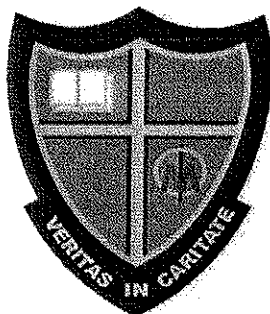


# ST BENEDICT'S COLLEGE



<b>SUBJECT</b>	Mathematics Paper 2
<b>GRADE</b>	12
<b>EXAMINER</b>	Mrs Sillman
<b>NAME</b>	MEMO
<b>CLASS</b>	

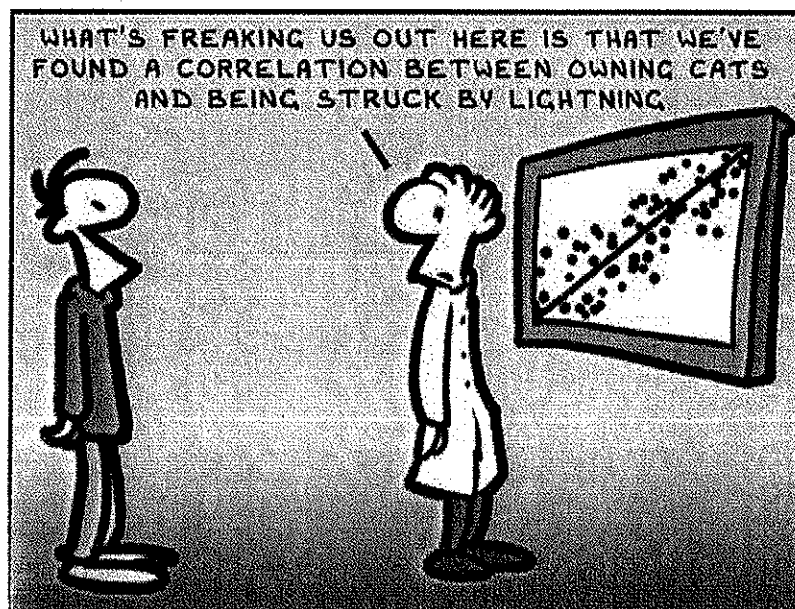
<b>DATE</b>	28 July 2014
<b>MARKS</b>	150
<b>MODERATOR</b>	Mrs Povall, Mrs Eckert
<b>DURATION</b>	3 hrs

QUESTION NO.	ASS STANDARD DESCRIPTION	TOTAL	ACTUAL
1 - 3	Analytical Geometry	23	
4 - 5	Data Handling	14	
6 - 7	Euclidean Geometry	13	
8	Trigonometry	20	
9 - 11	Trigonometry	31	
12	Data Handling	11	
13 - 14	Analytical Geometry	15	
15 - 16	Euclidean Geometry	23	
		125	

INSTRUCTIONS



1. This question paper consists of 16 questions and 24 pages (including coversheet and extra writing paper). Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
5. Round off your answers to one decimal digit where necessary.
6. All the necessary working details must be clearly shown. Answers only, without relevant calculations, may incur penalties.
7. It is in your own interest to write legibly and to present your work neatly.
8. There is an extra page at the back for answers in case you run out of space or make a mistake. Ensure the marker knows to go to the back and look for your answer by writing a note at the relevant question.

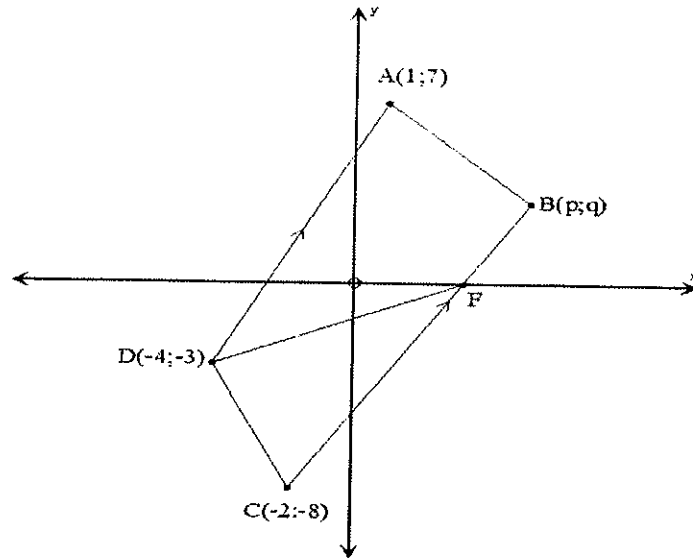


**QUESTION 1**

In the diagram, trapezium ABCD with  $AD \parallel BC$  is drawn. The co-ordinates of the vertices are:

