2015

P1

PRELIM MEMO

Q1

a. i $6x^2 - 10x - 4 = 0$

$3x^2 - 5x - 2 = 0$

$(3x+1)(x-2) = 0$

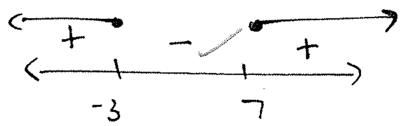
$\therefore x = -\frac{1}{3}$ or $x = 2$ (2)

ii $(x-2)^2 \geq 25$

$x^2 - 4x + 4 - 25 \geq 0$

$x^2 - 4x - 21 \geq 0$

$(x-7)(x+3) \geq 0$



$x \leq -3$ or $x \geq 7$ (4)

iii $4^y \cdot 3^{9x} = 7 \cdot 4^y$

$3^{9x} = 7$

$\log 3^{9x} = \log 7$

$9x \log 3 = \log 7$

$9x = \dfrac{\log 7}{\log 3}$

$9x = 1,77\ldots$ (4)

$x = 2$

b i. $\sqrt{x} + 3x = 2$

$\sqrt{x} = 2 - 3x$

$x = 4 - 12x + 9x^2$

$9x^2 - 13x + 4 = 0$

$(9x-4)(x-1) = 0$ (3)

ii $\sqrt{x} + 3x = 2$

$x = 4/9$ or $x = 1$ (2)

not valid.

c. $1{,}524157884 \times 10^{18} - 1{,}524157884 \times 10^{18}$

$= 0$ (3)

Q2

a. $T_3 + T_4 = 10$ $\qquad$ $T_{21} = 110$

$a + 2d + a + 3d = 10$ $\qquad$ $a + 20d = 110$

$2a + 5d = 10 \ldots (1)$ $\qquad$ $a = 110 - 20d \ldots (2)$

subs (2) into (1)

$2(110 - 20d) + 5d = 10 \ldots (1)$

$220 - 40d + 5d = 10$

$-35d = -210$

$d = 6$

subs $d = 6$ (2)

$a = 110 - 20(6)$

$a = T_1 = -10$ (6)

b. $T_{20} = -10 + 19(6)$

$T_{20} = 104$ (2)

c. $16;\ 12;\ 8$ $\therefore$ AS

$a = 16$; $d = -4$

$S_p = \dfrac{p}{2}[2(16) + (p-1)(-4)] = -20$

$\dfrac{p}{2}[32 - 4p + 4] = -20$

$18p - 2p^2 = -20$

$2p^2 - 18p + 20 = 0$

$p^2 - 9p + 10 = 0$

$(p-10)(p+1) = 0$

$\therefore p = 10$; $p = -1$ N/A (6)

2.

di. $T_3 = 10$; $T_6 = 1{,}25$

$T_3 = ar^2 = 10$ ① $T_6 = ar^5 = 1{,}25$ -- ②

$T_6 \div T_3$

$ar^5 = 1{,}25$ --- ②

$ar^2 = 10$ --- ①

$r^3 = 0{,}125$

$r = \frac{1}{2}$

Subs $r = \frac{1}{2}$ into ①

$a\left(\frac{1}{2}\right)^2 = 10$

$a = 40$

T_1 = first piece is 40cm

(6)

ii $S_{10} = \frac{40\left[1-\left(\frac{1}{2}\right)^{10}\right]}{1-\frac{1}{2}}$

$S_{10} = 79{,}92$cm

(2)

e. $S_\infty = 40$

$S_\infty = \frac{20}{1-r} = 40$

$20 = 40 - 40r$

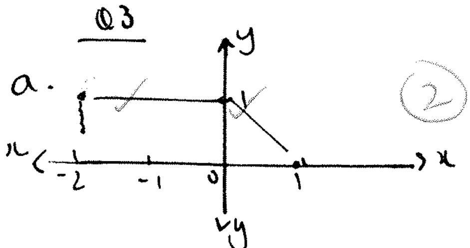
$-20 = -40r$

$\therefore r = \frac{1}{2}$

(4)

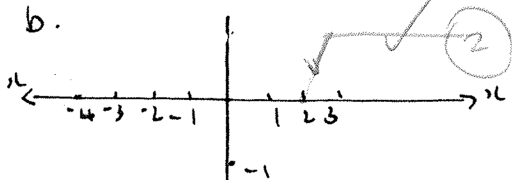
Q3

a.



