Durban Girls' College
Towards Tomorrow's Years

GRADE 12 TRIALS EXAMINATION
AUGUST 2015

MATHEMATICS PAPER 2

Time: 3 hours 150 marks
Examiners: Miss Eastes, Mrs. Jacobsz, Mrs. Dwyer

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. Read the questions carefully. Answer all the questions.
2. Number your answers exactly as the questions are numbered.
3. You may use an approved, non-programmable, and non-graphical calculator, unless otherwise stated.
4. Round off your answers to **ONE DECIMAL PLACE**, where necessary unless otherwise indicated. All the necessary working details must be clearly shown.
5. It is in your own interest to write legibly and to present your work neatly.
6. Diagrams are not drawn to scale.
7. Please note that there is an information sheet provided.

Name:________________________________ Teacher:__________________________

Marking Grid (for Educators' use only)

QUESTION	1	2	3	4	5	6	7
ACHIEVED							
POSSIBLE	**20**	**13**	**6**	**8**	**9**	**8**	**15**

8	9	10	11	**12**	**13**	**14**	**Total**
25	**11**	**13**	**7**	**6**	**4**	**5**	**150**

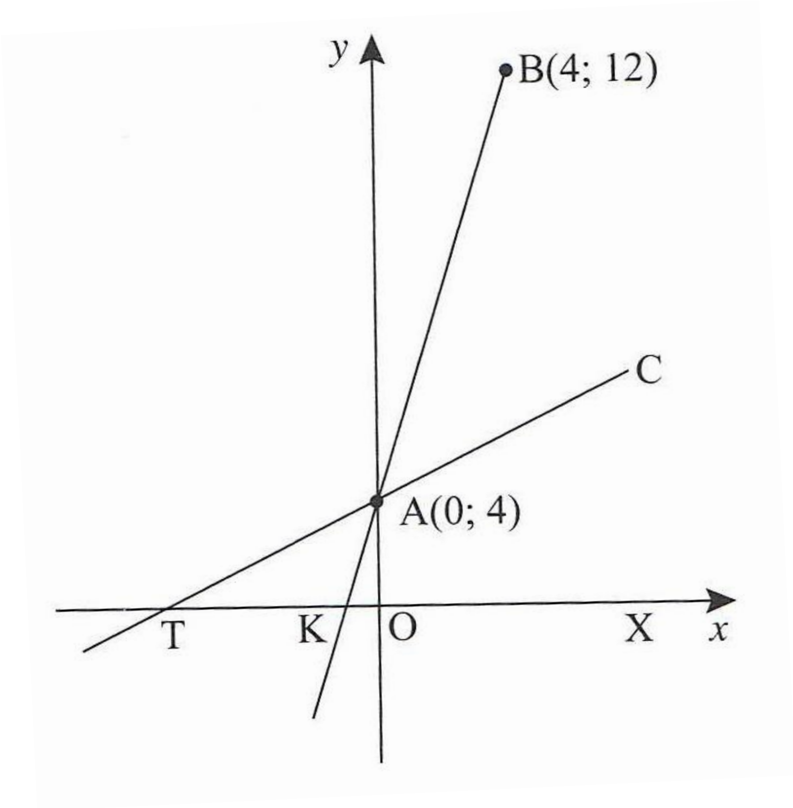
SECTION A

Question 1

1.1 In the diagram, A is the point (0;4) and B is the point (4;12).

The straight line CAT has a gradient of $\frac{1}{3}$.

KAB is a straight line.



Determine:

1.1.1 $C\hat{T}X$ (2)

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1.1.2 $B\hat{A}C$, giving reasons. (5)

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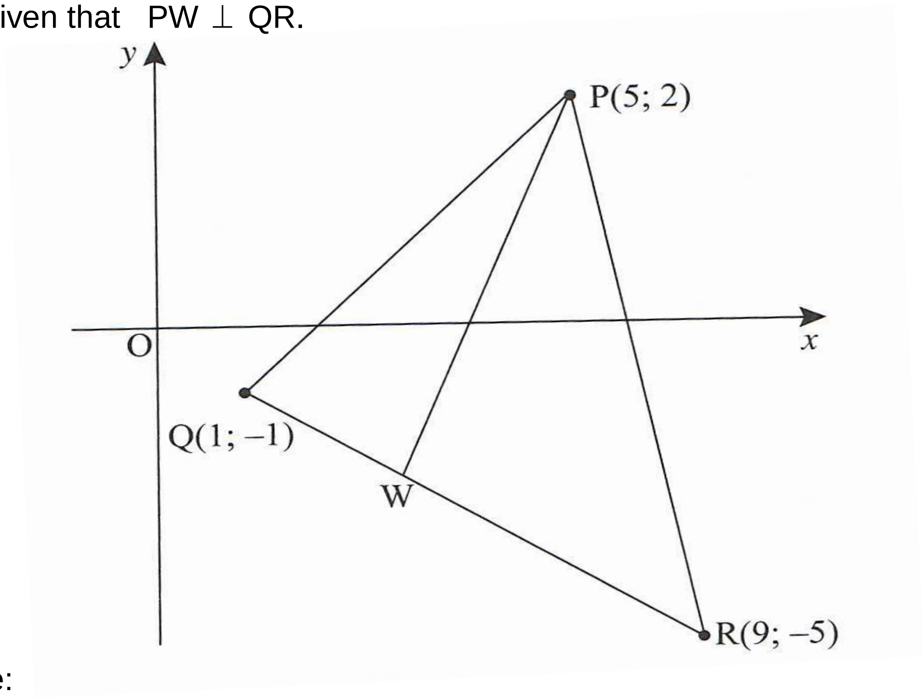
1.2 In the diagram $P(5;2), Q(1;-1)$ and $R(9;-5)$ are the vertices of the triangle PQR.

