



**MATHEMATICS PAPER 1**

Time: 3 hours

Examiners: Miss Eastes, Mrs. Jacobsz, Mrs. Dwyer

150 marks

Name: \_\_\_\_\_ Teacher: \_\_\_\_\_

**PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY**

1. Read the questions carefully. Answer all the questions.  
Show all necessary working details.
2. Number your answers exactly as the questions are numbered.
3. You may use an approved, non-programmable, and non-graphical calculator, unless otherwise stated.
4. Round off your answers to **ONE DECIMAL PLACE where necessary**, unless otherwise indicated.
5. It is in your own interest to write legibly and to present your work neatly.
6. Diagrams are not drawn to scale.
7. Please note that there is an information sheet supplied.

## SECTION A

### QUESTION 1

[16]

Solve for  $x$  if  $x \in R$ :

(a)  $(x-3)^2 = 4x^2$  (4)

(b)  $\sqrt{9x-18} + 6 = x$  (5)

(c)  $4^{2x} = 8^{3x-5}$  (3)

(d)  $3x^2 + x - 2 \leq 0$  (4)

### QUESTION 2

[6]

The price of a rugby test ticket has increased over the past years as follows :

2009	2010	2011	2012	2013
100	140	200	280	380

(a) If the pattern continues in the same way, what will the price of the ticket be this year, i.e. in 2015? (1)

(b) Consider the number sequence : 100 ; 140 ; 200 ; 280 ; 380 ; .....

(i) What type of sequence is represented? (1)

(ii) Determine the  $n^{\text{th}}$  term of the sequence. (4)

### QUESTION 3

[13]

(a) A car, costing R210 000, depreciates at a rate of  $a$  % per annum, on a reducing balance basis. Calculate the value of  $a$  if it takes 8 years for the value of the car to decrease to R80 000. (4)

(b) Ms Samuel has finally decided to buy an apartment. She takes out a loan of R500 000 from the bank and plans to repay the loan in monthly instalments over a period of 20 years. The interest rate charged is 9% p.a., compounded monthly.

(1) Calculate her monthly repayments. (4)

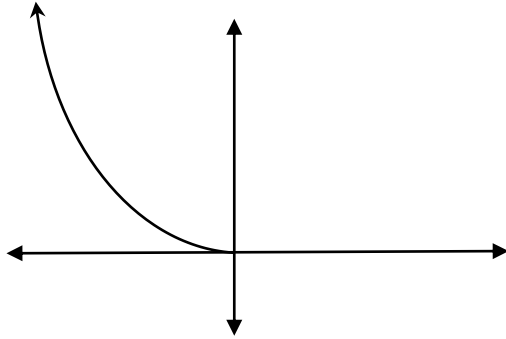
(2) What is the total amount she will have paid for the apartment after 20 years? (2)

(3) After 15 years, Ms Samuel wins the lotto and decides to clear the account. What will the outstanding balance on the loan be? (3)

**QUESTION 4**

**[10]**

(a) Given below is a sketch of  $y = 3x^2 ; x \leq 0$ .

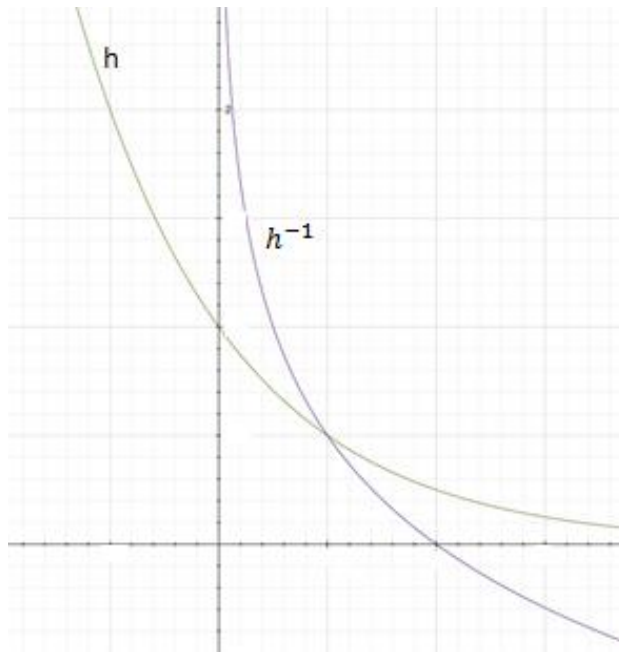


(i) Sketch the inverse graph of  $y = 3x^2 ; x \leq 0$  on your answer sheet (2)

(ii) Is the inverse graph a function? Explain your answer. (2)

(b) In the given diagram,  $h(x) = k^x$ , where  $k > 0$  and  $h^{-1}(x)$  is also drawn.

