

## MATHEMATICS: PAPER I

TRIAL EXAMINATION

28 AUGUST 2015

TIME: 3 HOURS

TOTAL: 150 MARKS

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EXAMINATION NUMBER: \_\_\_\_\_

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### PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. Write your examination number on the paper.
2. This question paper consists of 20 pages and an Information sheet. Please check that your question paper is complete.
3. Read the questions carefully.
4. Answer **ALL** the questions on the question paper and hand this in at the end of the examination.
5. Diagrams are not necessarily drawn to scale.
6. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
7. All necessary working details must be clearly shown.
8. Round off your answers to **one decimal digit** where necessary, unless otherwise stated.
9. Ensure that your calculator is in **DEGREE** mode.
10. It is in your own interest to write legibly and to present your work neatly.

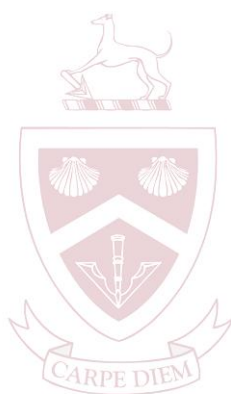
**SECTION A (38 Marks)**

Examination Number: \_\_\_\_\_

**QUESTION 1**Solve for  $x$ 

(a)  $2x^2 = 3x$

(3) (b)  $\sqrt{2x-1} + x = 2$  (4)



(c)  $\frac{4}{3} = \log_{x+2} 16$ , without the use of a calculator (4)

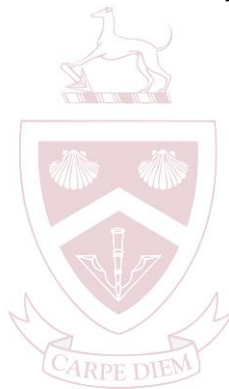
**QUESTION 2**

Simplify the following, without using a calculator:

$$\left(\sqrt{71} - \sqrt{7}\right)^{\frac{1}{3}} \left(\sqrt{7} + \sqrt{71}\right)^{\frac{1}{3}}$$

**[3]****QUESTION 3**

Solve simultaneously for  $x$  and  $y$  if  $3^y = 81^x$  and  $x^2 - 6x - y = -9$

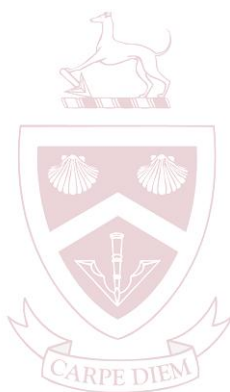
**[5]**

**QUESTION 4**

If it is given that  $A = \sqrt{12 - x^2 - 4x}$

(a) For which values of  $x$  will A be non-real? (4)

(b) Determine the maximum value of A. (3)

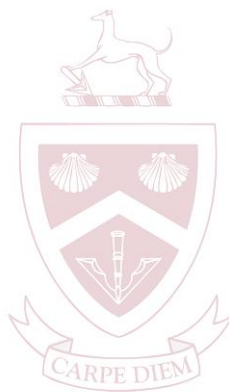
**[7]**

**QUESTION 5**

Given:  $\sum_{t=0}^{99} (3t - 1)$

(a) Write down the first three terms of the series. (1)

(b) Using an appropriate formula, calculate the sum of the series. Show all your working details. (3)

**[4]**

**QUESTION 6**

Differentiate  $f(x) = -\frac{2}{x}$  from first principles.

**[4]****QUESTION 7**

In an arithmetic sequence, the  $n^{\text{th}}$  term is given as  $T_n$  and the sum of the first  $n$  terms is  $S_n$ .

$$T_{10} - T_9 = 6$$

It is given that:

$$S_{10} - S_9 = 57$$

Find the value of  $T_1$ .

**[4]**

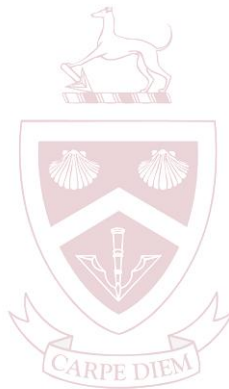
**SECTION B (36 Marks)**

Examination Number: \_\_\_\_\_

**QUESTION 8**

In a converging geometric series  $S_{\infty} = \frac{40}{3}$  and  $T_2 = \frac{5}{2}$ .

Calculate the possible value(s) of the first term of the series.

