Parkland College of Education

June Examinations - Autumn Quarter 2015

GRADE 12 MATHEMATICS PAPER 1

SOLUTIONS

QUESTION 1

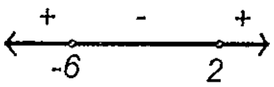
1.1.1. $\dfrac{10}{x} - 2x = 1$ (5)

$10 - 2x^2 = x$
$2x^2 + x - 10 = 0$
$(2x+5)(x-2) = 0$

Then $x = -\dfrac{5}{2}$ or $x = 2$

1.1.2. $x(4+x) \le 12$ (4)

$x^2 + 4x - 12 \le 0$
$(x+6)(x-2) \le 0$



Then $-6 \le x \le 2$

1.1.3. $2x^2 - 15x + 5 = 0$ (4)

$$x = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(2)(5)}}{2(2)}$$

$$x = \frac{15 \pm \sqrt{185}}{4}$$

Then $x = 7{,}15$ or $x = 0{,}35$

1.1.4. $5^x + 5^x + 5^x = \dfrac{3}{25}$ (3)

$5^x(1+1+1) = \dfrac{3}{25}$

$5^x(3) = \dfrac{3}{25}$

$5^x = \dfrac{1}{25} = \dfrac{1}{5^2} = 5^{-2}$

Then $x = -2$

1.2. Solve for x and y in the simultaneous equations:

$x-3y=-5$; $3x^2-5xy-2y^2=0$ (7)

$x=3y-5$ (1) ;

Sub (1) : $3(3y-5)^2-5y(3y-5)-2y^2=0$

$3(9y^2-30y+25)-15y^2+25y-2y^2=0$

$27y^2-90y+75-15y^2+25y-2y^2=0$

$10y^2-65y+75=0$

$2y^2-13y+15=0$

$(2y-3)(y-5)=0$

Then $y=1\frac{1}{2}$ or $y=5$

Then $x=3\left(\frac{3}{2}\right)-5$ or $x=3(5)-5$

Then $x=-\frac{1}{2}$ or $x=10$

1.3. Without solving for x, determine for which values of p the roots of the following equation will be real:

$(2x+3)(x+1)=p$ (5)

$2x^2+5x+3=p$

$2x^2+5x+(3-p)=0$

$\Delta=(5)^2-4(2)(3-p)$

$\Delta=25-24+8p$

$\Delta=1+8p\geq 0$

$8p\geq -1$

$p\geq -\frac{1}{8}$

1.4 Show that the roots of the equation $k^2x^2=4kx-4$ are equal for all real values of k, $k\neq 0$, without solving for x. (4)

[32]

$k^2x^2-4kx+4=0$

$\Delta=(-4k)^2-4(k^2)(4)$

$=16k^2-16k^2$

$=0$

Therefore the roots are equal

QUESTION 2

2.1. The *fourth* ($4th$) term of an Arithmetic sequence is 34, and the *twenty-sixth* ($26th$) term is -32.

2.1.1. Show that the constant difference is -3. (3)

$a+3d=34$ (1)

$a+25d=-32$ (2)

$(2)-(1)$: $22d=-66$; $d=-3$

2.1.2. Calculate the value of the *fortieth* ($40th$) term. (2)

$a=34-3(-3)=43$;
$T_{40}=43+(39)(-3)=-74$

2.1.3. Calculate the Sum of the Series to 200 terms. (3)

$$S_{200}=\frac{200}{2}[2(43)+(199)(-3)]$$
$$=-51100$$

2.2. Calculate the value of $18+12+8+...+1\frac{47}{81}$. (7)

Let $T_n=1\frac{47}{81}=\frac{128}{81}$

Then $18\left(\frac{2}{3}\right)^{n-1}=\frac{128}{81}$

$$\left(\frac{2}{3}\right)^{n-1}=\frac{128}{81}\div 18$$
$$\left(\frac{2}{3}\right)^{n-1}=\frac{64}{729}=\left(\frac{2}{3}\right)^{6}$$