$(-2;16)$  is a point on the graph of  $h$



Use the graph and above information to answer the following questions :

(i) Determine the value of  $k$ . (2)

(ii) Determine the equation of  $h^{-1}(x)$  in the form  $y = \dots$ . (2)

(iii) For what value of  $x$  is  $h^{-1}(x) \geq 0$ ? (2)

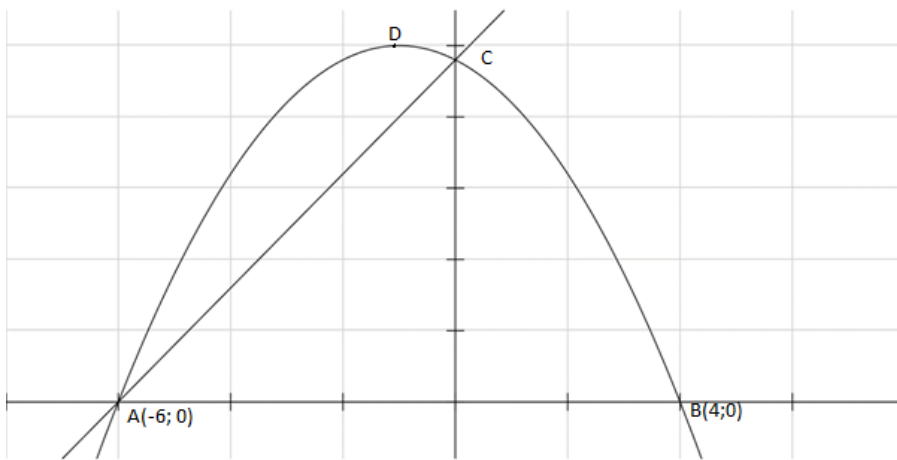
## QUESTION 5

[11]

In the diagram below, A(-6;0) and B(4;0) are the x-intercepts of the graph

$f(x) = ax^2 + bx + c$  and C is the y-intercept. **This diagram is not drawn to scale.**

The straight line is given by the equation  $h(x) = 8x + q$



Use the graph and above information to answer the following questions.

- (a) Determine the value of  $q$ . (2)
- (b) Prove that the equation of  $f(x)$  is given by the equation (4)  
$$f(x) = -2x^2 - 4x + 48$$
- (c) Calculate the coordinate of D, the turning point of  $f(x)$ . (3)
- (d) For what values of  $k$  will  $f(x) = k$  have no real roots. (2)

## QUESTION 6

[18]

- (a) (1) Determine  $f'(x)$  from first principles if  $f(x) = x^2 - 3x$  (4)
- (2) Determine the equation of the tangent to the graph of  $f$  at the point where  $f'(x) = -1$  (5)
- (b) Determine  $\frac{dy}{dx}$  if  $y = 4\sqrt{x} - \frac{8}{x} + \pi x^3$  (leave your answer in positive exponents) (4)
- (c)  $f(x) = ax^3 + bx^2 + cx + d$ . If  $f'(x) = 18x^2 + 14x - 8$  and  $f(0) = -7$ , determine the values of  $a$ ,  $b$ ,  $c$  and  $d$ . (5)

## SECTION B

### QUESTION 7

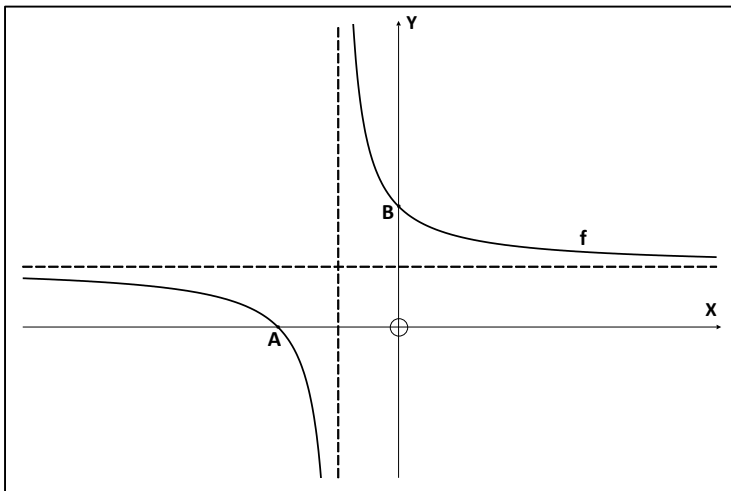
[18]

- (a) The 5<sup>th</sup> term of arithmetic series is 14. The sum of the first 10 terms is 160.  
Determine the first term and the common difference of the series. (5)
- (b)  $\log 2$  and  $\log 4$  are the first 2 terms of an arithmetic as well as a geometric sequence.  
Determine an expression for the  $n^{\text{th}}$  term of each sequence. Simplify each answer to one term. (6)
- (c) Determine the value of  $p$  if : 
$$\sum_{k=1}^{\infty} 27p^k = \sum_{t=1}^{12} (24 - 3t)$$
 (7)

