PRELIM P1 AUGUST 2015

1) $\sqrt{5-2x}-2x=1$

$\sqrt{5-2x}=1+2x$ ✓

$5-2x=1+4x+4x^2$ ✓

$0=4x^2+6x-4$

$0=2x^2+3x-2$ ✓

$0=(2x-1)(x+2)$

$x=\frac{1}{2}$ or $x=-2$ ✓

invalid ✓ [5]

2) a) Cost $=(2\times10^6)$ rands

(i) Deposit $=300\,000$

$\therefore$ Loan value $=1{,}7\times10^6$ ✓

$P=x\left[\frac{1-(1+i)^{-n}}{i}\right]$

$1{,}7\times10^6=x\left[\frac{1-\left(1+\frac{0{,}095}{12}\right)^{-240}}{\frac{0{,}095}{12}}\right]$ ✓ ✓

$x=R$ ✓✓ (5)

(ii) $1+i_{eff}=(1+i_{nom})^n$ ✓

$1+i_{eff}=\left(1+\frac{0{,}095}{12}\right)^{12}$ ✓

$i_{eff}=9{,}92\%$ ✓ (3)

b) $A=P(1+i)^n$

$46\,000=6000\left(1+\frac{0{,}116}{4}\right)^{4n}$ ✓ ✓

$\frac{23}{3}=\left(1+\frac{0{,}116}{4}\right)^{4n}$

$\log_{\left(1+\frac{0{,}116}{4}\right)}\frac{23}{3}=4n$ ✓

$71{,}250\ldots=4n$

$17{,}81\ldots=n$ ✓

$\therefore$ After 18 years ✓ (5)

[13]

3) a) $y=1$ ✓ (1)

b) $f(x)=k^x+1$

$3=k^{1/2}+1$ ✓

$2=k^{1/2}$

$4=k$ ✓

$\therefore f(x)=4^x+1$ ✓ (3)

c) The eqn of g is $y=a(x-p)^2+q$ ✓

$\therefore y=a(x-\frac{1}{2})^2+3$ ✓

At $(-1;0)$: $0=a(-1-1/2)^2+3$ ✓

$-3=\frac{9a}{4}$ ✓

$-\frac{4}{3}=a$ (4)

d) $\therefore y=-\frac{4}{3}(x-\frac{1}{2})^2+3$ OR $y=-\frac{4}{3}(0-\frac{1}{2})^2+3$

$=-\frac{4}{3}(x^2-x+\frac{1}{4})+3$ $=\frac{8}{3}$

$=-\frac{4}{3}x^2+\frac{4}{3}x+\frac{8}{3}$ ✓

$\therefore C(0;\frac{8}{3})$ ✓ (2)

e) $y=4^x+1$

For f^{-1}: $x=4^y+1$ ✓

$x-1=4^y$ ✓

$y=\log_4(x-1)$

$\therefore f^{-1}(x)=\log_4(x-1)$ ✓ (3)

(-1) for incorrect notation

f) $x-1>0$ ✓ $\therefore x>1$ ✓ (2)

g) (i) $g(x)\geq f(x)$ & $x\geq0$

$x\in[0;\frac{1}{2}]$ ✓✓ (2)

(ii) $f(x).g(x)\leq0$

$E(2;0)$ ✓ by symm

$x\in(-\infty;-1]\cup[2;\infty)$ ✓ ✓ (3)

[20]

4 (a) $f(x) = \frac{a}{x-p} + q$

(i) $\therefore f(x) = \frac{a}{x-1} + 3$ ✓

At $(0;1)$:

$1 = \frac{a}{0-1} + 3$

$-2 = \frac{a}{-1}$ ✓

$\therefore a = 2$

$f(x) = \frac{2}{x-1} + 3$ ✓ (3)