Then $n-1=6$; Therefore $n=7$

$$18+12+8+...+1\frac{47}{81}=S_7=\frac{18\left(1-\left(\frac{2}{3}\right)^7\right)}{1-\frac{2}{3}}$$
$$=\frac{4118}{81}=50\frac{68}{81}$$

2.3. Calculate the value of x if $\sum_{k=1}^{\infty} x\left(-\frac{2}{5}\right)^{k-1} = 10$. (3)

$$\sum_{k=1}^{\infty} x\left(-\frac{2}{5}\right)^{k-1} = x - \frac{2}{5}x + \frac{4}{5}x + ...$$

Then $\dfrac{x}{1-\left(-\frac{2}{5}\right)} = 10$;

Therefore $x = 10\left(\frac{7}{5}\right) = 14$

2.4. If $S_n = 5n - 3n^2$, for a certain Series, determine the value of T_{12}. (3)

[21]

$$\begin{aligned} T_{12} &= S_{12} - S_{11} \\ &= [5(12) - 3(12)^2] - [5(11) - 3(11)^2] \\ &= -64 \end{aligned}$$

QUESTION 3

The quadratic sequence $x+2y; 2x+y; 3x-2y; 5x-3y; ...$ is given.

3.1. Prove that $x = -4y$. (4)

d_1 : $(2x+y)-(x+2y); (3x-2y)-(2x+y); (5x-3y)-(3x-2y); ...$

$2x+y-x-2y; 3x-2y-2x-y; 5x-3y-3x+2y; ...$

$x-y; x-3y; 2x-y; ...$

d_2 : $(x-3y)-(x-y) = (2x-y)-(x-3y); ...$

$x-3y-x+y = 2x-y-x+3y$

$-2y = x+2y$

$x+2y = -2y$

$x = -4y$

3.2. Hence, determine, determine the general $(n-th)$ term if $y=-1$. (5)

[9]

d_1 : $-4(-1)-(-1);-4(-1)-3(-1);2(-4(-1))-(-1)...$

$5;7;9...$

d_2 : $2;2;...$

$2a=2 \quad a=1$

$3(1)+b=5 \quad b=2$

$1+2+c=-2(-1)=2$

$c=-1$

$T_n=n^2+2n-1$

QUESTION 4

The functions f and g are defined as follows:

$f(x)=\frac{1}{2}x^2-x-4$ and $g(x)=\frac{-4}{x+2}-1$.

4.1. Write $f(x)$ in the form $y=a(x-p)^2+q$, by completing the square. (4)

$$
\begin{aligned}
f(x) &= \frac{1}{2}[x^2-2x-8] \\
&= \frac{1}{2}[x^2-2x+(-1)^2-8-(-1)^2] \\
&= \frac{1}{2}[(x-1)^2-8-1] \\
&= \frac{1}{2}[(x-1)^2-9] \\
&= \frac{1}{2}(x-1)^2-4\frac{1}{2}
\end{aligned}
$$

4.2. On the same set of axes, draw neat sketch graphs of f and g, showing all the necessary calculations, and indicate all the intercepts with the axes, axis of symmetry, asymptotes and turning point. [Sketch graphs must be drawn on the DIAGRAM SHEET.] (11)