It is also given that $PW \perp QR$.



Calculate:

1.2.1 the length of QR (*leave answer in simplest surd form*) (2)

1.2.2 the equation of QR (4)

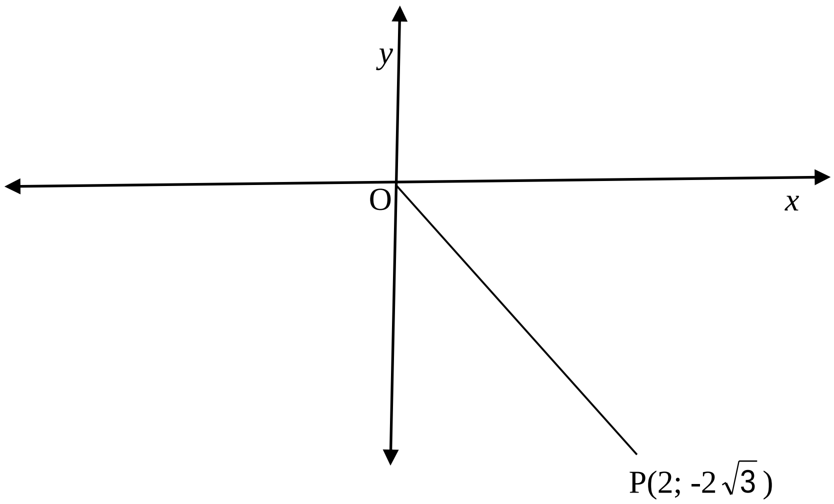
1.2.3 the equation of the line PW (3)

1.2.4 the coordinates of W (4)

[20]

Question 2

2.1 In the diagram, P is the point $(2; -2\sqrt{3})$. Reflex $X\hat{O}P = A$.



Determine, leaving your answers in surd form where necessary:

2.1.1 the length of OP (2)

2.1.2 sin 2A (3)

2.2 Simplify (without using a calculator):

$$\frac{\sin 223^\circ . \sin(x - 90^\circ)}{\cos 47^\circ \cos(360^\circ + x)}$$ (5)

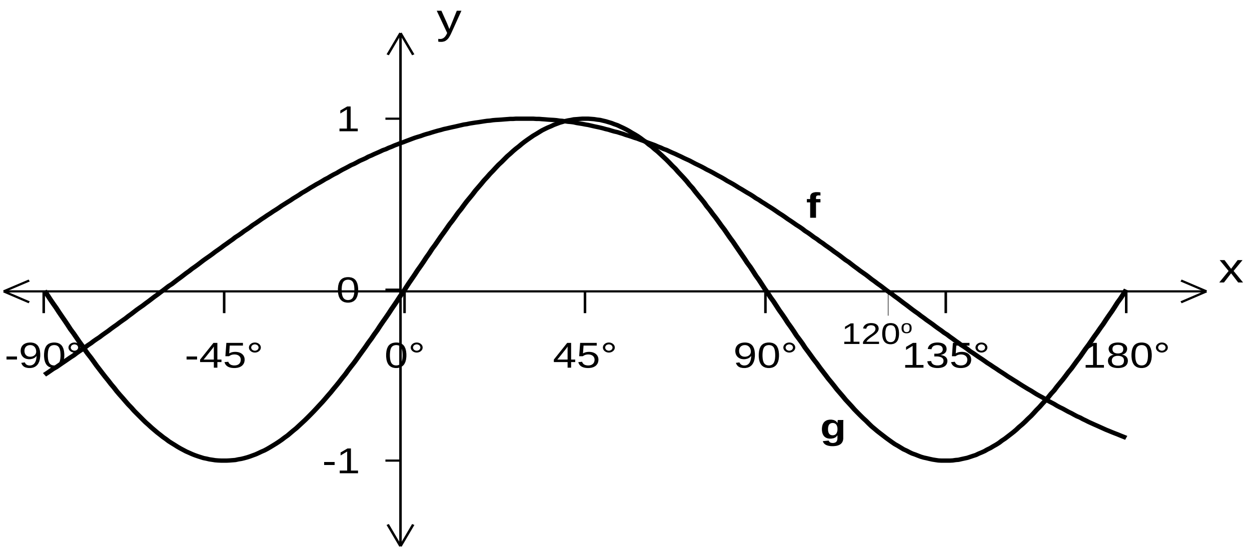
2.3 Solve for x (without using a calculator) :