$A(1;7)$   $B(p;q)$   $C(-2;-8)$  and  $D(-4;-3)$

BC intersects the x-axis at F.  $\widehat{DCB} = \alpha$

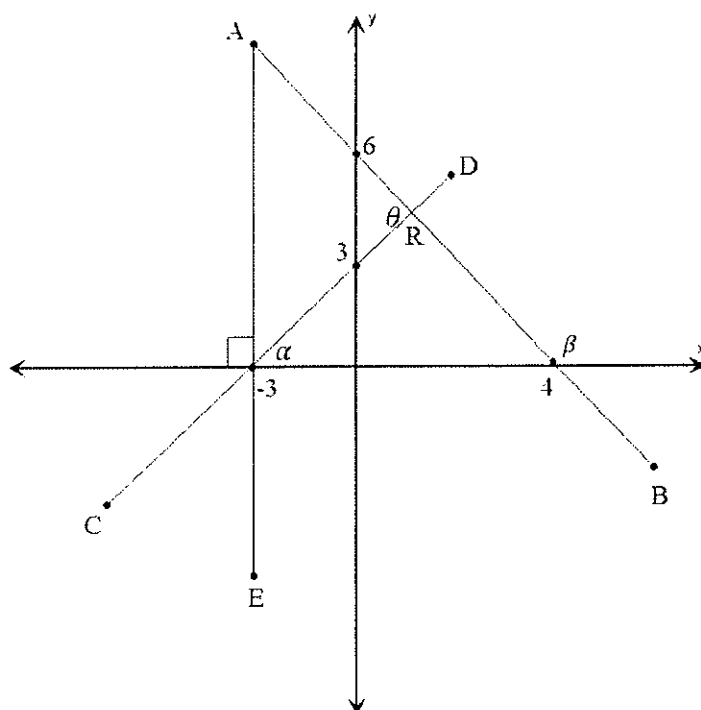


- Calculate the gradient of AD (1)
  - Find the mid-point of AD (2)
  - Determine the equation of BC in the form  $y = mx + c$  (2)
  - Determine the co-ordinate of the point F (2)
  - $AB'CD$  is a parallelogram with  $B'$  on BC. Determine the co-ordinate of  $B'$  (2)
- 9-

a) $m_{AD} = \frac{7+3}{1+4} = \frac{10}{5}$ $= 2$ ✓	d) $0 = 2x - 4$ $4 = 2x$ $2 = x$ ✓
b) $\left(\frac{1-4}{2}, \frac{7-3}{2}\right)$ $= \left(-\frac{3}{2}, 2\right)$ ✓ ✓	$F(2,0)$ ✓
c) $m=2$ $(-2;-8)$ $y + 8 = 2(x + 2)$ $y = 2x + 4 - 8$ $y = 2x - 4$ ✓ ✓	e) $B(3;2)$ ✓ ✓

QUESTION 2

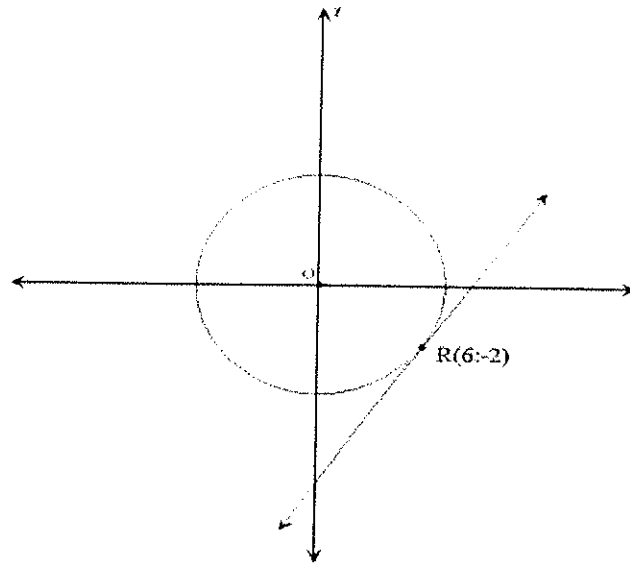
a)	Determine the size of $\alpha$ , $\beta$ and $\theta$ .	(6)
b)	Determine the size of $E\hat{A}B$	(3)



a)	$m_{AB} = \frac{0-6}{4-0}$	$m_{CD} = \frac{3-0}{0+3}$
	$= -\frac{3}{2} \checkmark$	$= 1 \checkmark$
	$\tan \beta = -\frac{3}{2}$	$\tan \alpha = 1$
	$\therefore \beta = 123,7^\circ \checkmark$	$\alpha = 45^\circ \checkmark$
	$\therefore \theta = 45^\circ + 56,3^\circ \checkmark$	ext $\angle$ of $\Delta$
	$= 101,3^\circ \checkmark$	
b)	$E\hat{A}B + 101,3^\circ + 45^\circ = 180^\circ$	LS of $\Delta$
	$\therefore E\hat{A}B = 33,7^\circ \checkmark$	
OR	$E\hat{A}B = 123,7^\circ - 90^\circ$	
	$= 33,7^\circ$	

**QUESTION 3**

PRQ is a tangent to the circle with centre O at the point R (6; -2)



Calculate the equation of:

- a) the circle (2)  
 b) the tangent (3)

a) $OR = \sqrt{(0-6)^2 + (0+2)^2}$
$= \sqrt{36+4}$
$= \sqrt{40} \quad \checkmark (2\sqrt{10})$
$\therefore x^2 + y^2 = 40 \quad \checkmark$
b) $m_{OR} = \frac{-2}{6} = -\frac{1}{3} \quad \checkmark$
$\therefore m_{TAN} = 3 \quad \checkmark (6, -2)$
$y + 2 = 3(x - 6)$
$y = 3x - 18 - 2$
$y = 3x - 20 \quad \checkmark$

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**QUESTION 4**

The following table gives the frequency distribution of the daily travelling time (in minutes) from home to work for the teachers at St Benedict's College.

Daily travelling time ( $x$ ) in minutes	Number of teachers
$0 \leq x < 10$	20
$10 \leq x < 20$	35
$20 \leq x < 30$	30
$30 \leq x < 40$	10
$40 \leq x < 50$	5

a) Circle the correct answer to the following questions:

(i) The estimated mean time, in minutes, taken by the teachers is:

A 14,5      **B 19,5** ✓      C 16,7      D 24,5      (2)

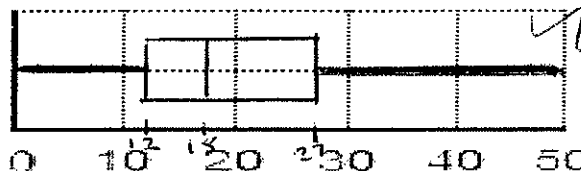
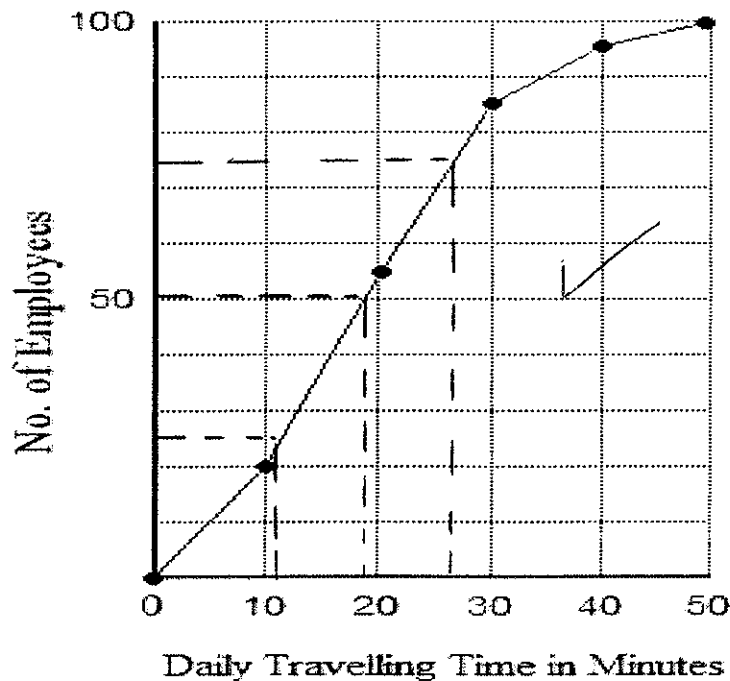
(ii) The estimated standard deviation for the time, in minutes is:

A 10,57      B 14,14      C 114,75      **D 10,71** ✓ ✓      (2)

b) An ogive was constructed from the given data.

Construct a box-and-whisker plot on the scaled axes below the ogive to summarise the data. Show on the ogive (by means of dotted lines), where you have read off the data required for the box-and-whisker.

(3)



c) State whether the following statements are TRUE or FALSE:

The Inter-quartile range for this data is 25

FALSE ✓ (1)

Only 35 teachers take less than 20 minutes to travel to school

FALSE ✓ (1)

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### QUESTION 5

A group of students wrote a stats test and the following data was obtained:

Std deviation	8
Mean	72
Median	64

- a) Is the data skewed? If so, in which direction and give a reason for your answer. (2)
- b) The teacher decided to add 5 marks to each person's mark. For the new set of marks, write down the:
- (i) Mean
  - (ii) Standard deviation
  - (iii) Median (3)

a) Yes, Mean > median ✓ ∴ positively skewed ✓
b) (i) 77 ✓
(ii) 8 ✓
(iii) 69 ✓

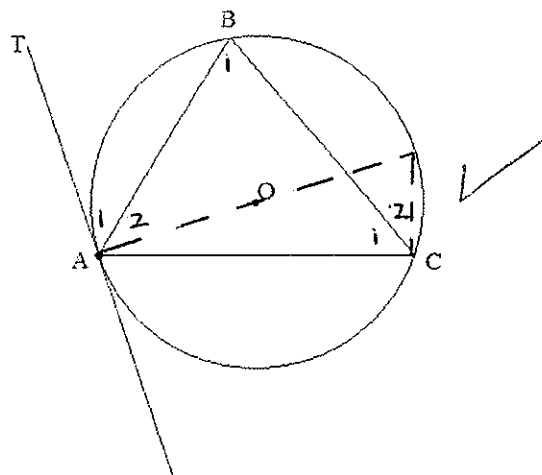
-5-

QUESTION 6

Prove the following theorem using the given sketch:

If TA is a tangent to the circle with centre O at A, then  $\hat{TAB} = \hat{ACB}$ .