**[6]**

**QUESTION 9**

$$f(x) = ax^2 + bx + c \text{ and } g(x) = \frac{k}{x-p} + q$$

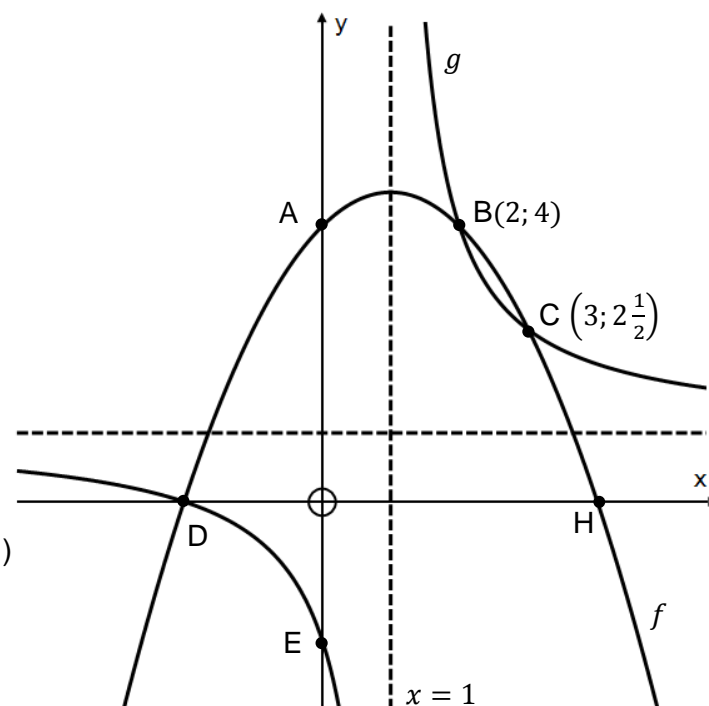
are sketched alongside.

Points A and B are symmetrical to each other in the line  $x = 1$ .

$f$  and  $g$  intersect each other at  $B(2; 4)$  ;  
 $C\left(3; 2\frac{1}{2}\right)$  and D.

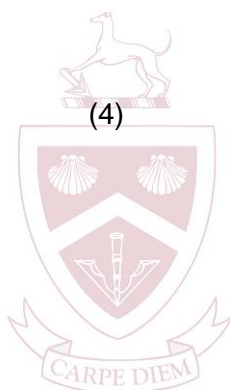
(a) Write down the value of  $p$

(1)



(b) Determine the values of  $k$  and  $q$

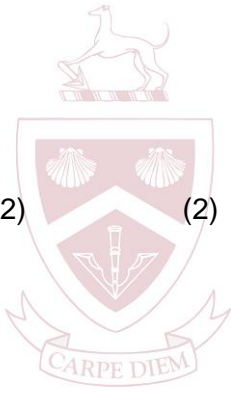
(4)



(c) Write down the co-ordinates of A.

(1)

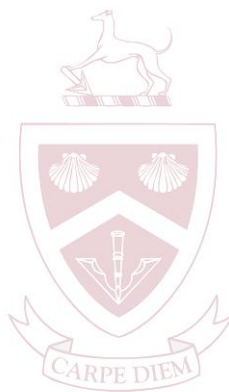


- (d) Determine the co-ordinates of D. (2)
- (e) Determine the values of  $a$  and  $b$ . (4)
- (f) For which values of  $x$  is :
- (1)  $f(x) \geq 0$  (2)  $f(x) < g(x)$  (3)
- 
- (3)  $\frac{f(x)}{g(x)} = 1$  where  $x > 0$  (2)

**QUESTION 10**

Given  $f'(x) = \frac{1}{\sqrt{x}}$  and  $f(4) = 3$

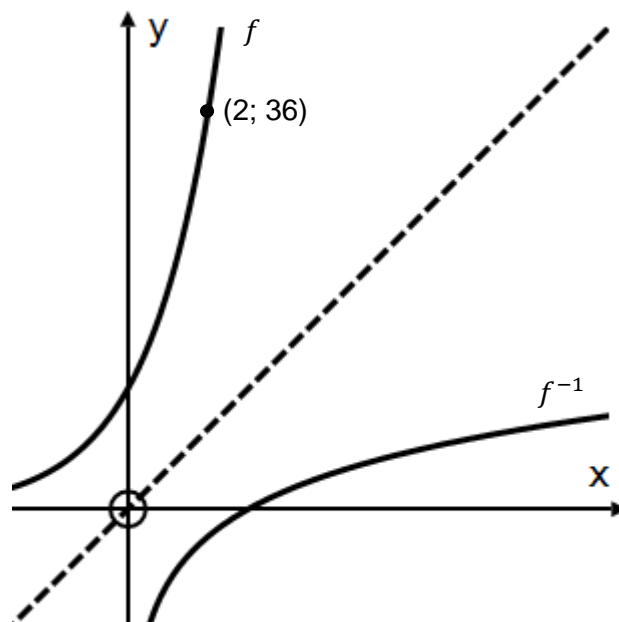
Find the equation of the *normal* (a line perpendicular to the tangent) to the graph of  $f$  at  $x = 4$ .