$\sin x = - \sin 50^\circ$ (3)

[13]

Question 3

The graphs of $f(x) = \cos(x + \mathbf{a})$ and $g(x) = \sin \mathbf{b}x$ are shown above for $\mathbf{x} \in \mathbf{[-90°; 180°]}$.



3.1 Determine:

3.1.1 the value of a (1)

3.1.2 the value of b (1)

3.1.3 the amplitude of *f* (1)

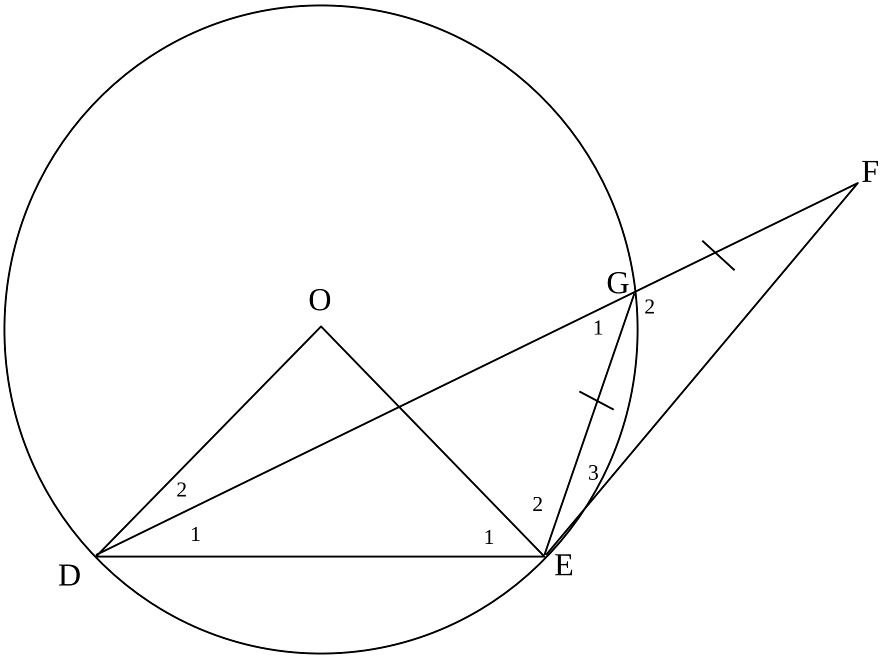
3.1.4 the period of g (1)

3.2 If g is moved down 2 units, what will its equation change to? (2)

[6]

Question 4

In the figure below, FE is a tangent to the circle with centre O.
D and F are joined so that EG = GF.



4.1 If $\hat{E}_3 = x$, name, with reasons, two other angles each equal to x. (3)

4.2 Prove that DE = EF. Give reasons for your answers (1)

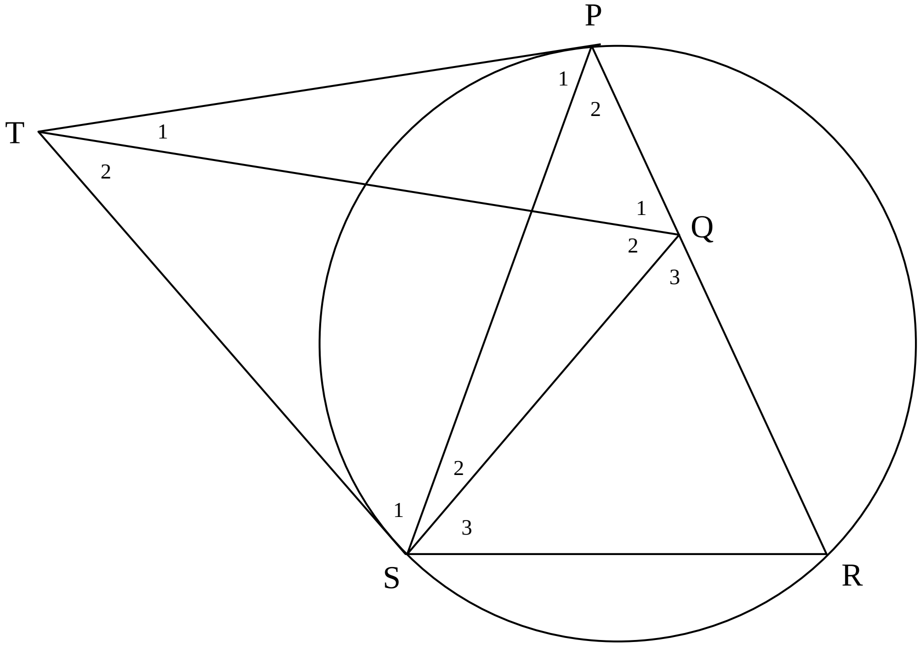
4.3 Express $D\hat{O}E$ in terms of x. Give reasons for your answers (4)