Show all construction lines.



(5)

Given: Tangent TA
RTP. $\hat{TAB} = \hat{ACB}$
PROOF: Draw diameter AC(D) ✓
and join BC(D) ✓
$\hat{A}_1 + \hat{A}_2 = 90^\circ$ ✓ tan ⊥ rad ✓
$\hat{C}_1 + \hat{C}_2 = 90^\circ$ ✓ ∠ in semi-circle ✓
but $\hat{A}_2 = \hat{C}_2$ ✓ L's in same segment ✓
$\therefore \hat{C}_1 = \hat{A}_1$ ✓
$\therefore \hat{TAB} = \hat{ACB}$

-5-



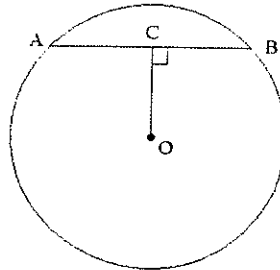
**QUESTION 7**

Complete the following:

- a) A line from the centre of a circle to the mid-point of a chord is (1)

perpendicular ✓ to the chord.

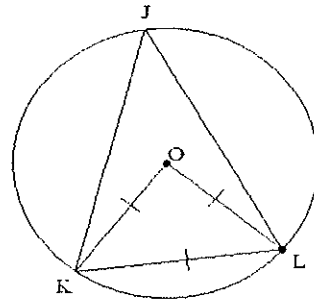
- b) Given  $OC \perp AB$ ; O is the centre of the circle. (1)



$AC = \frac{1}{2}$  AB ✓

- c) Given  $OK = KL = LO$

$\angle =$   $30^\circ$  ✓  
 Reason:  $\angle$  at centre =  
 $2 \times \angle$  at circumf ✓



(2)

- d) Given: O is the centre of the circle.  $\widehat{AOC} = 140^\circ$

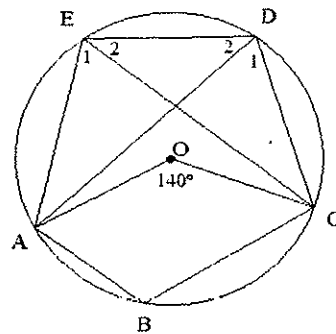
True or false:

- (i)  $\widehat{B} = 70^\circ$   
 (ii)  $\widehat{E}_1 = 70^\circ$   
 (iii)  $\widehat{D}_1 = \widehat{B}$

(i) FALSE ✓

(ii) TRUE ✓

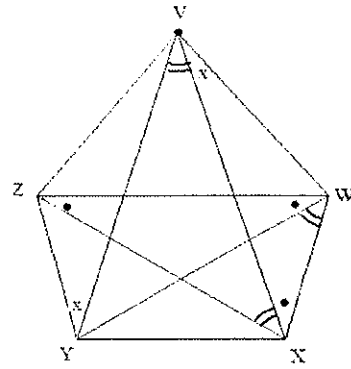
(iii) FALSE ✓



(3)

e) The only cyclic quadrilateral in the diagram is:

VYXW ✓



(1)  
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**QUESTION 8**

a) If  $\sin A = \frac{3}{5}$  and  $90^\circ < A < 270^\circ$ , determine, by means of a sketch (and without a calculator) the value of:

$$\frac{\sin 2A}{\cos^2 A}$$

(5)

$x^2 + y^2 = r^2$   
 $x^2 + 3^2 = 5^2$  ✓  
 $x = 4$  ✓  
 $\therefore x = -4$   
 $y = 3$   
 $r = 5$

$$\frac{\sin 2A}{\cos^2 A} = \frac{2 \sin A \cos A}{\cos^2 A}$$

$$= \frac{2 \sin A}{\cos A}$$

$$= \frac{2 \left(\frac{3}{5}\right)}{-\frac{4}{5}} \checkmark$$

$$= \frac{6}{5} \times -\frac{5}{4} = -\frac{6}{4} = -\frac{3}{2} \checkmark$$

b) Given:

$$\sin 27^\circ \cdot \cos 13^\circ = a$$

$$\sin 13^\circ \cdot \cos 27^\circ = b$$

$$\cos 27^\circ \cdot \cos 13^\circ = c$$

$$\sin 27^\circ \cdot \sin 13^\circ = d$$

Express the following in terms of a, b, c and d:

(i)  $\cos 40^\circ$       (ii)  $\sin 14^\circ$

(6)

