FILE

Paper II – Memo Aug 2015

Q1.

a) $m_{AB} = \frac{-4-2}{2+1}$ ✓

$= -2$ ✓ (2)

b) $\frac{-6+3}{k-4} = -2$ ✓

$-3 = 2k + 8$

$-11 = 2k$

$\frac{11}{2} = k$ ✓ (2)

Q2

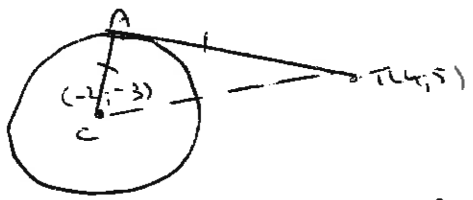
a) $(x+2)^2 + (y+3)^2 = 37 + 4 + 9$ ✓✓

$(x+2)^2 + (y+3)^2 = 50$

$\therefore C(-2;-3)$ ✓

$r = 5\sqrt{2}$ ✓ (4)

b)




$TA^2 + CA^2 = CT^2$ ✓ (Pythag)

$TA^2 + 50 = (6)^2 + (8)^2$ ✓

$TA^2 = 50$ (4)

$TA = 5\sqrt{2}$ ✓✓

$= CA$

c) $\therefore C\hat{T}A = 45°$ ✓ (1)

Q3

a) $m_{ON} = m_{OA} = \frac{3}{4}$ ✓

$y = \frac{3}{4}x$ ✓ (2)

b) $m_{MN} = -\frac{4}{3}$ ✓

$y - 3 = -\frac{4}{3}(x - 4)$ ✓

$y = -\frac{4}{3}x + \frac{25}{3}$ ✓ (4)

c) $OA = 5$ $(3;4;5\ \Delta)$ ✓ (1)

$\therefore OP = 5$ ✓

$\therefore x_P = x_M = 5$ ✓

Let $M(5;b)$

$\therefore b = -\frac{4}{3}(5) + \frac{25}{3} = \frac{5}{3}$ ✓✓ (5)

$\therefore a = 5;\ b = \frac{5}{3}$ ✓

Q4

a) $m_{AP} = m_{PD}$ ✓

$\therefore \frac{2}{5} = \frac{2-a}{-1+a^2}$ ✓

$\therefore -2 + 2a^2 = 10 - 5a$

$\therefore 2a^2 + 5a - 12 = 0$ ✓ (5)

$\therefore (2a - 3)(a + 4) = 0$

$\therefore a = +\frac{3}{2}$ or $a = -4$ ✓✓

MA

b) $AP^2 = (8)^2 + (5)^2 = 49$ ✓✓

$BP^2 = (8)^2 + (2)^2 = 68$ ✓✓ (5)

$\therefore$ A gets there first ✓

Q5

a) How many tested in total? ✓ (1)

Ill | Healthy etc small sample

b) How significant? ✓ (1)

age? etc.

**ANSWER ALL THE QUESTIONS WHICH FOLLOW IN THIS BOOKLET.**

**QUESTION 6** **[9]**

a) Prove that $\dfrac{(1+\tan^2\theta)\sin(90°+\theta)}{1-\tan\theta} = \dfrac{1}{\cos\theta-\sin\theta}$ (5)

$$LHS = \frac{\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)\cdot\left(\frac{\cos\theta}{1}\right)}{1-\left(\frac{\sin\theta}{\cos\theta}\right)} \qquad RHS = \frac{1}{\cos\theta-\sin\theta}$$

$$= \frac{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}\cdot\frac{\cos\theta}{1}}{\frac{(\cos\theta-\sin\theta)}{\cos\theta}}$$

$$= \frac{1}{\cancel{\cos\theta}} \times \frac{\cancel{\cos\theta}}{\cos\theta-\sin\theta}$$

$$= \frac{1}{\cos\theta-\sin\theta}$$

b) For which value(s) of $\theta$ is the above identity undefined? (4)

$\tan\theta = 1$ ; $\cos\theta = 0$

$\theta = 45° + k180°$ or $\theta = \pm 90° + k360°$ ; $k \in \mathbb{Z}$

$-1$ No $k \in \mathbb{Z}$

**QUESTION 7** **[7]**

a) Simplify cos5β cos 3β + sin5β sin 3β to a trigonometric expression involving only cosβ. (2)