(ii) $0 = \frac{2}{x-1} + 3$ ✓

$0 = 2 + 3x - 3$

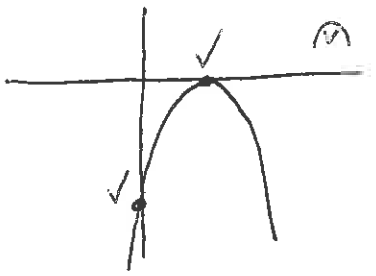
$1 = 3x$

$\frac{1}{3} = x$ ✓

$(\frac{1}{3}; 0)$ is x-intercept (2)

(b) $y = ax^2 + bx + c$

$a < 0$; $c < 0$; $b^2 - 4ac = 0$



(3)

[8]

5 (a) $S_n = \frac{n}{2}[a + l]$

$1240 = \frac{n}{2}[30 + 50]$ ✓

$1240 = 40n$ ✓

$31 = n$ ✓ (3)

(b) 3; 1; 3; 2; 3; 4; ; 8;

1; 2; 4; 8; . . . ✓

$S_n = \frac{a(r^n - 1)}{r - 1}$ ✓

$S_n = \frac{1(2^{10} - 1)}{2 - 1} = 1023$ ✓

3; 3; 3; .... (10 terms)

$\therefore S_{20} = 1053$ ✓ (4)

(c) (i) $27(x+3)^2$; $9(x+3)^3$; $3(x+3)^4$; ...

$r = \frac{x+3}{3}$ ✓

Converges if $-1 < \frac{x+3}{3} < 1$ ✓

$-3 < x + 3 < 3$

$-6 < x < 0$ ✓ (3)

(ii) $S_\infty = \frac{a}{1-r}$

$= \frac{27(-1+3)^2}{1 - \left(\frac{-1+3}{3}\right)}$ ✓ ✓

$= 324$ ✓ (3)

(d) $\sum_{k=1}^{n} T_k = 3n^2 + 2$

(i) $\therefore T_1 + T_2 + T_3 + \ldots + T_n = 3n^2 + 2$ ✓

ie. $S_n = 3n^2 + 2$

$\therefore S_6 = 3(6)^2 + 2 = 110$ ✓ (2)

(ii) $T_6 = S_6 - S_5$ ✓

$= 110 - [3(5)^2 + 2]$ ✓

$= 110 - 77$

$= 33$ ✓ (3)

[18]

6 (a) $\frac{x - \frac{1}{x}}{1 + \frac{1}{x}} = 1$

$x - \frac{1}{x} = 1 + \frac{1}{x}$ ✓ $x \neq 0$

$x^2 - 1 = x + 1$ ✓

$x^2 - x - 2 = 0$ ✓

$(x-2)(x+1) = 0$ ✓

$x = 2$ or $x = -1$ invalid ✓ (5)

6b) $x^2+4x+5-p^2=0$

$\Delta = b^2-4ac$

$= 4^2-4(1)(5-p^2)$ ✓

$= 16-20+4p^2$

$= 4p^2-4$ ✓

For non-real roots $\Delta<0$ ✓

$4p^2-4<0$

$p^2-1<0$

$(p-1)(p+1)<0$

$\therefore p\in(-1;1)$ ✓✓ (5)

c) $\dfrac{\sqrt{7^{2015}}}{\sqrt{7^{2013}}-\sqrt{7^{2011}}}$

Let $2015=k$

$\therefore \dfrac{\sqrt{7^k}}{\sqrt{7^{k-2}}-\sqrt{7^{k-4}}}$ ✓

$= \dfrac{7^{k/2}}{7^{\frac{k-2}{2}}-7^{\frac{k-4}{2}}}$ ✓

$= \dfrac{7^{k/2}}{7^{k/2}.7^{-1}-7^{k/2}.7^{-2}}$

$= \dfrac{7^{k/2}}{7^{k/2}(7^{-1}-7^{-2})}$ ✓

$= \dfrac{1}{\frac{1}{7}-\frac{1}{49}}$

$= \dfrac{1}{\frac{6}{49}}$

$= \dfrac{49}{6}$ ✓

or $\dfrac{\sqrt{7^{2013}}.\sqrt{7^2}}{\sqrt{7^{2013}}-\sqrt{7^{2013}}.\sqrt{7^{-2}}}$ (4)