(i) $\cos 40^\circ = \cos (27^\circ + 13^\circ)$ ✓
$= \cos 27^\circ \cos 13^\circ - \sin 27^\circ \sin 13^\circ$ ✓
$= c - d$ ✓
(ii) $\sin 14^\circ = \sin (27^\circ - 13^\circ)$ ✓
$= \sin 27^\circ \cos 13^\circ - \cos 27^\circ \sin 13^\circ$ ✓
$= a - b$ ✓

c) Simplify without the use of a calculator:

$$\frac{\cos(360^\circ + \theta) \cdot \tan(360^\circ - \theta)}{\sin(180^\circ + \theta)} + \frac{\sin(90^\circ + \theta)}{\cos(\theta - 360^\circ)}$$

(5)

$= \frac{\cos \theta \cdot -\tan \theta}{-\sin \theta} + \frac{\cos \theta}{\cos \theta}$ ✓
$= \frac{\cos \theta \cdot \frac{-\sin \theta}{\cos \theta}}{-\sin \theta} + 1$ ✓
$= \frac{-\sin \theta}{-\sin \theta} + 1$
$= 1 + 1$
$= 2$ ✓

d) Consider the identity  $\frac{1-\sin 2x}{\sin x - \cos x} = \sin x - \cos x$

For which values of  $x$  is this identity undefined?

(4)

$\sin x - \cos x = 0$ ✓
$\sin x = \cos x$
$\tan x = 1$ ✓
$C.A = 45^\circ$
$\therefore x = 45^\circ + k \cdot 180^\circ ; k \in \mathbb{Z}$

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**SECTION B**

**QUESTION 9**

Given  $f(x) = 2 \sin x$  and  $g(x) = \cos(x + 30^\circ)$

- a) Show that the equation  $2 \sin x = \cos(x + 30^\circ)$  can also be expressed as (5)

$$\tan x = \frac{\sqrt{3}}{5}$$

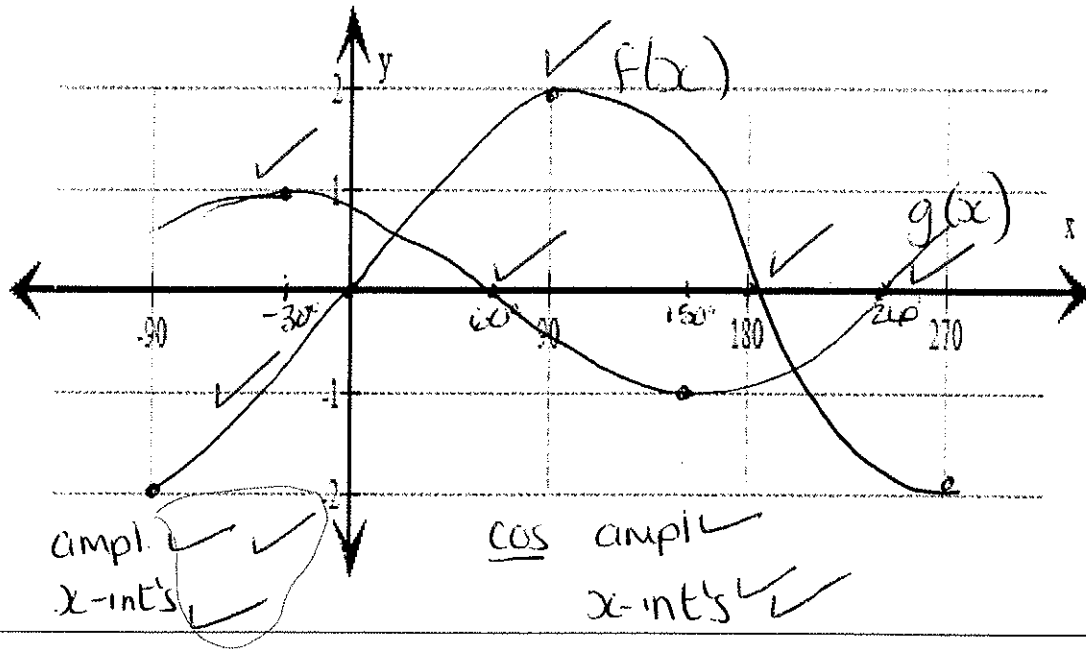
- b) Hence, determine the value(s) of  $x \in [-90^\circ; 270^\circ]$ , rounded to ONE decimal digit where  $f(x) = g(x)$  (3)

- c) Draw sketch graphs of the curves of  $f$  and  $g$  for  $x \in [-90^\circ; 270^\circ]$ , clearly showing all intercepts and turning points. (6)