For f, the X-intercepts : $\frac{1}{2}x^2 - x - 4 = 0$

$$x^2 - 2x - 8 = 0$$

$$(x+2)(x-4) = 0$$

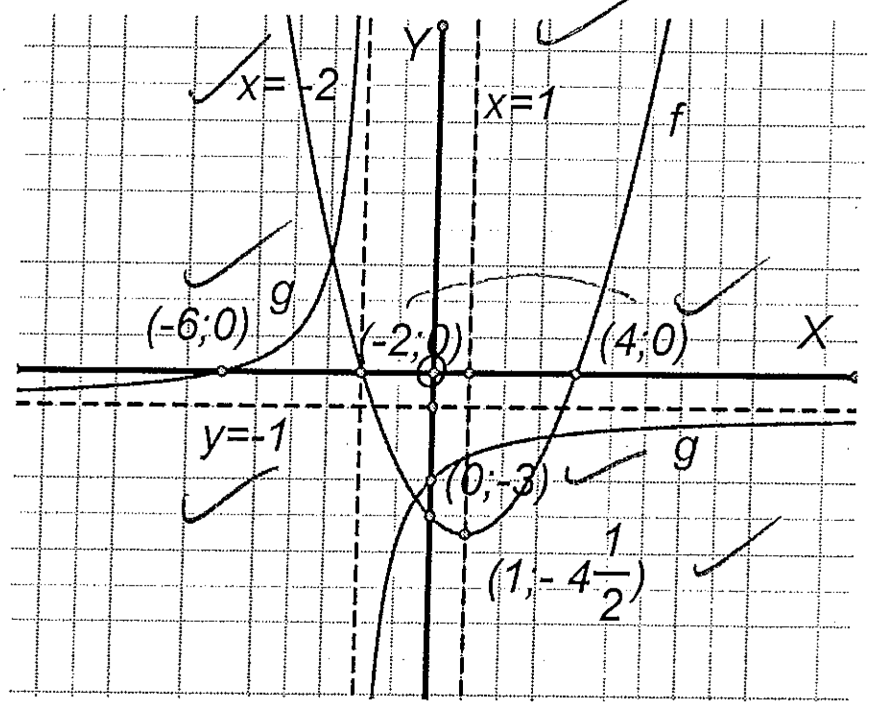
$x = -2$ or $x = 4$

For g, we have a: $x = -2$; $y = -1$

Y-intercept : $= \frac{-4}{0+2} - 1 = -3$

X-intercepts : $\frac{-4}{x+2} = 1$ $\quad \frac{x+2}{-4} = 1$

$x + 2 = -4$; $x = -6$



4.3. How must the graph of f be shifted vertically so that $f(0)-g(0)=3$? (2)

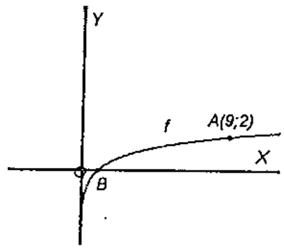
4 units up

4.4. Determine the new equation of graph g, if shifted by 2 units to the right and 2 units up. (2)
[19]

$y=\frac{-4}{x}+1$

QUESTION 5

The function $f(x)=\log_a x$ is represented in the diagram, with $A(9;2)$ a point on the graph.



5.1. Determine the value of a. (2)

$(9;2):\ 2=\log_a 9$
Then $a^2=9$
Then $a=3$

5.2. Write down the equation of the inverse of f^{-1} in the form $y=...$ (2)

$y=3^x$

5.3. Write down the *Range* of f^{-1}. (2)

$y\in R; y>0$

5.4. For which values of x will $f(x)<-1$? (3)
[9]

$\log_3 x<-1$
$x<3^{-1}$
$0<x<\frac{1}{3}$

QUESTION 6

6.1. The price of an article depreciates by a fixed rate, $12{,}95\,\%$ p.a., according to the method of reducing balance. The original price was $R\,80\,500$, and after n years the price is $R\,40\,500$. Determine the value of n. (3)

$$R\,80\,500\left(1-\frac{12{,}95}{100}\right)^n = R\,40\,500$$

$$(0{,}8705)^n = \frac{40\,500}{80\,500}$$

$$n = \log_{0{,}8705}\frac{40\,500}{80\,500}$$

$$n = 4{,}95$$

6.2. If an investment is made at $9\,\%$ p.a., interest compounded quarterly, calculate the *effective* annual interest rate. (4)

$$\left(1+\frac{r}{100}\right)=\left(1+\frac{9}{400}\right)^4$$

$$\frac{r}{100}=\left(1+\frac{9}{400}\right)^4-1$$

$$r = 100[(1{,}0225)^4-1]$$

$$r = 9{,}31\,\%$$

6.3. A loan of $R\,850\,000$ must be settled by paying interest at $11{,}5\,\%$ p.a., interest compounded monthly, and paying over a period of 20 years. Calculate the monthly payment if the first payment is made one month after the loan has been granted. (4)