$= \cos(5\beta - 3\beta)$ ✓
$= \cos 2\beta$
$= 2\cos^2\beta - 1$ ✓

b) Hence solve for β if cos5βcos 3β +sin5βsin 3β + cosβ = 0 (5)

$2\cos^2\beta - 1 + \cos\beta = 0$
$2\cos^2\beta + \cos\beta - 1 = 0$ ✓
$(2\cos\beta - 1)(\cos\beta + 1) = 0$
$\cos\beta = \frac{1}{2}$ or $\cos\beta = -1$ ✓✓
$\beta = \pm 60° + 360°k$ ✓ or $\beta = \pm 180° + 360°k$ ✓ $k \in \mathbb{Z}$

**QUESTION 8** **[16]**

a) Show that sin(x − 30°) = 2cosx can be rewritten as $\sqrt{3}\sin x = 5\cos x$. (3)

$\sin x \cos 30° - \cos x \sin 30° = 2\cos x$ ✓
$\therefore \frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x = 2\cos x$ ✓ (sp. ∠s)
$\frac{\sqrt{3}}{2}\sin x = \frac{5}{2}\cos x$ ✓ (collecting)
$\sqrt{3}\sin x = 5\cos x$

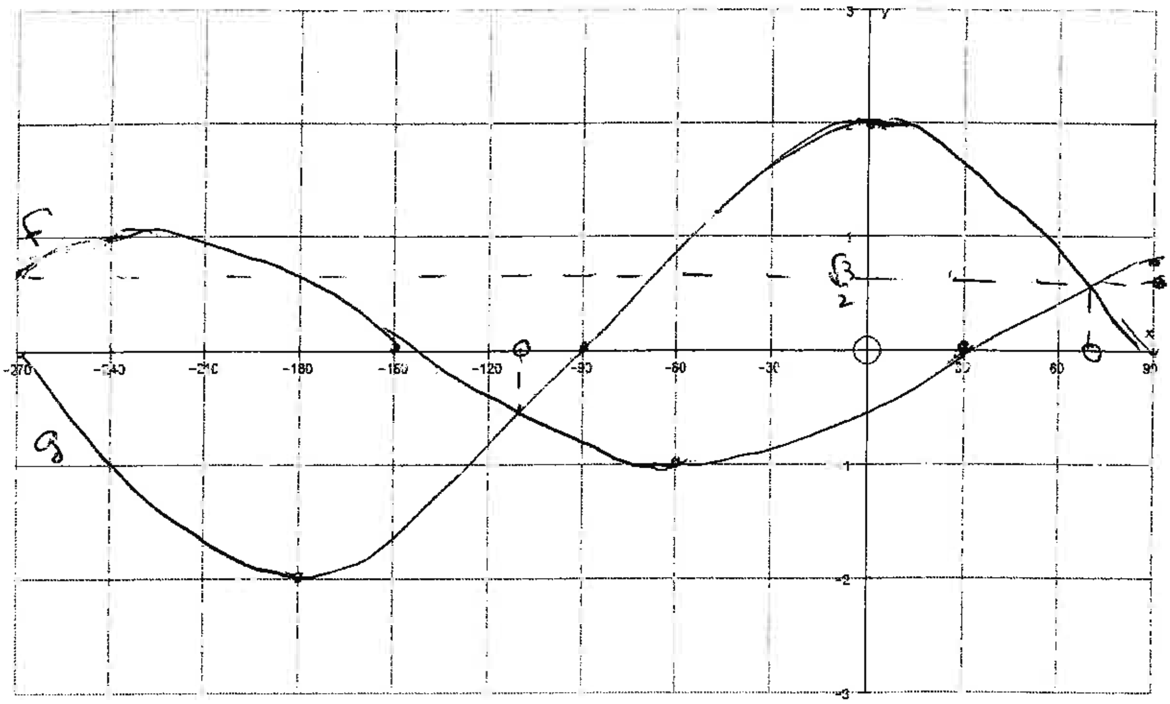
b) Hence solve for x ∈ [−270° ; 90°] if sin(x −30°) = 2cosx (4)

$\frac{\sin x}{\cos x} = \frac{5}{\sqrt{3}}$

$\therefore x = 70{,}89° + 180° k \; ; \; k \in \mathbb{Z}$

$\therefore x = 70{,}89° \; ; \; -109{,}11°$

c) **On the set of axes provided BELOW** sketch the graphs of
f(x) = sin(x − 30°) and g(x) = 2cosx for x ∈ [−270° ; 90°].
CLEARLY indicate the end points and any other relevant points. (7)