$= \dfrac{\sqrt{7^{2013}}.7}{\sqrt{7^{2013}}(1-7^{-1})}$ ✓ $= \dfrac{7}{1-\frac{1}{7}}$ ✓ $= \dfrac{49}{6}$ ✓

7a)(i) $f(x) = 5x.x^{1/2} = 5x^{3/2}$ ✓

$\therefore f'(x) = \frac{15}{2}x^{1/2}$ ✓ (2)

(ii) $g(x) = \dfrac{x^2-5}{x^2}$; $x\neq 0$

$= 1-5x^{-2}$ ✓✓ (3)

$g'(x) = 10x^{-3} = \dfrac{10}{x^3}$ ✓

(b) $x^2 = 20x-4k$

$4k = 20x-x^2$

$k = 5x-\frac{1}{4}x^2$ ✓

$\dfrac{dk}{dx} = 5$ ✓ $- \dfrac{1}{2}x$ ✓ (3)

(c) $y = -2x^3+6x^2+mx+4$

$\dfrac{dy}{dx} = -6x^2+12x+m$ ✓

At stat pts, $\dfrac{dy}{dx}=0$ ✓

$\therefore -6x^2+12x+m=0$ ✓

for equal roots, $\Delta=0$ ✓ (5)

$12^2-4(-6)(m)=0$

$24m=-144$ $\therefore m=-6$ ✓

[13]

8 a) $f(x) = a(x+4)(x+1)(x-2)$ ✓

$8 = a(0+4)(0+1)(0-2)$ ✓

$8 = -8a$

$-1 = a$ ✓

$\therefore f(x) = -1(x+4)(x^2-x-2)$

$= -(x^3-x^2-2x+4x^2-4x-8)$

$= -(x^3+3x^2-6x-8)$

$= -x^3-3x^2+6x+8$. ✓

$a=-1$; $b=-3$; $c=6$; $d=8$ ✓ (5)

(8b) $f'(x) = -3x^2 - 6x + 6$

$f''(x) = -6x - 6$

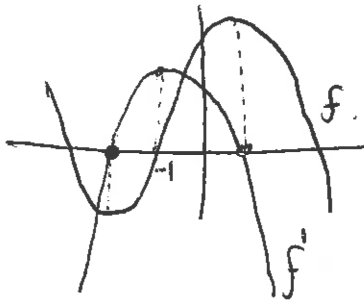
At pt of inflection $f''(x) = 0$

$-6x - 6 = 0$

$x = -1$

$\therefore$ pt of inflection $(-1; 0)$ (4)

(c)



$f'(x)$
Roots
TP

(9)(a) $V = (4x)(x)y = 4x^2y$ (1)

(b) $SA = 2xy + 2(4xy) + 4x^2$

$6 = 4x^2 + 10xy$

$6 - 4x^2 = 10xy$

$\frac{6}{10x} - \frac{4x^2}{10x} = y$

$y = \frac{3}{5x} - \frac{2x}{5}$ (4)

(c) $V = 4x^2\left(\frac{3}{5x} - \frac{2x}{5}\right)$

$= \frac{12}{5}x - \frac{8}{5}x^3$

$\frac{dV}{dx} = \frac{12}{5} - \frac{24}{5}x^2$

For max volume $\frac{dV}{dx} = 0$

$\frac{12}{5} - \frac{24}{5}x^2 = 0$

$\frac{1}{2} = x^2$

$\sqrt{\frac{1}{2}} = x$

$x = 0{,}71$m.