[8]

[8]

Question 5

In the figure below: TP and TS are tangents to the circle. R is a point on the circle and SR and PR are joined. Q is a point on PR so that $\hat{P}_1 = \hat{Q}_1$. S and Q are joined



Prove that:

5.1 TQ // SR (Give reasons for your answers) (3)

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5.2 QPTS is a cyclic quadrilateral. (Give reasons for your answers) (3)

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5.3 TQ bisects $S\hat{Q}P$. (Give reasons for yoru answers) (3)

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____ [9]

Question 6

The heights (*in cm*) of a group of basketball players are recorded as follows:

178; 184; 186; 186; 192; 194; 195; 195; 197; 198; 201

6.1 Determine the mean height of the players. (2)

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6.2.1 Determine the standard deviation. (2)

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6.2.2 Determine the interval of the heights within one standard deviation of the mean. (2)

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6.2.3 Determine the percentage of players, whose heights, are within one standard deviation of the mean. (2)

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____ **[8]**

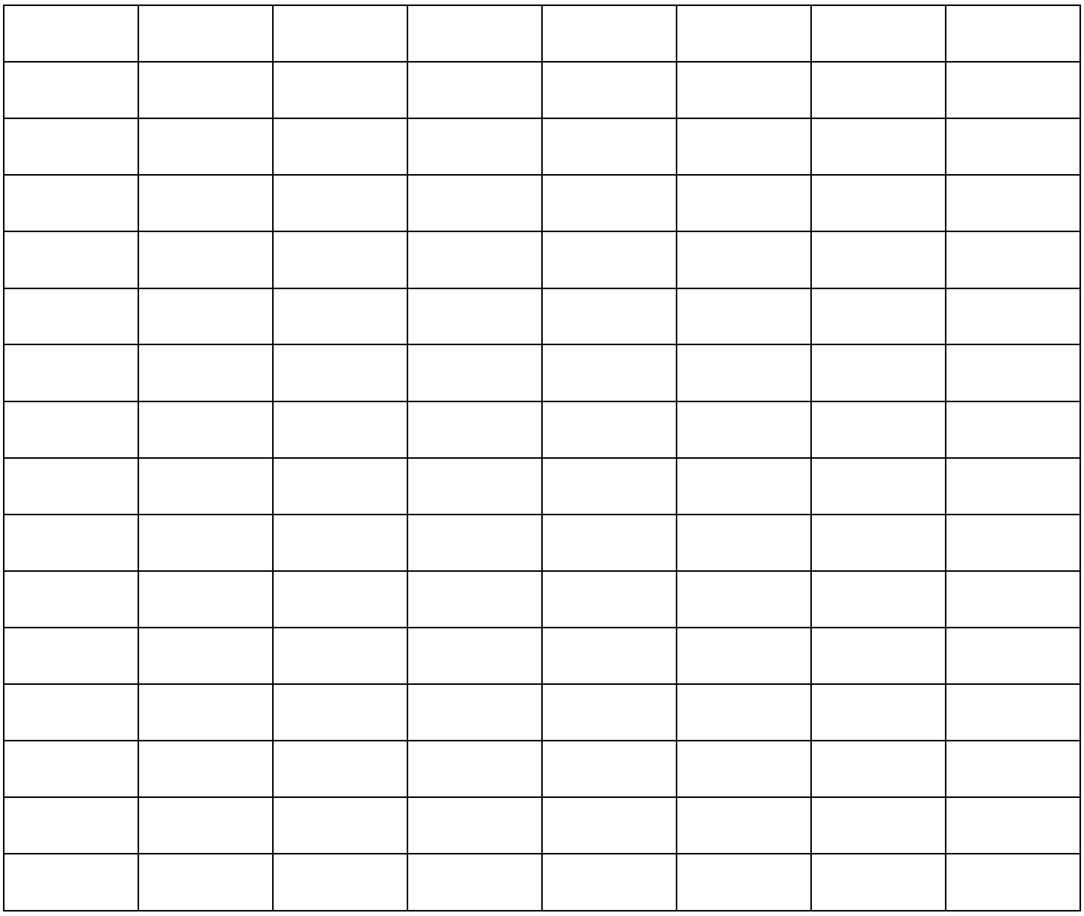
Question 7

The following frequency table shows the distribution of the marks of 200 students in a Mathematics test out of 60.

