KEARSNEY COLLEGE
Founded in 1921

MATHEMATICS: PAPER II

TRIAL EXAMINATION **11 SEPTEMBER 2015**

TIME: 3 HOURS **TOTAL: 150 MARKS**

MEMO

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

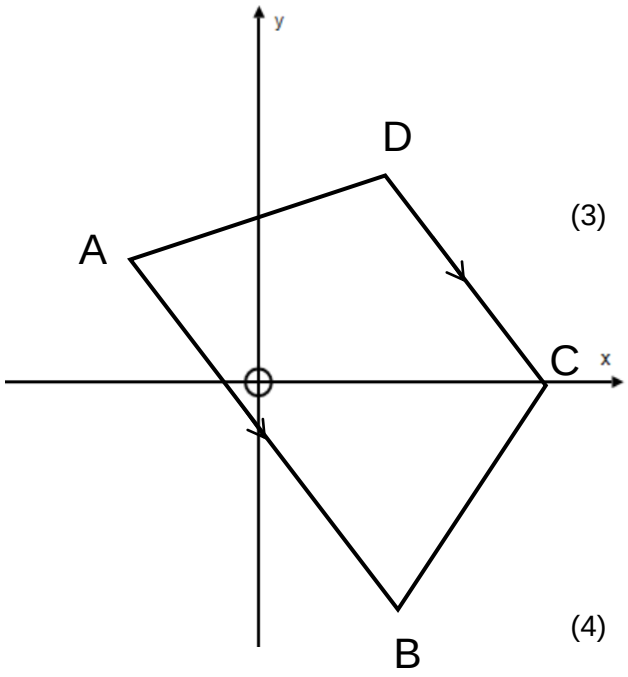
1. Write your examination number on the paper.
2. This question paper consists of **21 pages** and an Information sheet.
   Please check that your question paper is complete.
3. Read the questions carefully.
4. Answer **ALL** the questions on the question paper and hand this in at the end of the examination.
5. Diagrams are not necessarily drawn to scale.
6. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
7. All necessary working details must be clearly shown.
8. Round off your answers to **one decimal digit** where necessary, unless otherwise stated.
9. Ensure that your calculator is in **DEGREE** mode.
10. It is in your own interest to write legibly and to present your work neatly.

SECTION A (38 MARKS)

EXAMINATION NUMBER:_____

QUESTION 1

A $(-2;1)$, B $(p;\ -4)$, C$(5;0)$ and D $(3;2)$ are vertices of a trapezium ABCD with AB//DC.



(a) Show that $p = 3$. (3)

$$\frac{-4-1}{p+2} = \frac{2-0}{3-5}$$

$$-\frac{5}{p+2} = -1$$

$$\therefore p = 3$$

(b) Calculate AB:CD in its simplest form (4)

$$AB = \sqrt{(-2-3)^2 + (1+4)^2} = 5\sqrt{2}$$

$$CD = \sqrt{(5-3)^2 + (0-2)^2} = 2\sqrt{2}$$

$$AB:CD = 5\sqrt{2}:2\sqrt{2} = 5:2$$

(c) N $(x;y)$ lies on the line AB so that NBCD is a parallelogram. Determine the coordinates of N. (2)

$$(1;\ -2)$$

(e) R $(-1;q)$ is collinear with A and C. Determine the value of q. (2)

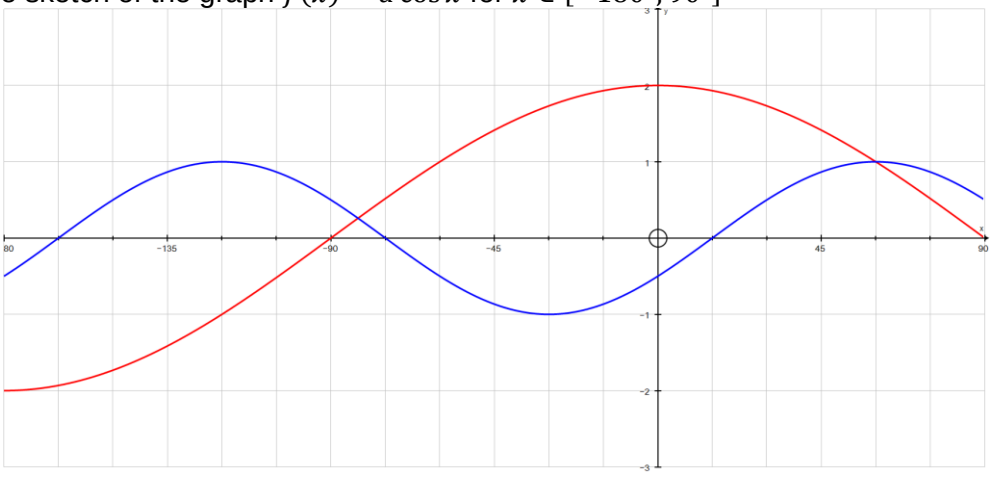
$$\frac{1}{-7} = q - 1$$

$$q = \frac{6}{7}$$

[11]

QUESTION 2

Below is the sketch of the graph $f(x) = a\cos x$ for $x \in [-180°; 90°]$



(a) Write down the value of a. (1)

$$a = 2$$

(b) Sketch the graph of $g(x) = \sin(2x - 30°)$ on the axes above for $x \in [-180°; 90°]$ (3)

(c) Write down the period of g. (1)

$$180°$$

(d) Use the graph to determine the value of x if $f(x) = g(x)$ for $x \in [0°;\ 90°]$ (1)

$$x = 60°$$

(e) For what values of x is $\frac{f(x)}{g(x)} \leq 0$ for $x \in [-180°; -90°]$ (3)