- d) For which values of  $x$  is  $f(x)$  increasing and negative? (2)

a)	$2 \sin x = \cos(x + 30^\circ)$
	$2 \sin x = \cos x \cos 30^\circ - \sin x \sin 30^\circ$
	$2 \sin x = \cos x \cdot \frac{\sqrt{3}}{2} - \sin x \cdot \frac{1}{2}$
	$4 \sin x = \sqrt{3} \cos x - \sin x$
	$5 \sin x = \sqrt{3} \cos x$
	$5 \tan x = \sqrt{3}$
	$\tan x = \frac{\sqrt{3}}{5}$
b)	$\tan x = \frac{\sqrt{3}}{5}$
	C.A = $19,1^\circ$
	$\therefore x = 19,1^\circ + k \cdot 180^\circ; k \in \mathbb{Z}$
	$x \in \{19,1^\circ; 199,1^\circ\}$

c)



d)

$$-90^\circ \leq x < 0$$

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QUESTION 10

a) Evaluate:

$$\frac{\sin 6x}{\sin 2x} - \frac{\cos 6x}{\cos 2x}$$

(4)

$$= \frac{\sin 6x \cos 2x - \cos 6x \sin 2x}{\sin 2x \cos 2x}$$

$$= \frac{\sin(6x - 2x)}{\sin 2x \cos 2x}$$

$$= \frac{2 \sin 4x}{\sin 2x \cos 2x}$$

$$= \frac{2 \sin 2x \cos 2x}{\sin 2x \cos 2x}$$

$$= 2$$

b) Express  $\cos^2 A$  in terms of  $\cos 2A$  and then show that  $\cos 15^\circ = \frac{\sqrt{2+\sqrt{3}}}{2}$  (6)

$$2\cos^2 A - 1 = \cos 2A \quad \checkmark$$

$$2\cos^2 A = \cos 2A + 1$$

$$\cos^2 A = \frac{\cos 2A + 1}{2} \quad \checkmark$$

$$\cos^2 15^\circ = \frac{\cos 30^\circ + 1}{2} \quad \checkmark$$

$$\cos^2 15^\circ = \frac{\frac{\sqrt{3}}{2} + 1}{2} \quad \checkmark$$

$$\cos^2 15^\circ = \frac{\frac{\sqrt{3} + 2}{2}}{2}$$

$$\cos^2 15^\circ = \frac{\sqrt{3} + 2}{2} \times \frac{1}{2} \quad \checkmark$$

$$\cos^2 15^\circ = \frac{\sqrt{3} + 2}{4} \quad \checkmark$$

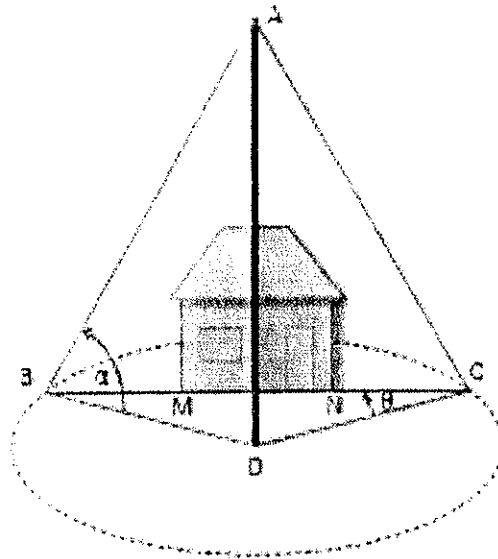
$$\cos 15^\circ = \sqrt{\frac{\sqrt{3} + 2}{4}} \quad \checkmark$$

$$= \frac{\sqrt{\sqrt{3} + 2}}{2}$$

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QUESTION 11

Building regulations require the provision of lightening protection for thatched roof structures. In the diagram, AD represents a vertical lightening mast constructed on the same horizontal plane as the house.



B and C are two points on the ground which are in line with the front of the house (MN). B and C lie on the circle of lightning protection provided by the lightning mast.  $\widehat{DBA} = \alpha$  is an angle of elevation from B to the top of the mast at A.  $\widehat{DCB} = \theta$ .  $AB = p$

- a) Express BD, the radius of protection, in terms of p and  $\alpha$  (1)  
 b) Hence, show that  $BC = 2p \cos \alpha \cos \theta$  (4)

a) In $\triangle ABD$ , $\cos \alpha = \frac{BD}{p}$ ✓
$\therefore p \cos \alpha = BD$
b) $\widehat{BDC} = 180 - 2\theta$ ✓
$\therefore \frac{BC}{\sin(180 - 2\theta)} = \frac{BD}{\sin \theta}$ ✓
$BC = \frac{BD \sin 2\theta}{\sin \theta}$
$BC = \frac{p \cos \alpha \cdot 2 \sin \theta \cos \theta}{\sin \theta}$ ✓
$= 2p \cos \alpha \cos \theta$ ✓

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**QUESTION 12**

Consider the table below which gives the number of hours studied and the results obtained in a Maths exam for seven boys.