Mathematics Mark	Frequency	Cumulative frequency
$0 \leq x \leq 10$	20	
$10 < x \leq 20$	40	
$20 < x \leq 30$	60	
$30 < x \leq 40$	50	
$40 < x \leq 50$	20	
$50 < x \leq 60$	10	

7.1 Complete the cumulative frequency table in the space provided (1)

7.2 Draw the cumulative frequency ogive on the grid below (3)

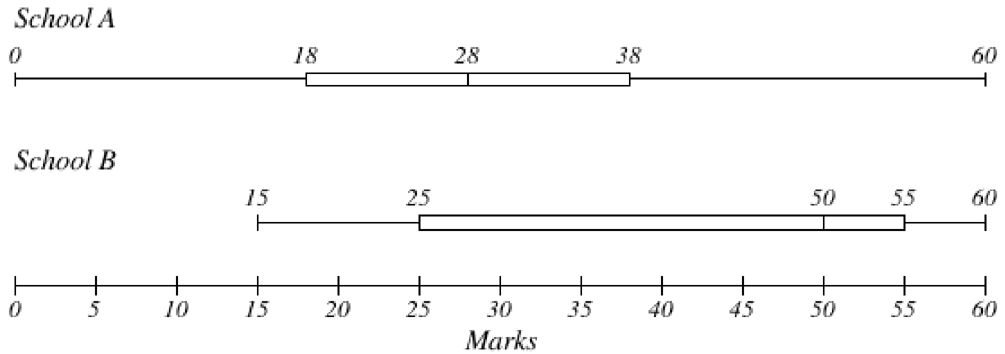


7.3 Use your graph to estimate the interquartile range. (3)

7.4 The top 40% of the students won't need to rewrite the test.
Use the graph to determine the cut-off mark. (2)

7.5 It is given that the standard deviation of the data is 12,9 and the mean is 27.
The teacher found that the marks were too low. He added 20 to each mark.
Write down the new mean and the new standard deviation of the new set of scores. (2)

7.6 Below are two box and whisker diagrams representing the marks of 200 learners each from two different schools, for the same test out of 60, answer the following questions:



7.6.1 What percentage of School B's results were above 55 out of 60 (1)

7.6.2 School A claims that their overall results are better than School B's,
Is this true? Explain your answer referring to the summary statistics? (3)

[15]

SECTION B

Question 8

8.1 Prove, using fundamental identities, that:

$$\frac{1}{\cos \mathbf{A} - \sin \mathbf{A}} + \frac{1}{\cos \mathbf{A} + \sin \mathbf{A}} = \frac{\tan \mathbf{2A}}{\sin \mathbf{A}}$$ (7)

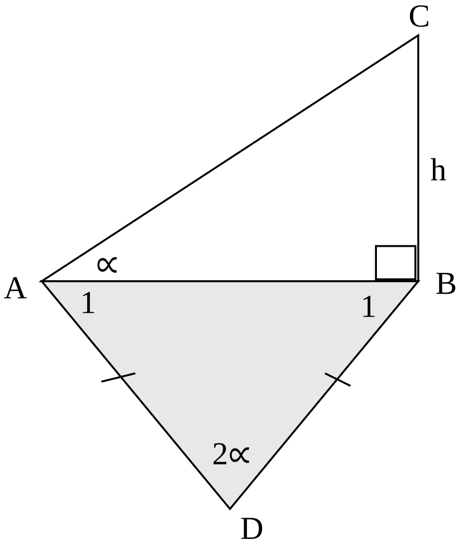
8.2.1 Show that the equation 2cos θ = sin(θ+ 30°) is equivalent to 3 cos θ = $\sqrt{3}$ sin θ. You may not use your calculator. (5)

8.2.2 Now calculate θ if $\theta \in [-180°; 180°]$ (5)

8.3 ABD is a triangle in the horizontal plane.
BC is a pole perpendicular to this plane.
AD = BD.

The angle of elevation from A to C is α and $A\hat{D}B = 2\alpha$.