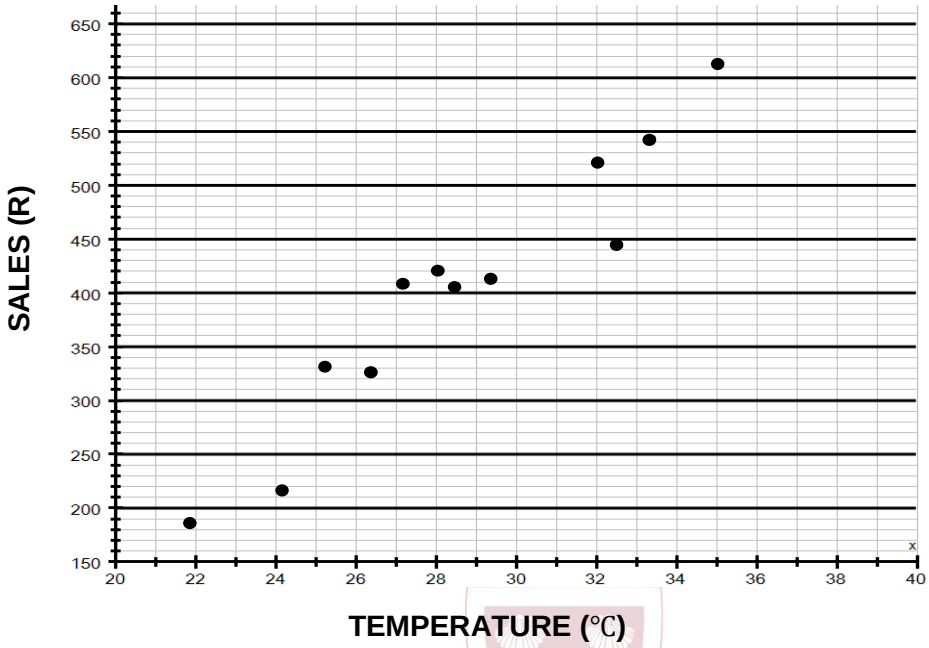
$$x \in (-165°; -90°]$$

[9]

QUESTION 3

On a certain summers day in Durban, the temperature was recorded at 12 different intervals together with the sales of ice cream at that time of day as shown in the table below:

Temperature in ℃	24,2	26,4	21,9	25,2	28,5	32,1	29,4	35,1	33,4	28,1	32,6	27,2
Sales of ice cream in Rands	215	325	185	332	406	522	412	614	544	421	445	408



(a) Describe the influence of temperature on the sales of ice cream in the scatter plot. (1)

As the temperature increases so does ice cream sales.

(b) Give a reason why this trend cannot continue indefinitely. (1)

Temperature cannot increase indefinitely.
Stock will run out

(c) Calculate an equation for the least squares regression line. (line of best fit) (2)
Round off your coefficients to 1 decimal digit.

$$y = 30{,}1x - 460{,}4$$

(d) Calculate the correlation coefficient and comment on the relationship between the 2 variables. (2)

$$r = 0{,}9575 \quad Strong\ positive\ correlation$$

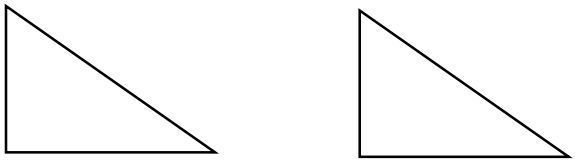
(e) Use your regression line to predict the sales of ice cream when the temperature reached 31℃. (1)

$$y = 30{,}1(32) - 460{,}4 =$$

[7]

QUESTION 4

(a) If A and B are acute angles such that $\sin A = \frac{1}{\sqrt{5}}$ and $\sin B = \frac{1}{\sqrt{10}}$, determine the value of $\sin(A + B)$ without solving for A and B. (5)



$$\sin(A + B) = \sin A \cos B + \sin B \cos A$$

$$= \frac{1}{\sqrt{5}} \times \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{10}} \times \frac{2}{\sqrt{5}}$$

$$= \frac{1}{\sqrt{2}}$$

(b) Solve for x if $\cos(200° + x) = \sin 120°$ where $x \in [-180°; 180°]$ (5)

$$\cos(200° + x) = \cos 30°$$

$$200° + x = \pm 30 + k360°$$

$$x = -170° + k.360° \qquad x = -230° + k.360°$$

$$x = \{-170°; 130°\}$$

[10]

SECTION B (38 MARKS)

EXAMINATION NUMBER:_____

QUESTION 5

In a survey, 50 university students were asked, "How many subject 'A's did you achieve in Matric?" Their results are recorded in the table below.

Number of subject 'A's	1	2	3	4	5	6	7
Number of students	5	3	7	8	9	10	8

(a) Calculate the mean and standard deviation of the distribution. (2)

$$\bar{x} = 4{,}5 \qquad sd = 1{,}9$$

(b) A normal distribution has approximately 68% of its data within one standard deviation of its mean. Use your answers in (a) to determine whether the above data is normally distributed. Show all working details. (3)

$$UB = 4{,}5 + 1{,}9 = 6{,}4$$

$$LB = 4{,}5 - 1{,}9 = 2{,}6$$

$$7 + 8 + 9 + 10 = 34 = 68\%$$

$$Normally\ distributed$$

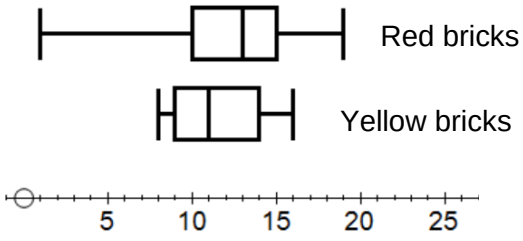
[5]

QUESTION 6

A builder can choose between two different types of brick that are coloured red or yellow. The box and whisker diagrams below illustrate the results of tests on the strength of the bricks.