Number of hours	1	3	5	6	8	10	11
Maths result (%)	35	55	60	65	75	70	80

- a) Calculate the correlation coefficient (2)

$$r = 0,93$$

- b) Comment on the strength of the relationship between the two variables. (2)

Very strong, positive correlation

- c) Find the line of best fit. (2)

$$y = 38,78 + 3,83x$$

- d) Use the equation to predict:

- (i) A boy's result if he studied for 3,5 hours  
 (ii) How many hours a boy studied if he obtained 100% (3)

(i) 52,2%

(ii)  $100 = 38,78 + 3,83x$   
 $61,22 = 3,83x$   
 $16 = x$   
 ∴ 16 hours

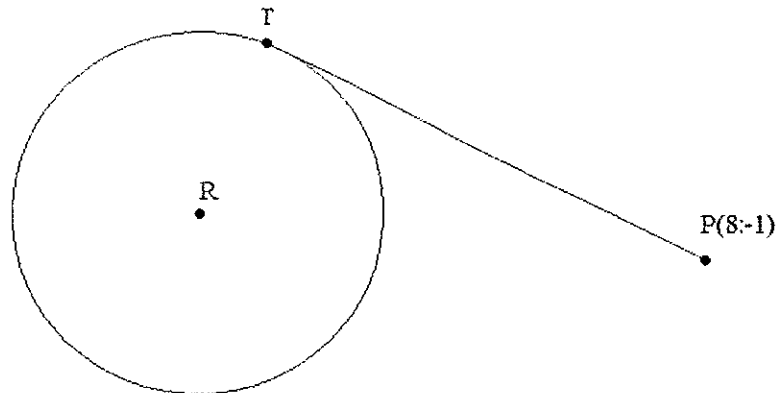
- e) Are the results obtained in (d) valid? Give a reason for your answer. (2)

Not necessarily. Just because a boy studies for 16 hours doesn't mean he will get 100%. Zero studying doesn't mean he will get 39%. Extrapolation not usually valid.

QUESTION 13

The equation of a circle in the Cartesian Plane is

$$x^2 + y^2 + 6x - 2y - 15 = 0 \text{ with centre R.}$$



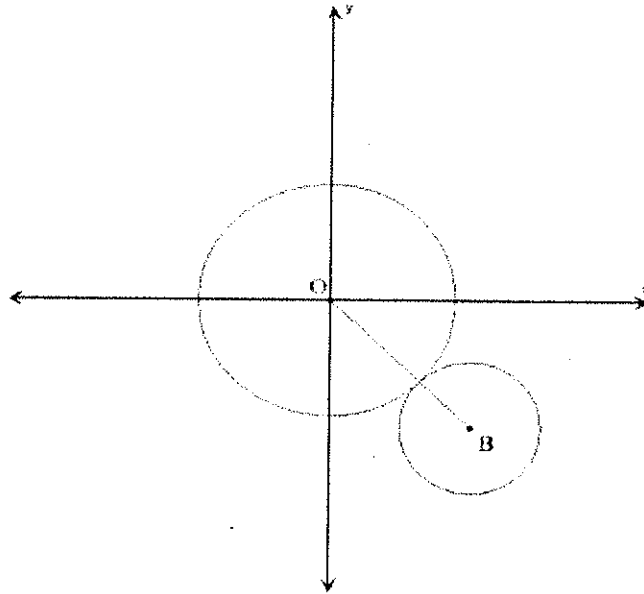
- a) Rewrite the equation in the form  $(x - p)^2 + (y - q)^2 = r^2$  (3)  
 b) Calculate the length of the tangent drawn to the circle from point P (8 ; -1) (6)

a) $x^2 + 6x + 9 + y^2 - 2y + 1 = 15 + 9 + 1$
$(x+3)^2 + (y-1)^2 = 25$
b) Centre (-3; 1)      radius (RT) = 5
$PR = \sqrt{(8+3)^2 + (-1-1)^2}$ ✓✓
$= \sqrt{121+4}$
$= \sqrt{125}$
$= 5\sqrt{5}$ ✓
$(5\sqrt{5})^2 = (5)^2 + PT^2$ ✓✓
$125 - 25 = PT^2$
$100 = PT^2$
$10 = PT$ ✓

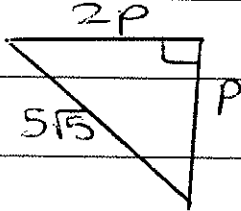
**QUESTION 14**

In the diagram below, two circles are drawn. Circle with centre O touches the circle with centre B externally.

The equation of the circle with centre O is given by  $x^2 + y^2 = 45$ . The equation of the circle with centre B is given by  $(x - 2p)^2 + (y + p)^2 = 20$

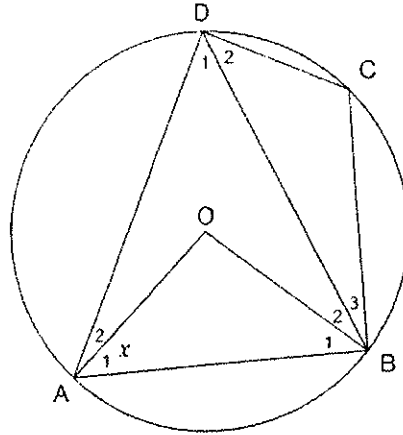


Determine the value of p.

