) $\Delta AIBX = \frac{1}{2}(40)(60 - 2x)$

$= 1200 - 40x.$

$\Delta XYC = \frac{1}{2}(2x)(60 - x)$

$= 40x - x^2$

$\Delta AOY = \frac{1}{2}(60)(x)$

$= 30x$

$\therefore$ Playground area: $A = 2400 - (1200 - 60x + 40x - x^2 + 30x)$

$A = 1200 - 30x + x^2$

(c) $A' = (2x - 30)$

$0 = 2x - 30$ ✓

$30 = 2x$

$15 = x$ ✓

$\therefore \quad A = (15)^2 - 30(15) + 1200$

$= 975 \text{ m}^2.$

Question 8:

(a) 2; 3; 6

1 3

2

$2a = 2$ $\quad a = 1$

$3a + b = 1$ $\quad 3(1) + b = 1$ $\quad \therefore b = -2$

$a + b + c = 2$ $\quad 1 - 2 + c = 2$ $\quad c = 3$

$\therefore T_n = n^2 - 2n + 3$

(b)(1) 600; 640; 484 $\quad \therefore r = 1,1$. (2) $S_7 = \frac{600(1,1^7 - 1)}{1,1 - 1}$

$T_7 = 600(1,1)^6$

$= R708,62$

$= R3\,794,87$

(c)(1) 10; 5; $\frac{5}{2}$ (2) $S_\infty = \frac{10}{1 - \frac{1}{2}}$

$= 20$ cm

$\therefore 110 + 20 = 130$ cm.

Question 9:

(a) $y = ax^2 + q$

$38,72 = a(17)^2 + q$

$38,72 = 289a + q$ ①

$34,91 = 1156a + q$ ①-②

$3,81 = -867a$

$a = -0,0044$

$\therefore y = -0,0044x^2 + q$

$38,72 = -0,0044(17)^2 + q$

$40 = q$

$34,91 = a(34)^2 + q$

$34,91 = 1156a + q$ ②

(b) $0 = -0,0044x^2 + 40$

$0,0044x^2 = 40$

$x = \pm 95,35$

$\therefore 190,7$ m.

(c) $y = -0,0044(-20)^2 + 40$

$y = 38,24$

$\therefore 38,24 + 3,43$

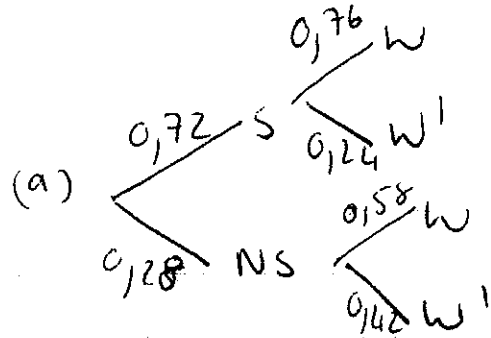
$= 41,67$

$t = \frac{41,67}{14,3}$

$= 2,91$ sec.

Question 10.

(a)



$0,72 \times 0,76 \rightarrow W$
$0,72 \times 0,24 \rightarrow W'$
$0,28 \times 0,58 \rightarrow W$
$0,28 \times 0,42 \rightarrow W'$.

(b) $W: \quad 0,72 \times 0,76 + 0,28 \times 0,58$
$= 0,7096.$

Question 11

a (1) $\dfrac{11!}{3!.2!.2!}$
$= 1\,663\,200$

(2) $\dfrac{10!.2!}{3!.2!.2!}$ ✓ Having the 2 l's together
$= 302\,400$ ✓

$\therefore \dfrac{302\,400}{1\,663\,200} = \dfrac{2}{11}$ ✓

∴ not having 2 "l"
$1 - \frac{2}{11} = \frac{9}{11}$ ✓

b (1) $P(A \cup B) = \frac{3}{8} + \frac{1}{4}$
$= \frac{5}{8}$

(2) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= \frac{3}{8} + \frac{1}{4} - \frac{3}{32}$
$= \frac{17}{32}$

$\therefore P(A \cap B) = \frac{3}{8} \times \frac{1}{4}$
$= \frac{3}{32}$