(a) Give a reason why the builder might prefer to use red bricks. (1)

- $Higher\ median$
- $Higher\ max$
- $Negatively\ skewed - higher\ mean$



(b) Give a reason why the builder might prefer to use the yellow bricks. (1)

- $Range\ is\ smaller$
- $Higher\ minimum$
- $Better\ dependability - smaller\ range$

[2]

QUESTION 7

Students were asked to report how far (in miles) they each live from school. The following distances were recorded.

	Student	**Distance**
1	Zane	0,4
2	Jackson	0,5
3	Benjamin	1,0
4	Bethany	1,0
5	Joe	2,4
6	Noelle	2,7
7	Tianye	3,0
8	Anthony	3,2
9	Amanda	4,1
10	Michaela	4,2
11	Miranda	4,4
12	Joseph	5,0
13	John	9,5

A summary of statistics for the distances are given below.

Minimum	**1st Quartile**	**2nd Quartile**	**3rd Quartile**	**Maximum**
0,4	1, 0	3, 0	4, 3	9,5

A data value can be considered an "outlier" if it is more than 1.5 times the IQR above Q_3 or more than 1.5 times the IQR below Q_1.

Using this description of an outlier, was John's distance from school considered an outlier?

$$IQR = 3{,}3$$

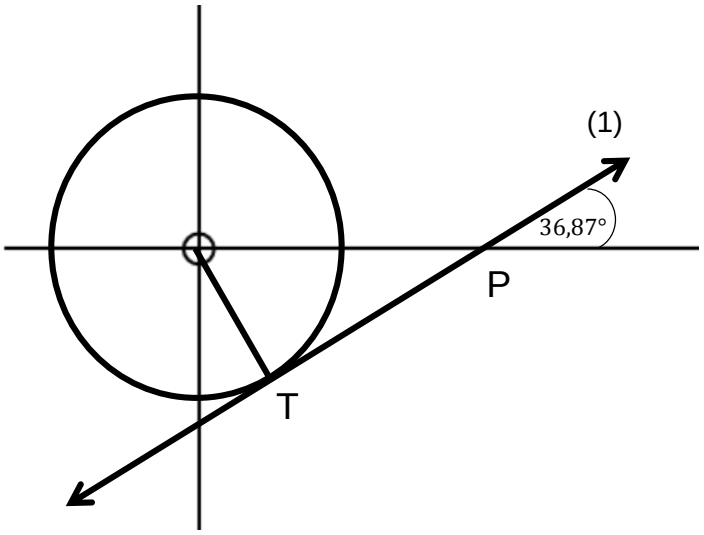
$$UB = 4{,}3 + 1{,}5(3{,}3) = 9{,}25$$

$$Johns\ disntacne > 9{,}25\ -\ outlier$$

[4]

QUESTION 8

The figure shows circle O centred at the origin with radius 15 units.
The line TP touches the circle at T and meets the x –axis at an angle of $36{,}87°$.



(a) Write down the equation of the circle. (1)

$x^2 + y^2 = 225$

(b) Calculate the length of OP. (3)

$$\sin\theta = \frac{15}{OP}$$

$$OP = \frac{15}{\sin 36{,}87} = 25$$

$$P(25; 0)$$

(c) Find the equation of the line TP. (2)

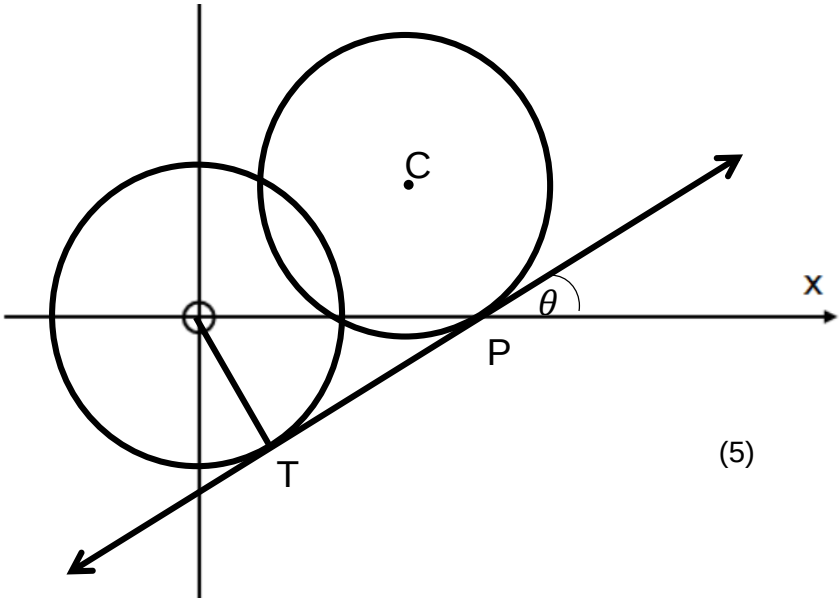
$$y = mx + c$$

$$m = \tan 36{,}87 = \frac{3}{4}$$

$$y = \frac{3}{4x} + \frac{75}{4}$$

(d) Another circle with centre C and radius 15 is drawn to touch TP at P. (1)
Write down the equation of the line OC.