### QUESTION 8

[10]

The sketch below represents the graph of the function  $f(x) = \frac{x+8}{x+2}$



- (a) Determine the coordinates of  $A$  and  $B$ , the  $x$  and  $y$  intercepts of  $f$ . (4)
- (b) By writing  $f$  in the form  $f(x) = \frac{a}{x+2} + q$ , determine the equations of the horizontal and vertical asymptotes of  $f$ . (4)
- (c) Give the equations of the asymptotes of  $h(x) = f(x+1) - 3$ . (2)

### QUESTION 9

[5]

A traveler buys a combination lock for his suitcase.

The lock consists of 3 wheels, each with numbers 0 to 9.



- (a) How many different possible combinations are there if the numbers can be repeated? (2)
- (b) The traveler has unfortunately forgotten his combination. What is the probability that he gets his suitcase opened if he used only odd, non-repeated numbers? (3)

### QUESTION 10

[7]

The letters that form the word MATHEMATICS are arranged as shown below on separate cards.



- (a) How many different arrangements can be done using all these cards if the **repeated letters are treated as identical** ? (3)
- (b) What is the probability that an arrangement made using all the above letters has all the vowels next to each other? (**The repeated letters are treated as different**), (4)

### QUESTION 11

[5].

The probability that the Springbok team has all its players fit to play is 70%.

The probability that they will win a game if all their players are fit is 90%.

When they are not fit, the probability of them winning becomes 45%.

Calculate the probability of them winning their next game.

## QUESTION 12

[9].

An environmental study in the south Durban basin has revealed that the water from a factory has polluted the water in the nearby pond of a nature reserve.

The pollution had an effect on the plant life growing on the water surface.

At first the polluted water caused the plants to grow profusely and later on it caused the plants to die and the area covered by the plants to decrease.

A mathematician worked out that the surface area,  $A$  (in  $m^2$ ), covered by the plants can be

approximated by the formula : 
$$A = -\frac{1}{2}t^3 + 12t^2$$

where  $t$  is the time in months after the start of the study, with  $0 \leq t \leq 24$ .

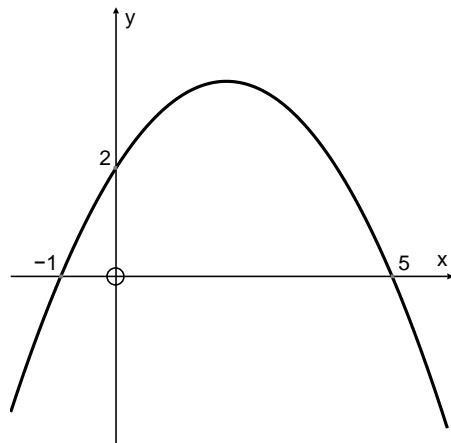
- (a) After how many months did the area covered by the plants reach a maximum? (4)
- (b) Calculate the maximum area that was covered by the plants. (2)
- (c) At what rate was the area increasing one month after the study began? (3)

## QUESTION 13

[7]

The function which is defined by  $y = f(x)$  is of the third degree.

**The parabola in the sketch represents the curve of  $y = f'(x)$ .**



- (a) What is the gradient of the tangent to the curve of  $f(x)$  if  $x = 0$ ? (1)
- (b) For which values of  $x$  will  $f(x)$  be increasing? (2)
- (c) Write down the  $x$  co-ordinates of the stationary points of  $f(x)$  and indicate whether those points will be local maxima or local minima. (4)

## **QUESTION 14**

**[6]**

The Richter scale measure of an earthquake is given by  $R = \log \left( \frac{I}{I_0} \right)$

where  $I$  is the intensity of the earthquake

and  $I_0$  is the intensity under normal conditions.

(a) An earthquake has an intensity 5 000 000 times the intensity under normal conditions,

$$\text{i.e. } \left( \frac{I}{I_0} \right) = 5\,000\,000$$

A second earthquake has intensity 5 000 times the intensity under normal conditions.

Determine how much greater, on the Richter scale, the first earthquake would be than to the second earthquake.

(4)

(b) An earthquake was recorded on the Richter scale as measuring 4.

Calculate how many times as intense it was compared to normal conditions.

(2)

## **QUESTION 15**

**[7]**

$$f(x) = x(x+a) - b; \quad a \neq 0; \quad b \neq 0$$

$$g(x) = 2x^2 + b - dx$$

If  $(x-a)$  is a factor of  $f(x)$  and  $(x-b)$  is a factor of  $g(x)$

Prove that  $d-1 = 4a^2$