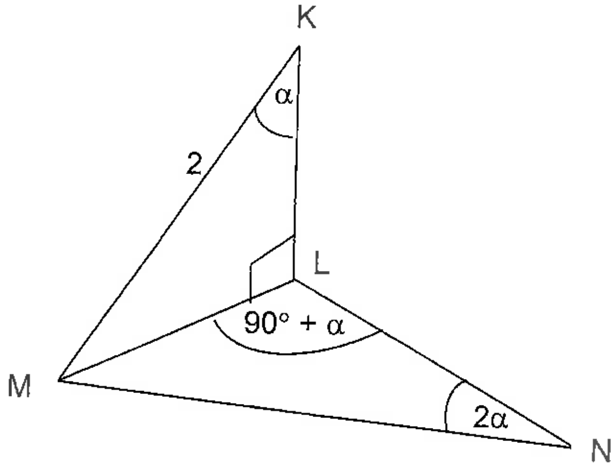



d) Using your answers in (b) and (c) above, determine x if 2cosx > sin(x − 30°)
for x ∈ [−270° ; 90°]. (2)

$x \in (70{,}89° \; ; \; -109{,}11°)$

## QUESTION 9

[9]




In the sketch, KL is a vertical tower and L, M and N are all points in the horizontal plane. $M\hat{L}N = (90° + \alpha)$; $\hat{N} = 2\alpha$ and $\hat{K} = \alpha$. KM = 2 units.

a) Express ML in terms of a trigonometric ratio of α and hence show that MN = 1 unit. (4)

$$\frac{ML}{2} = \sin\alpha$$

$$\therefore ML = 2\sin\alpha \quad ✓$$

$$\frac{MN}{\sin(90° + \alpha)} = \frac{2\sin\alpha}{\sin 2\alpha} \quad ✓$$

$$MN = \frac{2\sin\alpha\cos\alpha}{2\sin\alpha\cos\alpha} \quad ✓$$

$$= 1$$

b) Prove, using the sine rule, that LN = 1 – 4 sin²α (HINT: cos3α = 4cos³α – 3cosα) (5)

$$\frac{LN}{\sin(90° - 3\alpha)} = \frac{1}{\sin(90° + \alpha)} \quad ✓$$

$$LN = \frac{\cos 3\alpha}{\cos\alpha} \quad ✓ ✓$$

$$= \frac{4\cos^3\alpha - 3\cos\alpha}{\cos\alpha}$$

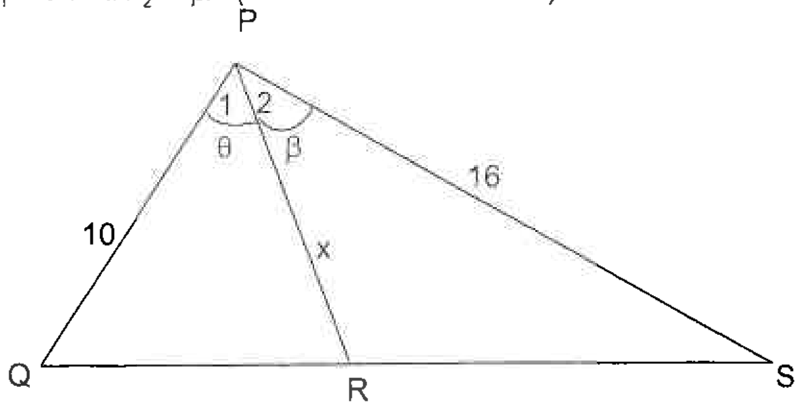
$$= 4\cos^2\alpha - 3 \quad ✓$$

$$= 4(1 - \sin^2\alpha) - 3 \quad ✓$$

$$= 1 - 4\sin^2\alpha$$

**QUESTION 10** **[9]**

$\Delta PQS$ has PQ = 10 units; PR = x units; PS = 16 units. Point R divides QS in the ratio 3 : 5. $\hat{P}_1 = \theta$ and $\hat{P}_2 = \beta$. (See the sketch below.)




a) Write down an expression for the area of $\Delta PQR$ in terms of x and $\theta$. (2)