$y = \frac{3}{4}x$



(e) Find the equation of circle C. (5)

$$m_{CP} = -\frac{4}{3}$$

$$0 = -\frac{4}{3}(25) + c$$

$$c = \frac{100}{3}$$

$$y = -\frac{4}{3}x + \frac{100}{3}$$

$$y = \frac{3}{4}x$$

$$x = 16 \qquad y = 12$$

$$(x - 16)^2 + (y - 12)^2 = 225$$

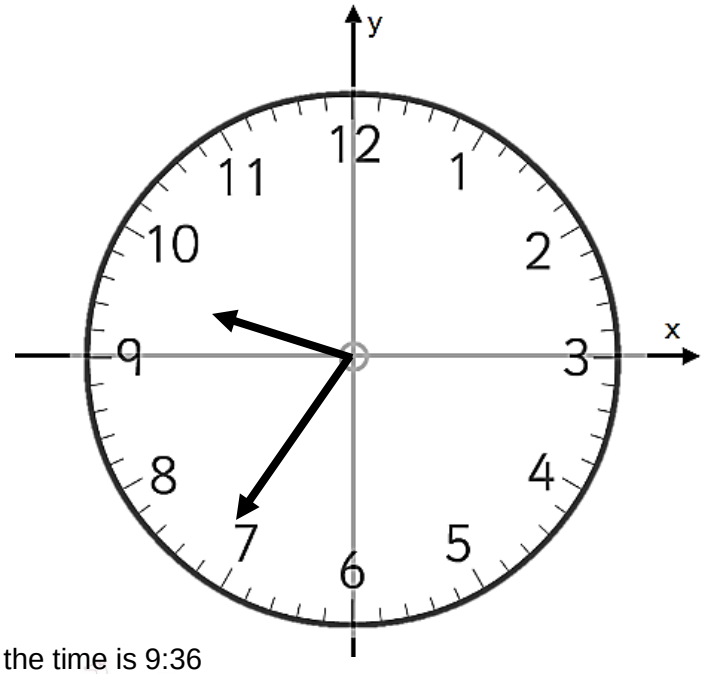
[12]

QUESTION 9

The length of the hour hand of a watch is 90mm and the length of the minute hand is 120mm

(a) Through how many degrees will the minute hand move from 9:35 to 9:37 (1)

$12°$



(b) If the hands on the watch shows that the time is 9:36

(1) Determine the acute angle that the hour hand makes with the $x-$ axis. (1)

$18°$

(3) Determine the distance from the ends of the hand at this time. (3)

$$angle\ between\ hands = 72°$$

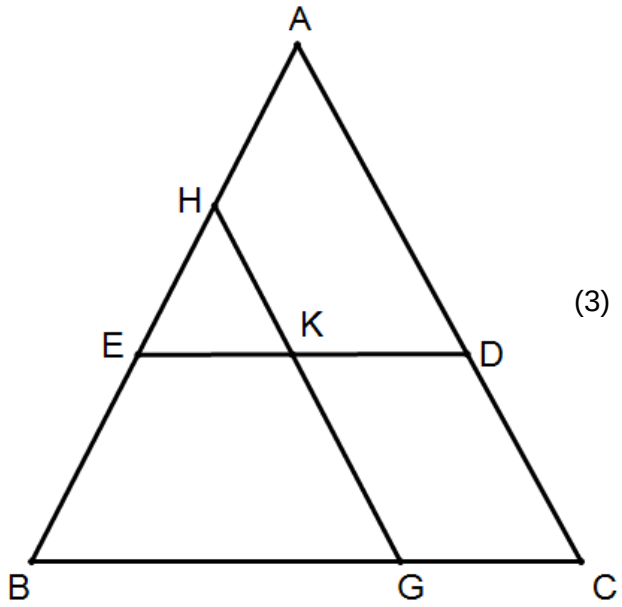
$$x^2 = 90^2 + 120^2 - 2(90)(120)\cos 72°$$

$$x = 125{,}8\ mm$$

[5]

QUESTION 10

In ΔABC, $HG \parallel AC$, $ED \parallel BC$, $\frac{AD}{DC} = \frac{3}{2}$ and $\frac{BG}{GC} = \frac{2}{1}$
If $AB = 15cm$, determine:



(a) The length of AE. (3)

$\frac{AE}{AB} = \frac{3}{5}$ $\ldots(Prop\ Theorem\ ED \parallel BC)$

$\therefore AE = 9$

(b) The length of AH. (4)

$\frac{AH}{HB} = \frac{1}{2}$ $\ldots(Prop\ Theorem\ HG \parallel AC)$

$\frac{AE}{EB} = \frac{3}{2}$ $\ldots(Prop\ Theorem\ ED \parallel BC)$

$3p = 15$

$AH = 5cm$

(c) The value of $\frac{GK}{KH}$ (3)

$\frac{GK}{KH} = \frac{BE}{EH} = \frac{3}{2}$ $\ldots$ $(Prop\ Theorem, EK \parallel BG)$

[10]

SECTION C (35 MARKS)

EXAMINATION NUMBER:_____

QUESTION 11

For each of the following questions, circle the most correct answer.

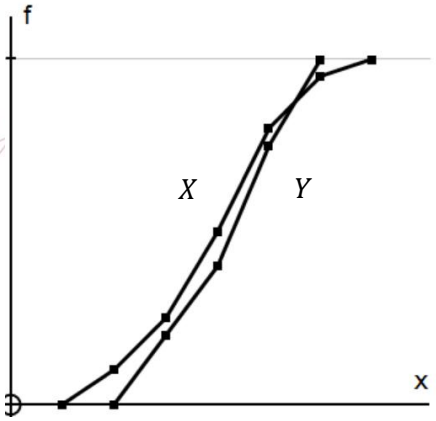
(a) The equation of the a circle is $2x^2 + 2y^2 + 8x - 12y + 3 = 0$.