(5)

[10]

(10)(a) (i) Yes. No repeated x values (2)

(ii) $y \in [1; \infty)$ (2)

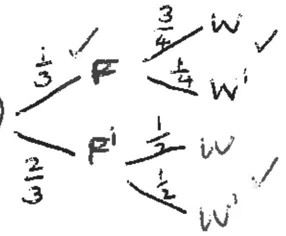
(iii) $y \in [-1; \infty)$ (2)

(iv) $x \leq 0$ or $x \in (-\infty; 0]$ (1)

(b) C (2)

[9]

(11)(a)(i)



(3)

(ii) P(win next game) $= \frac{1}{3} \times \frac{3}{4} + \frac{2}{3} \times \frac{1}{2}$

$= \frac{7}{12}$ (3)

(11) PARABOLA

(b)(i) number of arrangements $= \frac{8!}{3!} = 6720$ (3)

(ii) P(As tog) $= \frac{6!}{6720} = \frac{3}{28}$ $(= 0{,}11)$ (4)

[13]

(12) $P(A) = x$ & $P(B) = y$

$P(A) \cdot P(B) = P(A \cap B)$ (independent)

$\therefore xy = \frac{1}{3}$ $\boxed{\therefore x = \frac{1}{3y}}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{9}{10} = x + y - \frac{1}{3}$

$\frac{9}{10} = \frac{1}{3y} + y - \frac{1}{3}$

$27y = 10 + 30y^2 - 10y$ $\boxed{LCD = 30y ; y \neq}$

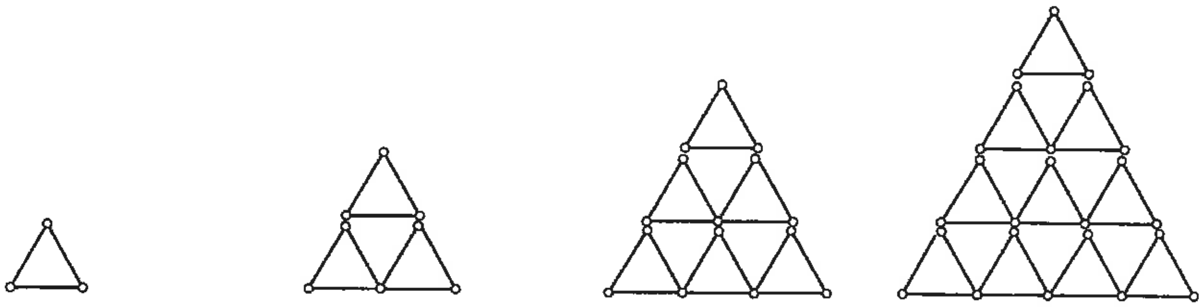
$37y = 30y^2 + 10$

[6]

Question 13

Congruent triangles made of matchsticks are stacked as shown in the figures below.

Fig. 1 Fig. 2 Fig.3 Fig.4



(a) Complete the table:
(write the answers on your folio paper – not on this sheet)

Figure	1	2	3	4	5
No.of matches	3	**(i)**	18	30	**(ii)**

(2)

(b) Determine a formula for the number of matches sides in the n^{th} figure. (5)

(c) Would it be possible to make a stack of triangles using exactly 135 matches? (3)

[10]

(a) (i) 9 ✓ (ii) 45 ✓ (2)

(b) 3; 9; 18; 30; 45

first diff 6 9 12 15

2nd diff 3 3 3 ✓

$\therefore a = \frac{3}{2}$ ✓

$T_0 = c = 0$ ✓

$\therefore T_n = \frac{3}{2}n^2 + bn$

$\boxed{T_1 = 3}$ $3 = \frac{3}{2} + b$

$b = 3/2$ ✓

$\therefore T_n = \frac{3}{2}n^2 + \frac{3}{2}n$ ✓ (5)

(c) $135 = \frac{3}{2}n^2 + \frac{3}{2}n$ ✓

$0 = 3n^2 + 3n - 270$

$0 = n^2 + n - 90$ ✓

$0 = (n+10)(n-9)$

$\therefore \underline{n = 9}$ ✓ (3)

Yes can make 9 with exactly 135 matches.

[10]