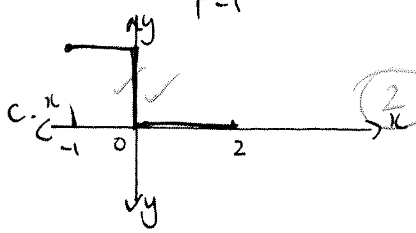
(2)

b.



(2)

c.



(2)

Q4

a. $y = a(x-3)^2 + 4$

Subs $(0;1)$

$1 = a(0-3)^2 + 4$

$-3 = 9a$

$a = -\frac{1}{3}$

$\therefore y = -\frac{1}{3}(x-3)^2 + 4$

$y = -\frac{1}{3}x^2 + 2x + 1$

(4)

b. x int $y = 0$

$0 = -\frac{1}{3}(x-3)^2 + 4$

$-4 = -\frac{1}{3}(x-3)^2$

$12 = (x-3)^2$

$\pm\sqrt{12} = (x-3)^2$

$\therefore x = 3 \pm\sqrt{12}$

$x = 6{,}46$

$\therefore$ No as C should be $(5{,}2;0)$

(4)

Q5

a. $f'(x) = \lim_{h\to 0} \frac{-2(x+h)^2 + 3(x+h) - (-2x^2 + 3x)}{h}$

$f'(x) = \lim_{h\to 0} \frac{-2x^2 - 4xh - 2h^2 + 3x + 3h + 2x^2 - 3x}{h}$

$f'(x) = \lim_{h\to 0} \frac{-4xh - 2h^2 + 3h}{h}$

$f'(x) = \lim_{h\to 0} -4x - 2h + 3$

$f'(x) = -4x + 3$

(5)

b $f(x) = \frac{1}{3}x^{3/5} + 10$

$f'(x) = \frac{1}{3} \times \frac{3}{5} x^{-2/5}$

$f'(x) = \frac{1}{5}x^{-2/5}$

OR $f'(x) = \frac{1}{5\sqrt[5]{x^2}}$

(3)

ii $f(x) = \dfrac{(2x^2-5)(3x+2)}{x^2}$

$f(x) = \dfrac{6x^3+4x^2-15x-10}{x^2}$

$f(x) = 6x + 4 - 15x^{-1} - 10x^{-2}$

$f'(x) = 6 + 15x^{-2} + 20x^{-3}$ 4

c. $y = 2x+p$ --- ① $f(x) = 2x^2-6x+20$ -- ②

① = ②

$2x+p = 2x^2-6x+20$

$2x^2 - 8x + 20 - p = 0$

For a tangent $\Delta = 0$

$\Delta = b^2 - 4ac = 0$

$\Delta = (-8)^2 - 4(2)(20-p) = 0$

$64 - 160 + 8p = 0$

$8p = 96$

$p = 12$

Subs $p = 12$ into $f(x)$

Point of contact : $f(2) = 2(2)^2 - 6(2) + 20$

$f(2) = 16$ 7

$\therefore (2;16)$

d. Let no & reciprocal be:

x & $1/x$

Sum of values $x + \frac{1}{x}$

$x + x^{-1}$

For min value: $\dfrac{dS}{dx} = 1 - x^{-2} = 0$

$1 - \frac{1}{x^2} = 0$ 6

$x^2 - 1 = 0$

$x = \pm 1$ $\boxed{\text{But } x > 0}$

$\therefore$ Minimum sum $1+1=2$

Q6

a. $-1 < x < 3$ or $(-1; 3)$ 2

b. $x < -3$ or $-3 < x < 3$ or 3

$(-\infty; -3)$ or $(-3; 3)$

c. $x < -3$ or $x > 3$ or 3

$(-\infty; -3)$ or $(3; \infty)$

d.i y-co-ord of E

$f(2) = -(2)^3 + 3(2)^2 + 9(2) - 27$

$f(2) = -5$

$E(2; -5)$

$f'(x) = -3x^2 + 6x + 9$

$f'(2) = -3(2)^2 + 6(2) + 9$

$f'(2) = 9$

$y - y_1 = m(x - x_1)$ subs $E(2; -5)$

$y + 5 = 9(x-2)$ 5

$y = 9x - 23$

ii $-x^3 + 3x^2 + 9x - 27 = 9x - 23$

$x^3 - 3x^2 + 4 = 0$

$(x-2)(x^2 - x - 2) = 0$

$\therefore (x-2)(x-2)(x+1) = 0$

$\therefore x = 2$ or $x = -1$

Subs $x = -1$ into $y = 9x - 23$

$y = 9(-1) - 23$

$y = -32$ $(-1; -32)$ Pt intersection.

5

Q7

a. $A = \dfrac{500\left[\left(1+\frac{0,15}{12}\right)^{120}-1\right]}{0,15/12}$ (4)

$A = R137\ 608,53$

b. Loan amount: 1 205 000 − 135 000

R 1 070 000

$1070000 = \dfrac{x\left[1-\left(1+\frac{0,09}{12}\right)^{-240}\right]}{0,09/12}$

$x = R\ 9\ 627,07$ (4)

ci. $A = \dfrac{9\ 627,07\left[1-\left(1+\frac{0,09}{12}\right)^{-144}\right]}{0,09/12}$

$A = R\ 845\ 941,15$ (4)

ii $845\ 941,15 = \dfrac{2500\left[1-\left(1+0,09/12\right)^{-n}\right]}{0,09/12}$

$-0,744217655 = -\left(1+0,09/12\right)^{-n}$

$-n = \log_{\left(1+\frac{0,09}{12}\right)} 0,746217655$

$-n = -39,18$

$n = 39,18$ months.

$n \approx 40$ months. (4)

Q8

a. $3\times2\times1 = 6$ (2)

b. $2\times1\times2\times1 = 4$ (2)

c. $\dfrac{10!}{2!2!2!2!} = 226\ 800$ (3)

d. $226\ 800 - \dfrac{9!\,2!}{2!2!2!2!} = 181\ 440$ (4)

Q9

a. $B\left(x, 4-\frac{x^2}{4}\right)$ (2)

b. Area $\triangle OBD = \frac{1}{2}(OD)(OB)$

$= \frac{1}{2}(x)\left(4-\frac{x^2}{4}\right)$

$= 2x - \dfrac{x^3}{8}$ (3)

c. Max area $\dfrac{dA}{dx} = 0$

$2 - \dfrac{3}{8}x^2 = 0$

$2 = 3/8\, x^2$

$16/3 = x^2$

$\left(4\sqrt{3}/3\right)$ $x = 2,3$ only +ve value. (5)

d. For max area subs x from (c) into (b)

Max area $= 2\times\dfrac{4\sqrt{3}}{3} - \left(\dfrac{4\sqrt{3}}{3}\right)^3 \div 8$

Max area $= \dfrac{16\sqrt{3}}{9}$

Max area = 3,1 square units (2)

Q10

a. For $t=0$: $S_A = 30(0)^2+20(0)+40$

$S_A = 40$

For $t=0$: $S_B = 10(0)^2+80(0)$

$S_B = 0$ (2)

∴ 40 units ahead

b. Cars are next to each other if:

$$S_A = S_B$$

$$30t^2 + 20t + 40 = 10t^2 + 80t$$

$$20t^2 - 60t + 40 = 0$$

$$\therefore t^2 - 3t + 2 = 0$$

$$\therefore (t-2)(t-1) = 0$$

$$\therefore t = 2 \text{ or } t = 1$$

3

c. $$\frac{dS_A}{dt} = \frac{dS_B}{dt}$$

$$60t + 20 = 20t + 80$$

$$40t = 60$$

$$t = 1{,}5$$

5

If $t = 1{,}5$ then velocities are the same:

$$S_A(1{,}5) = 30(1{,}5)^2 + 20(1{,}5) + 40 = 137{,}5$$

$$S_B(1{,}5) = 10(1{,}5)^2 + 80(1{,}5) = 142{,}5$$

$$\therefore S_B > S_A$$

Car B is ahead.