Which of the following is/ are true?

i. The coordinates of the centre are $(-2; 3)$
ii The radius of the circle is 7 units.
iii. The point $(2; 3)$ lies outside the circle.

A. i and ii only
(B.) i and iii only
C. ii and iii only
D. i, ii and iii

(b) The figure below shows the cumulative frequency graphs for 2 test scores (X and Y) of the same students in a class. The median (m), range (r) and standard deviation(s) for each data set is compared.



Which of the following is/ are true?

i. $m_X > m_Y$
ii. $r_X > r_Y$
iii. $s_X > s_Y$

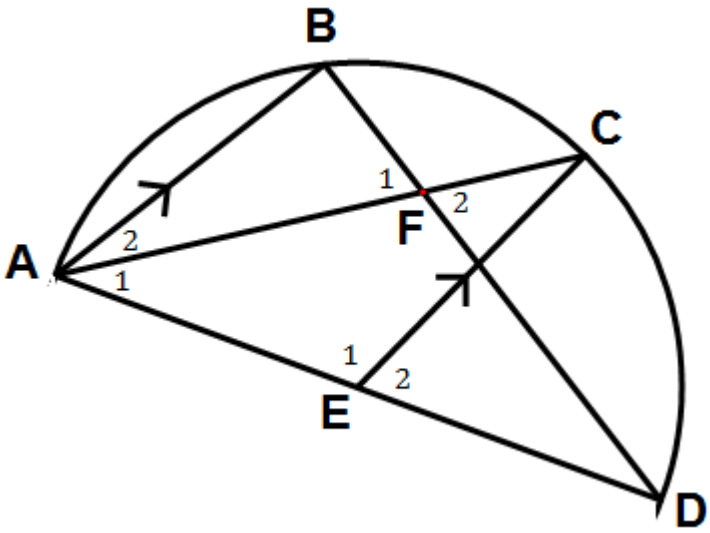
A. i and ii only
B. i and iii only
(C.) ii and iii only
D. i, ii and iii

[6]

QUESTION 12

In the figure, E is the centre of the semi-circle ABCD. AC and BD intersect at F.

AB // EC and $\hat{D} = 30$



(a) Write down the size of $\hat{B}$. (1)

$\hat{B} = 90°$ ... $(\angle\, in\ semi\ circle)$

(b) Determine the size of $\hat{F}_1$. (5)

$\hat{A} = 60°$ ... $(\angle\, sum\ \Delta)$

$\hat{E}_2 = 60°$ ... $(corresp\ \angle =; AB \parallel EC)$

$AE = CE$ ... $(radii)$

$\hat{C} = 30°$ ... $(ext\ \angle\ \Delta)$

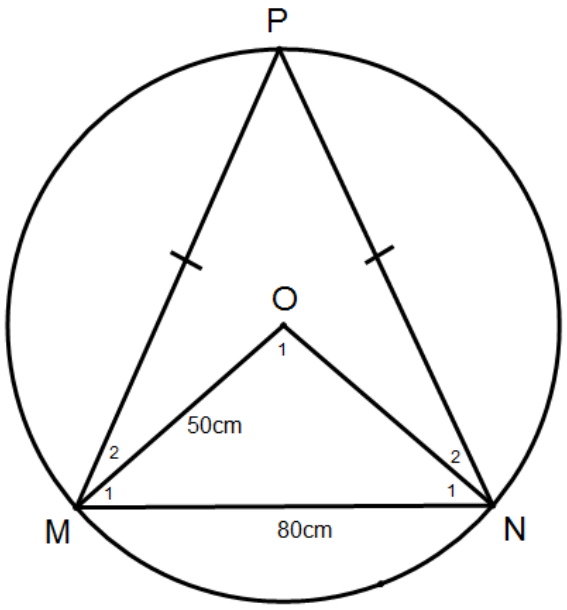
$\hat{A}_2 = 30°$ ... $(alt\ \angle =; AB \parallel CE)$

$\therefore\ \hat{F}_1 = 60°$ ... $(\angle\, sum\ \Delta)$

[6]

QUESTION 13

O is the centre of the circle and P, M and N lie on the circle. MN = 80cm. The radius of the circle is 50cm. PM = PN.



Determine:

(a) The size of $\hat{O}_1$, rounded to the nearest degree. (3)

$ON = 50cm$

$$\hat{O}_1 = \cos^{-1}\left(\frac{50^2 + 50^2 - 80^2}{2(50)(50)}\right) = 106°$$

(b) The length of PM, rounded off to one decimal digit. (4)

$$\hat{P} = 53°$$

$$\hat{N} = \frac{180 - 53}{2} = 63{,}5°$$

$$\frac{PM}{\sin 63{,}5} = \frac{80}{\sin 53}$$

$$PM = 89{,}6\ cm$$

(c) The area of ΔPMN, rounded off to the nearest cubic centimetre. (2)