$\Delta = \frac{1}{2}(10)(x)\sin\theta$ ✓

$= 5x\sin\theta$ ✓

b) (i) Complete the statement:

"Area of $\Delta PQR$ : Area of $\Delta PRS$ = 3 : 5" (1)

(ii) Hence, determine the ratio $\frac{\sin\theta}{\sin\beta}$. (2)

$\frac{5x\sin\theta}{8x\sin\beta} = \frac{3}{5}$ $\therefore \frac{\sin\theta}{\sin\beta} = \frac{24}{25}$

c) If $Q\hat{P}S = 90°$, calculate $\theta$ correct to one decimal place. (4)

$\frac{\sin\theta}{\sin(90-\theta)} = \frac{24}{25}$ ✓ $\quad \sin(90-\theta) = \cos\theta$

✓ $\tan\theta = 24/25$

$\therefore \theta = 43,83°$ ✓

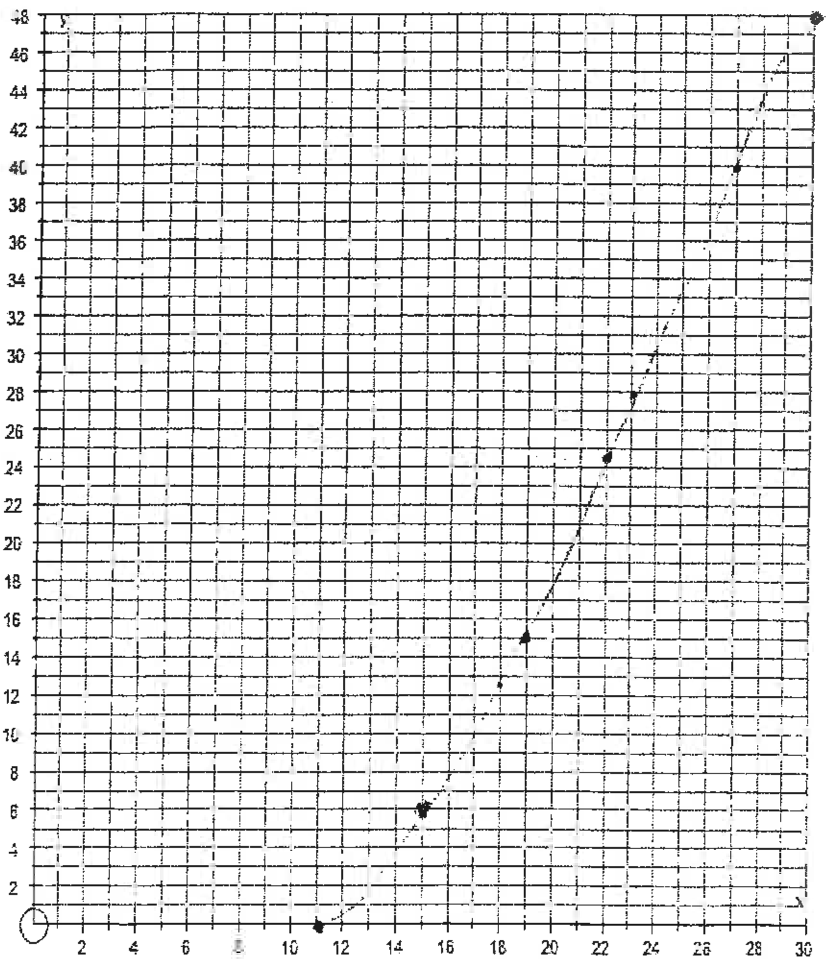
**QUESTION 11** **[9]**

A group of 48 ladies was asked to complete an obstacle course. The time taken in minutes to complete the course is given below: .

| Time (in minutes) | Frequency | Cumulative frequency |
|---|---|---|
| $11 \leq t < 15$ | 6 | 6 |
| $15 \leq t < 19$ | 9 | 15 |
| $19 \leq t < 23$ | 13 | 28 |
| $23 \leq t < 27$ | 12 | 40 |
| $27 \leq t < 30$ | 8 | 48 |

a) Complete the cumulative frequency column. (1)

b) Draw an ogive curve for the data. (3)



c) Determine from the ogive the :

i. median 23

ii. lower quartile 20

iii. upper quartile 25 (3)

d) Comment on the spread of the time taken to complete the race. (2)