**[5]**

**QUESTION 11**

Sketched are the graphs of  $f(x) = a^x$ , where  $a > 0$  and  $y = f^{-1}(x)$ .

The point  $(2; 36)$  is on  $f$



(a) Determine the value of  $a$ . (2)

(b) Give the equation of  $f^{-1}(x)$  in the form  $y = \dots\dots\dots$  (1)

(c) For which value(s) of  $x$  is  $f^{-1}(x) \leq 0$ ? (1)



(d) Write down the domain of  $h(x)$  if  $h(x) = f^{-1}(x - 2)$  (2)

**[6]**

**SECTION C (35 Marks)**

Examination Number: \_\_\_\_\_

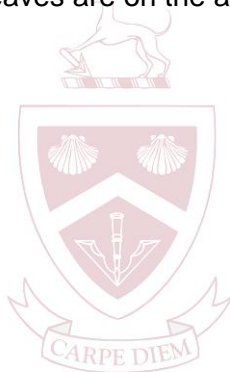
**QUESTION 12**

Frank sets off on a camping trip. He heads south and sets up his tent in the Addo Elephant Park. He opens the information booklet and analyses some of the information about the Eastern Cape Aloe.

	End of First year	End of second year	End of third year	End of fourth year
<b>Number of leaves on Aloe</b>	2	$x$	$2x + 1$	$4x$

Frank suspects that the pattern has a constant second difference.

- (a) Use this fact to calculate how many leaves are on the aloe at the end of the fourth year (4)



- (b) Determine an expression for the number of leaves on the aloe at the end of the  $n^{\text{th}}$  year. (3)

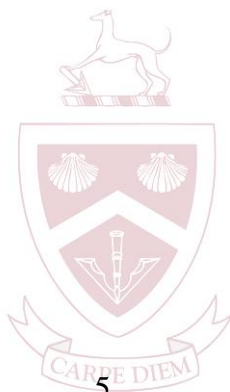
**QUESTION 13**

$$f(x) = -x^3 + 5x^2 + 8x - 12$$

(a) Show that  $(x - 1)$  is a factor of  $-x^3 + 5x^2 + 8x - 12$ . (2)

(b) Find the  $x$ - and  $y$ -intercepts of  $f$ . (2)

(c) Calculate the co-ordinates of the turning points of  $f$ . (4)



(d) Show that  $f$  has a point of inflection at  $x = \frac{5}{3}$ . (2)

(e) If  $y = 15x + c$  is a tangent to  $f$  at  $(a ; b)$ , determine the value of  $c$  if  $a, b \in Z$  (5)

**[15]**

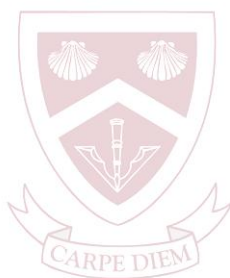
**QUESTION 14**

(a) Find  $\frac{d}{dx} \left[ \sum_{n=3}^5 (2n^2 x^2 - x) \right]$  (4)

(b) Find  $\frac{dy}{dx}$  given:

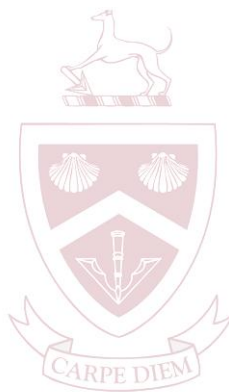
$$y = \frac{x}{3\sqrt{x^3}} + \frac{4\sqrt{x} - x^3}{2x^2}$$

(6)

**[10]**

**QUESTION 15**

Alex bought a laptop for R 12 500. It depreciated in value to R 5546,32 after 5 years. Calculate the annual depreciation rate using a reducing balance.

**[3]**

**SECTION D (41 Marks)**

Examination Number: \_\_\_\_\_

**QUESTION 16**

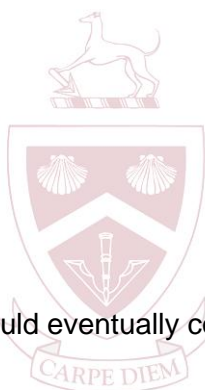
A couple take a mortgage loan on a house.  
The plan is to repay the loan monthly over a period of 30 years.