$$Area = \frac{1}{2}(89{,}6)(89{,}6)\sin 53 \ = \ 3206cm^2$$

Or

$$Area = \frac{1}{2}(80)(89{,}6)\sin 63{,}5 \ = \ 3207cm^2$$

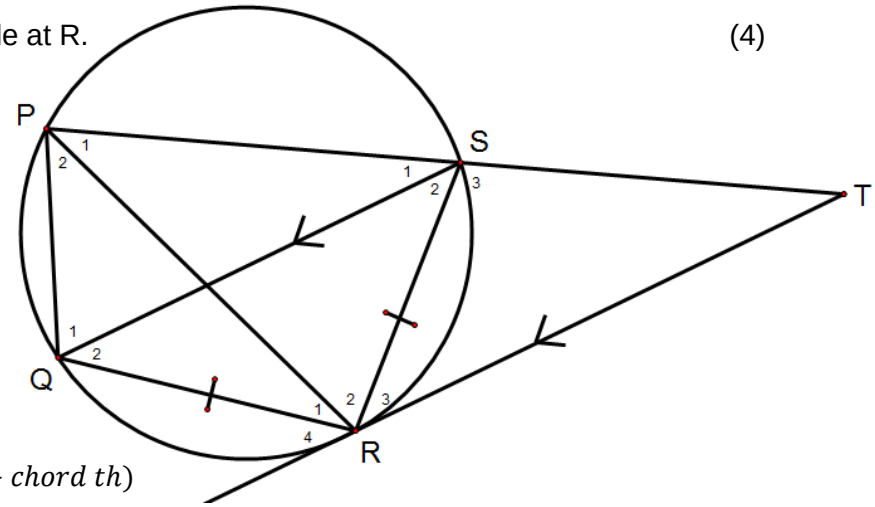
Or 3209

[9]

QUESTION 14

In the diagram, PQRS is a cyclic quad with RS = QR. RT // QS. Let $\hat{R}_3 = x$

(a) Prove that RT is a tangent to the circle at R. (4)



$\hat{R}_3 = \hat{S}_2$ ... $(alt \angle =; RT \ /\!/ \ QS)$

$\hat{S}_2 = \hat{Q}_2$ ... $(isos\ \Delta)$

$\therefore \hat{R}_3 = \hat{Q}_2$

$\therefore RT\ is\ a\ tangent$ ... $(converse\ tan - chord\ th)$

(b) Prove that $\hat{R}_1 = \hat{T}$ (3)

$\hat{R}_1 = \hat{S}_1$ ... $(\angle\ in\ same\ seg)$

$\hat{S}_1 = \hat{T}$ ... $(corresp\ \angle; QS \parallel TR)$

$\therefore\ \hat{R}_1 = \hat{T}$

(c) Prove that $\Delta\ RST \ /\!/\!/ \ \Delta PQR$ (4)

$In\ \Delta\ RST \ /\!/\!/ \ \Delta PQR$

1) $\hat{T} = \hat{R}_1$ ... $(proved)$

2) $\hat{S}_3 = \hat{Q}$ ... $(ext\ \angle\ of\ cyclic\ quad)$

3) $\hat{R}_3 = \hat{P}_2$ ... $(rem\ \angle)$

$\therefore \Delta\ RST \ /\!/\!/ \ \Delta PQR$ ... (AAA)

(d) If PQ= $4cm$ and ST= $9cm$, find the length of QR. (3)

$$\frac{PQ}{RS} = \frac{QR}{ST} = \frac{PR}{RT}$$

$$\therefore \frac{QR}{9} = \frac{4}{QR} \quad \ldots QR = SR$$

$$QR^2 = 36$$

$$QR = 6cm$$

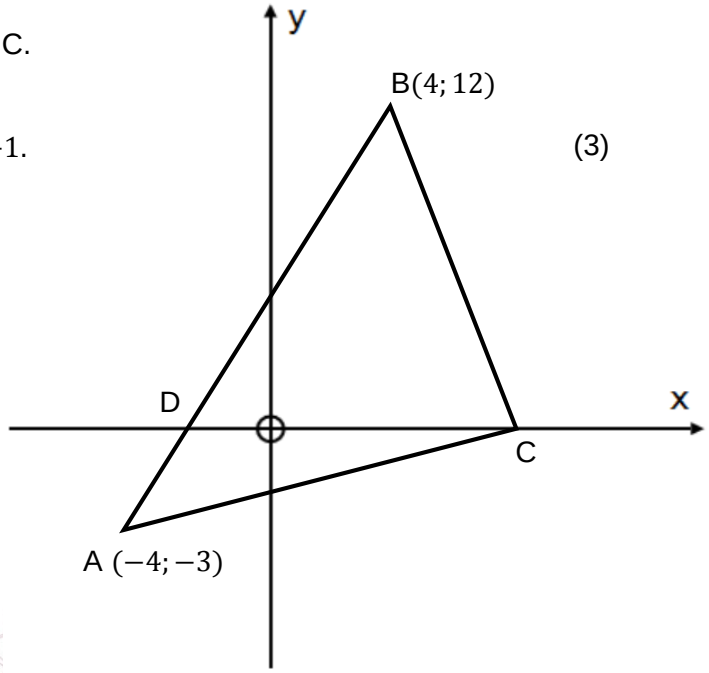
[14]

SECTION D (40 MARKS)

EXAMINATION NUMBER:_____

QUESTION 15

A is the point $(-4;\ -3)$ and B is the point $(4; 12)$.
AB meets the x –axis at D and BC meets the x –axis at C.