50% finished within a 5 min range

Median – Lowest $\approx 12$

Highest – Median $\approx 8$

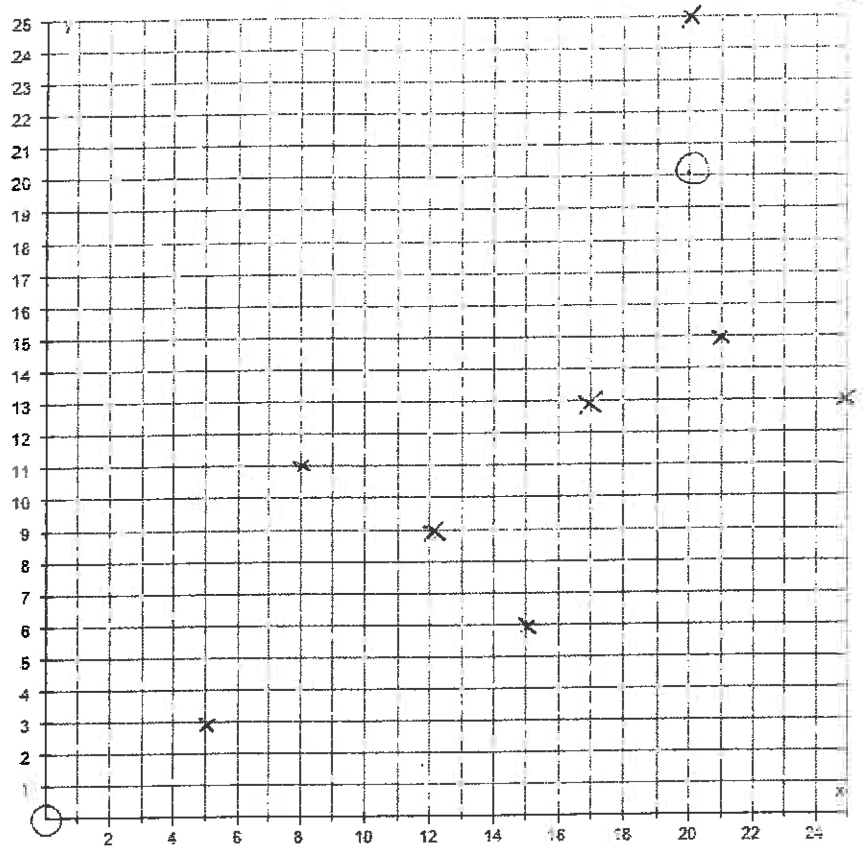
Data skewed to left

**QUESTION 12** **[11]**

A supermarket wants to survey how long in seconds (y) it takes to scan (x) items at a till point. They decide to select the results from 9 shoppers. The results are indicated on the table below:

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| x (number of items) | 5 | 8 | 12 | 15 | 15 | 17 | 20 | 21 | 25 |
| Y (time in seconds | 3 | 11 | 9 | 6 | 15 | 13 | 25 | 15 | 13 |

a) Represent the information in the table on the scatter plot below: (3)




b) Use your calculator to find the equation of the regression line and draw this line. (2)

$y = 2{,}68 + 0{,}62x$

c) Calculate the value of r, the correlation co-efficient for the data. (1)

$r = 0{,}63$

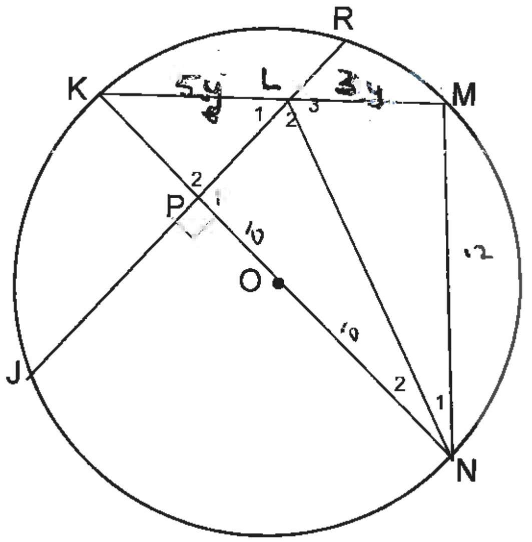
d) Estimate the time taken for the teller to scan 21 items at the till. (2)