radius of circle O = $\sqrt{45} = 3\sqrt{5}$
radius of circle B = $\sqrt{20} = 2\sqrt{5}$
$\therefore OB = 3\sqrt{5} + 2\sqrt{5}$
$= 5\sqrt{5}$ ✓✓
B(2p; -p)

$(2p)^2 + (p)^2 = (5\sqrt{5})^2$ ✓✓
$4p^2 + p^2 = 125$
$5p^2 = 125$
$p^2 = 25$
$\therefore p = 5$ ✓✓

QUESTION 15

Given: ABCD is a cyclic quadrilateral. O is the centre of the circle. OA bisects  $\hat{B}AD$ .  $\hat{O}AB = x$



- a) Name two other angles equal to  $x$  (2)

$\hat{B}_1 = x$  ✓  $\hat{A}_2 = x$  ✓

- b) Find  $\hat{B}CD$  in terms of  $x$ , giving a reason. (2)

$\hat{B}CD = 180 - 2x$  ✓ opp to cyclic quad ✓

- c) Express  $\hat{A}DB$  in terms of  $x$ , giving reasons. (4)

$\hat{A}DB = \frac{1}{2} \hat{A}OB$  ✓  $\angle$  at centre =  $2 \times \angle$  at circumf ✓  
 and  $\hat{A}OB = 180 - 2x$  ✓  $\angle$  in  $\Delta$  ✓  
 $\therefore \hat{A}DB = 90 - x$  ✓

- d) Can AD be a tangent to the circle through B, C and D? Motivate your answer. (2)

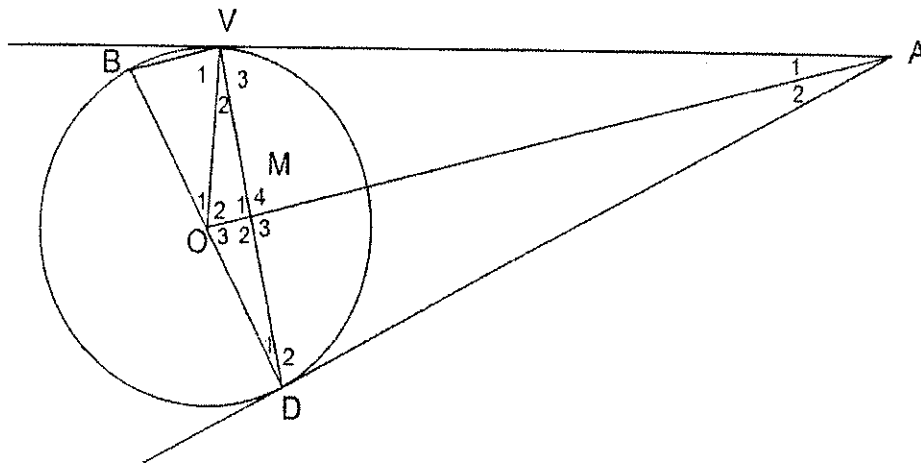
NO ✓  $\hat{B}_1$  would have to equal  $\hat{C}$  ✓

QUESTION 16

From a point A outside the circle, centre O, two tangents AD and AV are drawn. AO and VD meet at M.

BOD is a diameter of the circle. BV and VO are drawn.

Let  $\hat{A}_1 + \hat{A} = 40^\circ$



a) Complete:

$\hat{A}DO = 90^\circ$

$\hat{B}VD = 90^\circ$

Reason:

tan  $\perp$  rad ✓

$\angle$  in semi-circle ✓

(2)

b) Calculate, with reasons, the size of:

(i)  $\hat{D}_1$

(ii)  $\hat{O}_1$

(4)(2)

(i) $AV = AD$ ✓	equal tang. from common pt ✓
$\therefore \hat{V}_3 = \hat{D}_2$	
$\hat{V}_3 + \hat{D}_2 = 140^\circ$ ✓	L's ab $\Delta = 180^\circ$ ✓
$\therefore \hat{V}_3 = \hat{D}_2 = 70^\circ$ ✓	
$\therefore \hat{D}_1 = 20^\circ$ ✓	( $\hat{A}DO = 90^\circ$ ) ✓
(ii) $\hat{O}_1 = 2\hat{D}_1$ ✓	$\angle$ at centre = 2x $\angle$ at circ. ✓
$\therefore \hat{O}_1 = 40^\circ$ ✓	

\* OR ADO IS cyclic quad

$AC = AD$ ✓	equal tangents from common pt ✓
$OC = OD$ ✓	radii ✓
∴ $ACOD$ is a kite	
∴ $\hat{m}_1 = 90^\circ$ ✓ diagonals of a kite ✓	
∴ $B\hat{V}D + \hat{m}_1 = 180^\circ$ ✓	
∴ $BV \parallel OA$ ✓ CO-INT L's = $180^\circ$ ✓	

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$$\begin{aligned} & \cos 15^\circ \\ &= \cos (45^\circ - 30^\circ) \\ &= \cos \end{aligned}$$