(a) If $A\hat{B}C = 73{,}1°$, show that the gradient of BC is -1. (3)

$\tan \widehat{D}_1 = \frac{15}{8}$

$\widehat{D}_1 = 61{,}9°$

$B\hat{C}X = 61{,}9 + 73{,}1 = 135°$

$m_{BC} = \tan 135 = -1$

(b) Hence, or otherwise, find the co-ordinates of C. (3)

$\frac{12}{4 - x} = -1$

$x = 16$

$C(16; 0)$

(c) $\Delta\, ABC$ is rotated clockwise about point C until CA lies along the x –axis. What is the inclination of AB to the x – axis in this new position? (3)

$$m_{AC} = \frac{-3 - 0}{-4 - 12} = \frac{3}{16}$$

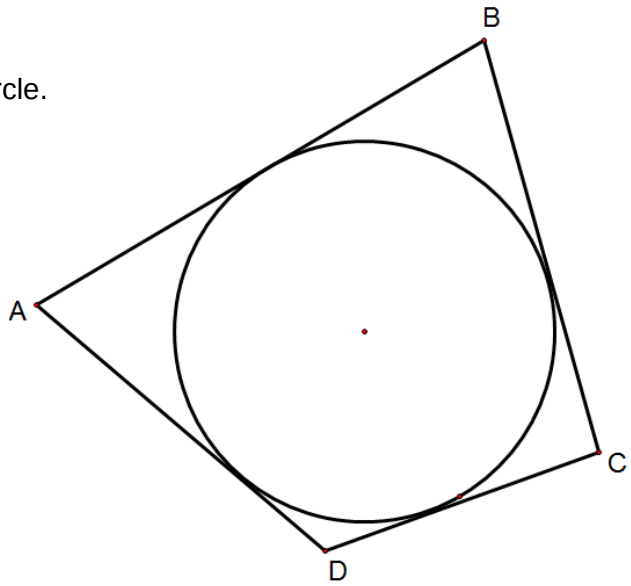
$$D\hat{C}A = 8{,}5°$$

$$\widehat{D}_1 = 61{,}9 - 8{,}5 = 53{,}4°$$

[9]

QUESTION 16

In the diagram, a circle is inscribed in a quadrilateral ABCD where AB, BC, CD and AD are tangents to the circle.
If BC $= 28$ and AD$= 22$



(a) Complete the statement:

Tangents to the circle from the same point are:

Are equal in length

(b) Determine the perimeter of ABCD.

$x + q = 28$
$y + p = 22$

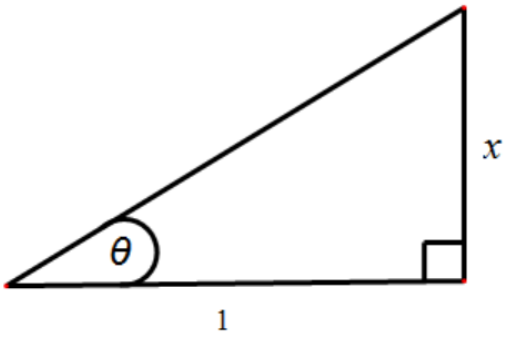
$x + y + p + q = 50$

$Perimeter = 100$

[4]

QUESTION 17

Given that $\tan\theta = x$,



(a) Show that $\sin 2\theta = \frac{2x}{x^2+1}$ (3)

$2\sin\theta\,.\cos\theta$

$= 2\frac{x}{\sqrt{x^2+1}} \times \frac{1}{\sqrt{x^2+1}}$

$= \frac{2x}{x^2+1}$

(b) Find the maximum value of $\frac{(x+1)^2}{x^2+1}$ (3)

$\frac{x^2+2x+1}{x^2+1}$

$= \frac{x^2+1}{x^2+1} + \frac{2x}{x^2+1}$

$= 1 + \sin 2\theta$

$Max = 2$

[6]

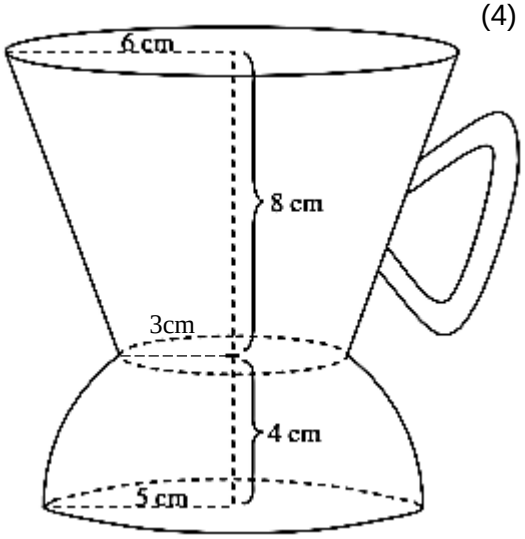
QUESTION 18

In the figure an empty coffee cup consists of 2 portions. The lower portion is half a sphere of radius, 5cm, with a portion removed. The upper portion is a frustum of a cone. The height of the frustum is 8cm and the radius of the top of the frustum is 6cm and the bottom is 3cm.

Coffee is meant to fill the bottom, from the spherical base.

$$V = \frac{4}{3}\pi r^3 \qquad V = \frac{1}{3}\pi r^2 h$$

(a) Show that the height of the original cone that the frustum comes from is 16cm. (4)



$\frac{x}{x+8} = \frac{3}{6}$ $\ldots (Prop\ theorem)$

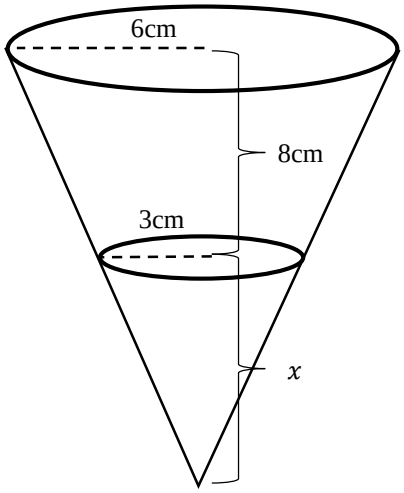
$6x = 3x + 24$

$x = 8$

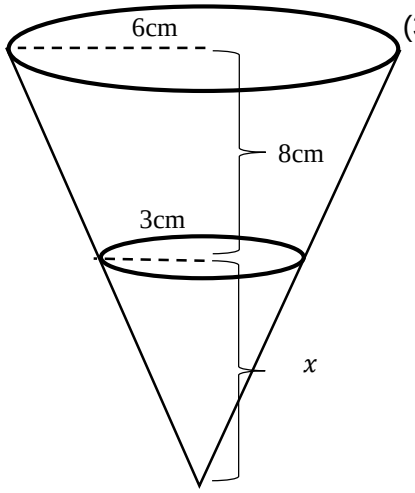
$original\ height = 16cm$

Or

Using trig



(b) Determine the volume of the frustum. (3)