Prove that AD = $\dfrac{h\cos\alpha}{2\sin^2\alpha}$

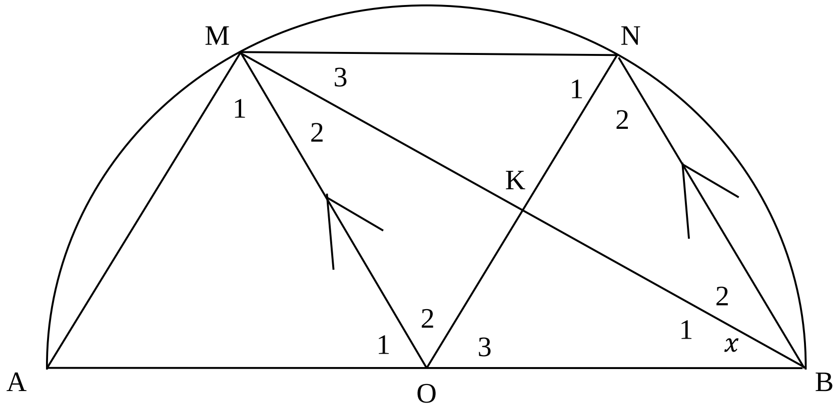


(8)

[25]

QUESTION 9

In the given figure, AOB is the diameter of the semi-circle, centre O, MO // NB, ON and MB intersect at K and $\hat{B}_1 = x$.



9.1 Prove that MB bisects $N\hat{B}O$. (Give reasons for your answers) (3)

9.2 Express the following in terms x. (Give reasons for your answers)

9.2.1 $M\hat{K}N$ (2)

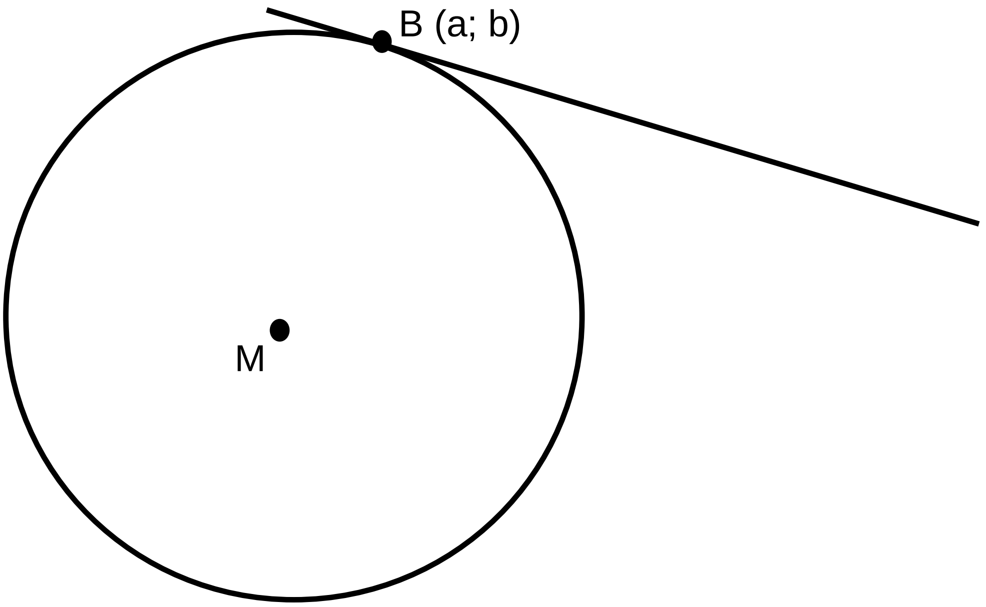
9.2.2 $\widehat{M_1}$ (2)

9.3 If $x = 30°$, calculate the sizes of the angles of ΔMKN. (Give reasons for your answers) (4)

[11]

Question 10

10.1 A circle, with centre M, is defined by the equation $(x+6)^2+(y+1)^2=20$.
A tangent is drawn, touching the circle at B (*a*; *b*).
The equation of this tangent is $2y+x-2=0$.



10.1.1 Determine the gradient of the tangent? (1)

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10.1.2 Show that B(-4 ; 3). (6)