15,7 sec

e) It was found that an error was made with the time value G(20;25). The point should have been (20;20). If the correlation co-efficient for the new regression line was now calculated, would it indicate a stronger or a weaker correlation? Validate your answer without any further calculations. (3)

Stronger (20;25) seems to be an outlier

**QUESTION 13** **[15]**




KON is a diameter of the circle centre O.
KO = 10 cm and MN = 12 cm. NL bisects $K\hat{N}M$, and chord RLJ cuts KN perpendicularly at P.

a) (i) Give, with a reason, another angle which is 90° (2)

$\hat{M}$ ; ∠ subt by diam / ∠ in semi circle

(ii) Write down a triangle which is similar to ΔPKL (1)

Δ MKN

(ii) Name **2** congruent triangles in the figure. State which case of congruency has been applied. (2)

ΔLPN ≡ ΔLMN AAS

b) Calculate with reasons

(i) KM (2)

By Pythag : $KM^2 = (20)^2 - (12)^2$

$KM = 16$

(ii) KL : LP (3)

ΔPKL ||| ΔMKN

$\frac{KL}{LP} = \frac{KN}{MN}$ Sim Δs (sides in prop)

$= \frac{20}{12}$

$= \frac{5}{3}$

(iii) LM (5)

LM = LP  corr $\Delta$s

$\therefore \frac{KL}{LM} = \frac{5}{3}$

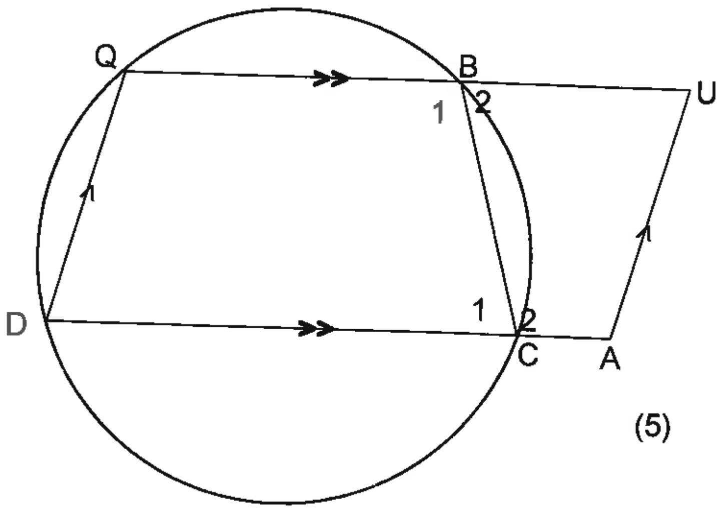
$\therefore LM = \frac{3}{8} KM$

$= \frac{3}{8} \times 16$

$= 6$

**QUESTION 14** **[5]**

1. QUAD is a parallelogram and QBCD is a cyclic quadrilateral. Prove that BUAC is also a cyclic quadrilateral.



(5)

$\hat{Q} = \hat{C}_2$ ext ∠ cyclic quad

$\hat{Q} = \hat{A}$ opp ∠s of parm

$\hat{C}_2 = \hat{B}_1$ alt ∠s QU ∥ DA

$\therefore \hat{B}_1 = \hat{A}$

$\therefore$ BUAC cyclic { conv. ext ∠ cyclic quad / ext ∠ = int opp ∠ }

OR

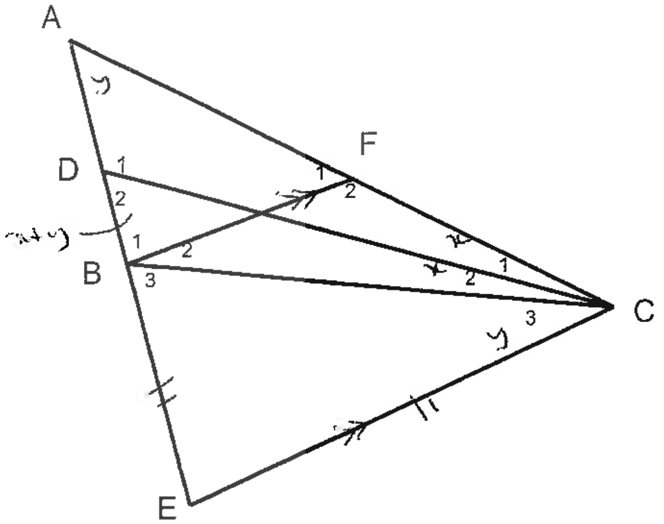
$\hat{Q} + \hat{C}_1 = 180$ coint ∠s QD ∥ UA