The value of the loan is R 500 000 and the interest is  
9% p.a., compounded monthly.

- (a) Calculate the monthly payment. (4)



- (b) What is the total amount the house would eventually cost? (2)



- (c) After 28 years the couple wants to clear the account. What would be the outstanding balance of the account? (4)

**[10]**



**QUESTION 17**

(a) Events A and B are mutually exclusive.

It is given that:

- $P(B) = 2P(A)$
- $P(A \text{ or } B) = 0,57$

Calculate  $P(B)$

(3)

(b) Given the word **S U M M E R**

Work out the factorials – e.g.  $4! = 24$

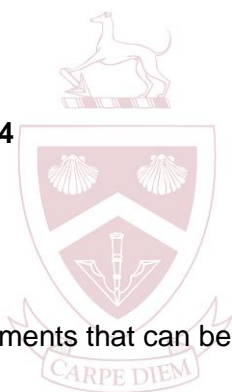
Determine:

i) the number of 6-letter arrangements that can be made

(2)

ii) the probability that a randomly selected 'word' will have an M at each end.

(3)

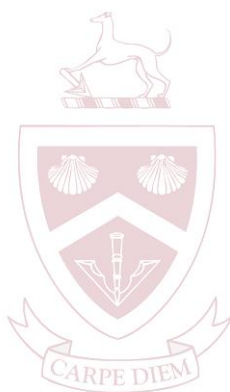


- (c) A survey was carried out to investigate the relationship between Maths results and extra Maths lessons. The results have been recorded in the table below.

		<b>A</b>	<b>A'</b>	
		<b>80% or More for Maths</b>	<b>Less than 80% for Maths</b>	<b>TOTAL</b>
<b>B</b>	<b>Extra Maths Lessons</b>	240	560	800
<b>B'</b>	<b>No Extra Maths Lessons</b>	60	140	200
<b>TOTAL</b>		300	700	1 000

Extra Maths teachers claim that learners who take extra Maths lessons are more likely to get more than 80% for Maths than those that don't. Are they correct?

Justify your answer with the necessary calculations to test for independence.



(5)

**[13]**

**QUESTION 18**

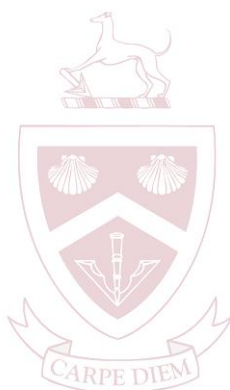
A bracelet is made by using 10 spheres and 10 cylinders. The radii,  $r$ , of the spheres and the cylinders are exactly the same. The height of each cylinder is  $h$ . The spheres and cylinders are to be coated in coloured paint. (Ignore the holes in the spheres and cylinders).



$V = \pi r^2 h$	$S = 2\pi r^2 + 2\pi r h$
$V = \frac{4}{3}\pi r^3$	$S = 4\pi r^2$

- (a) If  $h = \frac{6}{\pi r^2}$ , show that the total surface area ( $S$ ) of all the painted surfaces of the bracelet is equal

to: 
$$S = 60\pi r^2 + \frac{120}{r} \quad (3)$$



- (b) Determine the value of  $r$  so that the least amount of paint is used. (4)

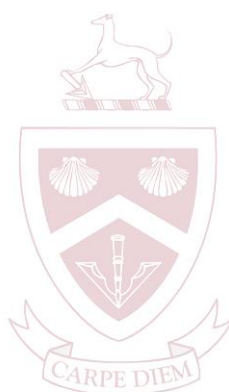
**[7]**

**QUESTION 19**

Andre is required in a test to find the derivative of a function  $f(x)$ .  
However, by mistake he finds the inverse instead.  
He finds that:

$$f^{-1}(x) = \sqrt[3]{\frac{x+7}{2}}.$$

Find the correct answer to the problem



[4]

**QUESTION 20**

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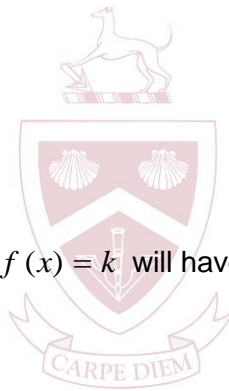
f

tics Paper I

The sketch of  $f(x) = x^3 - 3x - 2$  is given:

- (a) If  $h(x) = f(-x)$ , give the equation of  $h(x)$  (3)

- (b) Write down the values of  $k$  for which  $f(x) = k$  will have only one real root. (2)



- (c) Write down the values of  $x$  for which  $f(x) \cdot f'(x) > 0$  (2)

[7]