$$Volume = \frac{1}{3}\pi(6)^2(16) - \frac{1}{3}\pi(3)^2(8) = 168\pi$$

(c) Determine the total volume of liquid that the cup could hold if the portion removed from the sphere is $\frac{14}{3}\pi$ (3)

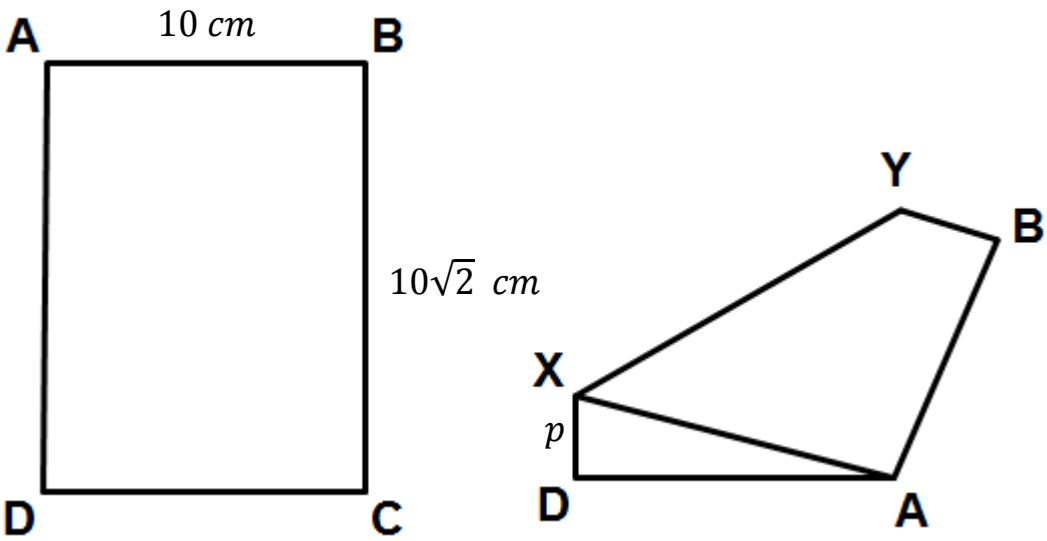
$$Volume = 168\pi - \frac{14}{3}\pi + \frac{4}{3}\pi(5)^3 \div 2 = \frac{740}{3}\pi$$

[10]

QUESTION 19

In the diagram, ABCD is a sheet of rectangular paper where AB $10cm$ and BC= $10\sqrt{2}cm$.
The paper is folded along XY such that A is placed on top of C.

Let XD $= p$



Find the length of XD, leave your answer in surd form.

$$x^2 + 10^2 = \left(10\sqrt{2} - x\right)^2$$

$$x^2 + 100 = 200 - 20\sqrt{x} + x^2$$

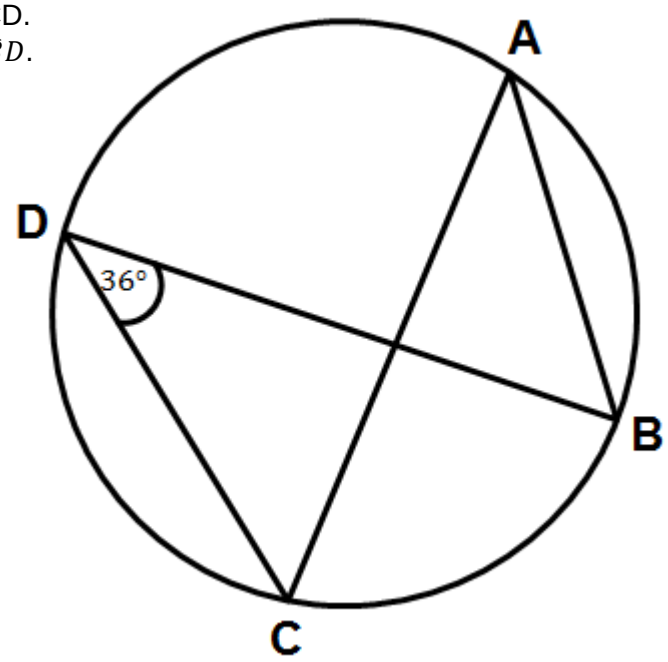
$$20\sqrt{2}x = 100$$

$$x = \frac{5}{\sqrt{2}}$$

[5]

QUESTION 20

In the figure, BD is the diameter of the circle ABCD.
If AB = AC and $\hat{D} = 36°$, determine the size of $A\hat{B}D$.



$Construct\ BC$

$D\hat{C}B = 90°$ $\ldots(\angle\ in\ semi\ circle)$

$\hat{A} = 36°$ $\ldots(\angle\ in\ same\ seg)$

$A\hat{B}C = A\hat{C}B$ $\ldots(isos\ \Delta)$

$A\hat{C}B = 72°$ $\ldots(\angle\ sum\ \Delta)$

$D\hat{C}A = 18°$

$A\hat{B}D = 18°$ $\ldots(\angle\ in\ semi\ circle)$

[6]