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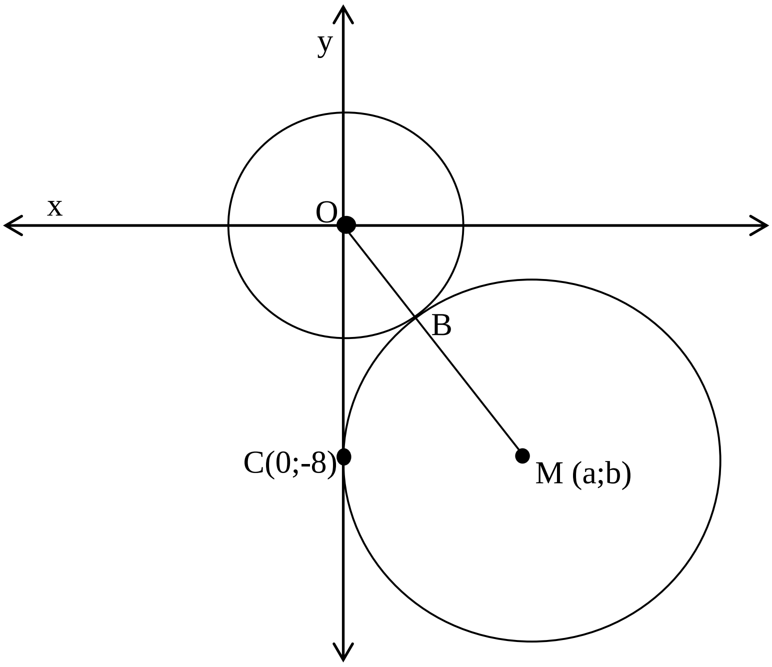
10.2 Two circles with centre O(0; 0) and M(a; b) touch externally at B.

The equation of the smaller circle with centre O is $x^2 + y^2 = 16$.

Circle centre M touches the y-axis at C(0; -8).

Determine the co-ordinates of M. (6)

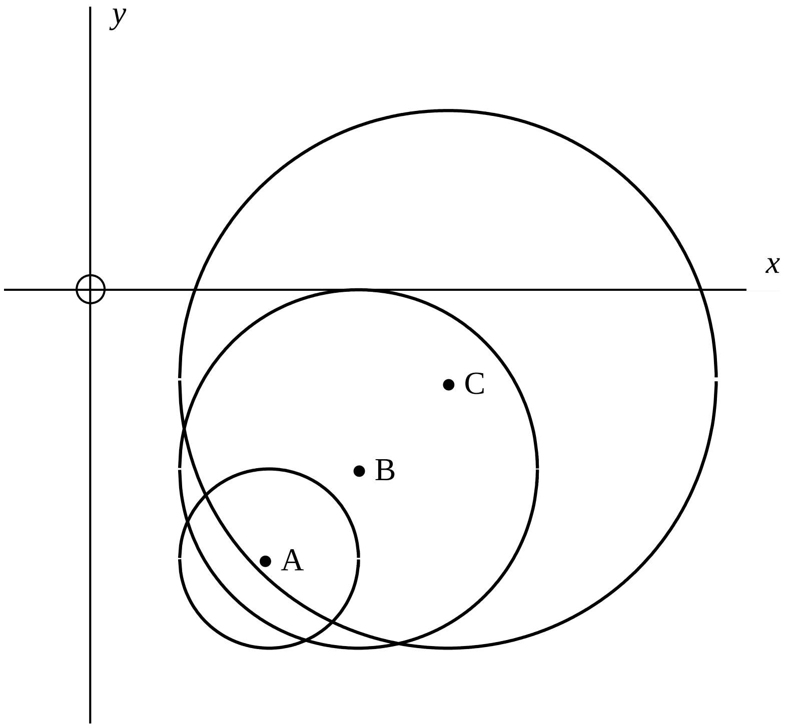


[13]

QUESTION 11

Three circles are sketched below, with centres A, B and C respectively.
The equation of the first, centred at A, is $x^2 + y^2 - 4x + 6y + 12 = 0$.

Note: The radius of the circle, centred at B, is 1 unit greater than the circle centred at A and the radius of the circle, centred at C (p; q), is 1 unit greater than the circle centred at B. Each circle centre is shifted 1 unit right and then 1 unit up to determine the next circle centre.