$\hat{Q} = \hat{C}_2$ ext ∠ cyclic quad

$\therefore \hat{C}_2 + \hat{C}_1 = 180$

$\therefore$ BUAC cyclic { conv. opp ∠s cyclic quad / opp ∠s of quad supp }

**QUESTION 15** **[9]**




In the figure, the bisector of $A\hat{C}B$ meets AB in D. AB is produced to E such that DE=CE ~~and BF∥EC~~ Prove that:

a) EC is a tangent to circle ABC (5)

$\hat{B}_2 = \hat{C}_2 + \hat{C}_3$ ✓ ΔECD isos

$\hat{D}_2 = \hat{A} + \hat{C}_1$ ✓ ext ∠ of Δ

Let $\hat{C}_1 = \hat{C}_2 = x$; $\hat{C}_3 = y$

$\therefore \hat{D}_2 = x + y = \hat{A} + x$ ✓

$\therefore \hat{A} = y$ ✓

∴ EC tan to ⊙ ABC conv of tan chord th ✓

b) $EC^2 = EB \cdot EA$ (4)

In ΔECB and ΔEAC

$\hat{E}$ common

$\hat{C}_3 = \hat{A}$ proved ✓✓

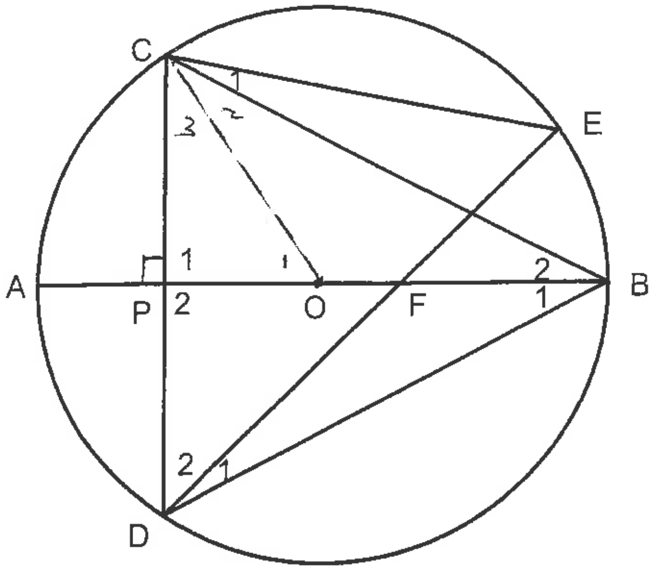
$\therefore \hat{B}_3 = E\hat{C}A$ rem ∠ of Δ

ΔECB ||| ΔEAC AAA ✓

$\therefore \frac{EC}{EA} = \frac{EB}{EC}$ sim Δs ✓

$\therefore EC^2 = EA \cdot EB$

**QUESTION 16** **[9]**




In the figure, O is the centre of the circle with AB as the diameter. CD ⊥ AB at P and chord DE cuts AB at F.

Prove that :

a) $\hat{B}_1 = \hat{B}_2$ (4)

CP = PD line from ⊙ ⊥ chord

∴ ΔCPB ≡ ΔDPB RHS

∴ $\hat{B}_1 = \hat{B}_2$ congruent Δs

b) $C\hat{E}D = 2\hat{B}_2$ (2)

$C\hat{E}D = \hat{B}_1 + \hat{B}_2$ ∠s in same seg subt AD

$= 2\hat{B}_2$

c) CEFO is a cyclic quadrilateral. (3)

$\hat{O}_1 = 2\hat{B}_2$ ∠ at centre = 2 ∠ at circum

$= C\hat{E}D$

∴ CEFO cyclic quad { conv. of ext ∠ of cyclic quad
{ ext ∠ of quad = int opp ∠

## QUESTION 17

[3]

If sinB + cosB = 1,2, evaluate **without using a calculator** sinBcosB. (3)

$(\sin B + \cos B)^2 = (1,2)^2$ ✓

$\sin^2 B + 2\sin B\cos B + \cos^2 B = 1,44$

$2\sin B\cos B = 1,44 - 1$ ✓

$\sin B\cos B = 0,22$ ✓