11.1 Determine the radius and the coordinates of the centre of the circle centred at A. (3)

11.2 Determine the equation of the circle, centred at C, in the form:

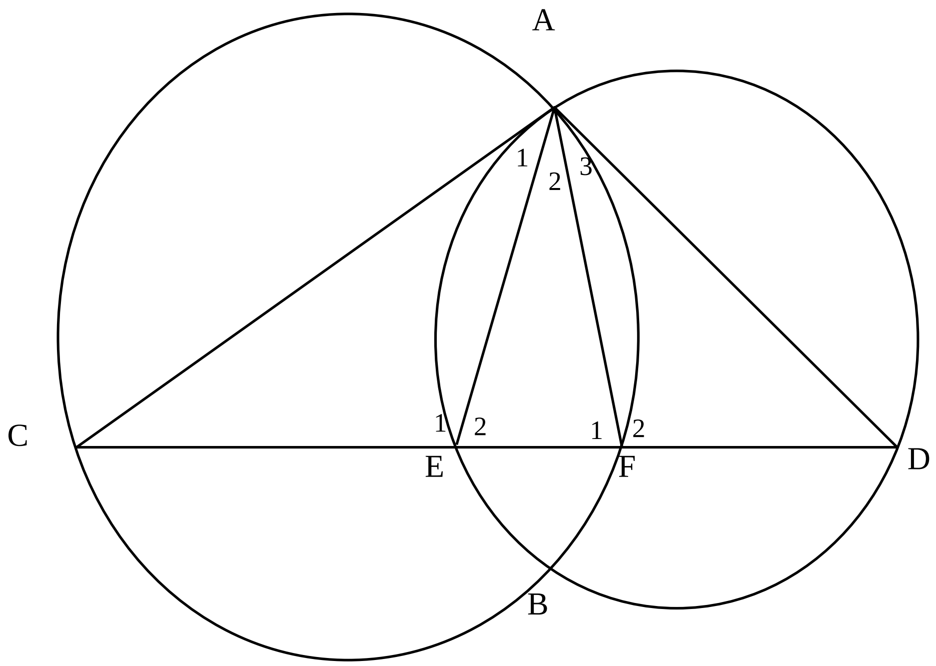
$$(x-p)^2 + (y-q)^2 = r^2$$

(4)

[7]

Question 12

Two circles intersect at A and B. AC is a tangent to circle ABD at A and AD is a tangent to the circle ACB at A. Straight line CEFD intersects the circles at E and F. AE = AF.



12.1 Prove: ΔACE /// ΔDAF (Give reasons for your answers) (3)

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12.2 Show: AC.DF = AD.AF (Give reasons for your answers) (3)

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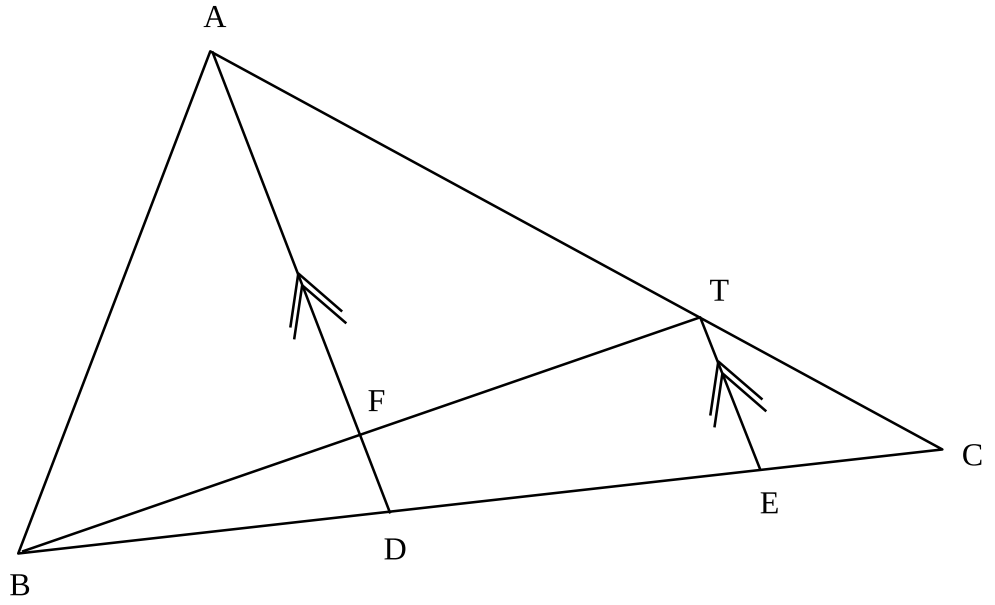
Question 13

In the figure below, ∆ABC has D and E on BC, BD = 6cm and DC = 9cm.
AT : TC = 2 : 1 and AD // TE.



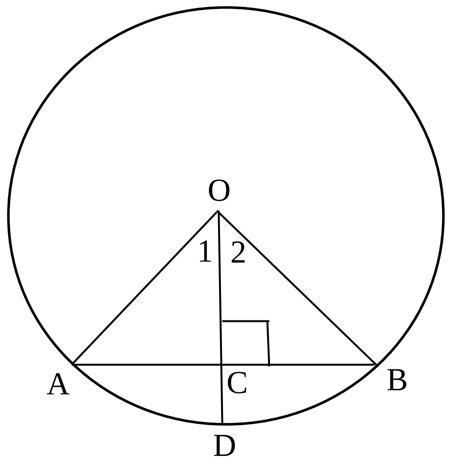
13.1 Write down the numerical value of $\frac{CE}{ED}$ (Give reasons for your answers) (2)

13.2 Show that D is the midpoint of BE. (2)

[4]

Question 14

O is the centre of the circle with radius = 1 unit. OD $\perp$ AB at C.
DC = p. $\hat{O}_1 = \hat{O}_2 = \theta$.



Giving reasons for your answers, prove that:

14.1 p = 1 – cos θ (3)

14.2 AB = 2 sin θ (2)

[5]