Let the monthly repayment be x *rands*

Then $$\frac{x\left(1-\left(1+\frac{11{,}5}{1200}\right)^{-12(20)}\right)}{\frac{11{,}5}{1200}} = 850\,000$$

Therefore $$x = R\,\frac{850\,000\times\frac{11{,}5}{1200}}{1-\left(1+\frac{11{,}5}{1200}\right)^{-240}} = R\,9\,064{,}65$$

6.4. The monthly payment on a sinking fund, paid over 6 years, is $R300$, and the interest rate is $9{,}5\,\%$ p.a., interest compounded monthly. Calculate the final value of the sinking fund. (4)

[15]

$$F = R\frac{300\left(\left(1+\frac{9{,}5}{1200}\right)^{12(6)}-1\right)}{\frac{9{,}5}{1200}}$$

$$F = R\,28\,963{,}05$$

QUESTION 7

7.1. $f(x) = 3-2x^2$

Determine $f'(x)$, from *first principles.* (5)

$$f(x) = 3-2x^2$$
$$f(x+h) = 3-2(x+h)^2 = 3-2(x^2+2xh+h^2)$$
$$= 3-2x^2-4xh-2h^2$$

Then $f(x+h)-f(x) = -4xh-2h^2$

$$\frac{f(x+h)-f(x)}{h} = \frac{-4xh-2h^2}{h}$$
$$= \frac{-4xh-2h^2}{h} = \frac{h(-4x-2h)}{h}$$
$$= (-4x-2h)$$

Then $f'(x) = \lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0}(-4x-2h)$$
$$= -4x$$

7.2. Determine the following:

7.2.1. $f'(x)$, if $y = f(x) = x^2\left(x-\frac{3}{2}\right)+2x-7$ (3)

$$f(x) = x^3-\frac{3}{2}x^2+2x-7$$
$$f'(x) = 3x^2-3x+2$$

7.2.2. The value(s) of x for which the equation of the tangent to f is $y=8x-17$ (4)

$$f'(x)=3x^2-3x+2=8$$
$$3x^2-3x-6=0$$
$$x^2-x-2=0$$
$$(x+1)(x-2)=0$$
$$x=-1 \text{ or } x=2$$

7.3. Determine $\frac{d}{dx}\left[\frac{3x^3-5x-2}{x^2}\right]$. (3)

$$=\frac{d}{dx}\left[3x-5x^{-1}-2x^{-2}\right]$$
$$=3+5x^{-2}+4x^{-3}$$
$$=3+\frac{5}{x^2}+\frac{4}{x^3}$$

[15]

QUESTION 8

$f(x)=3x^3+4x^2-17x-6$.

8.1. Determine the X-intercepts and the coordinates of the turning points of f. (12)

$X-intercepts$: $f(x)=0$;

$$f(2)=3(2)^3+4(2)^2-17(2)-6$$
$$=24+16-34-6=0$$

Then : $(x-2)$ is a factor of $f(x)$

2	3	4	- 17	- 6
		6	20	6
	3	10	3	0

(Long division can also be used)

$$f(x)=(x-2)(3x^2+10x+3)$$
$$=(x-2)(3x+1)(x+3)$$
$$=0 \text{ if } x=2 \;;\; x=-\frac{1}{3} \text{ or } x=-3$$

For Turning points : $f'(x)=0$;
$f'(x)=9x^2+8x-17=0$;
then $(9x+17)(x-1)=0$

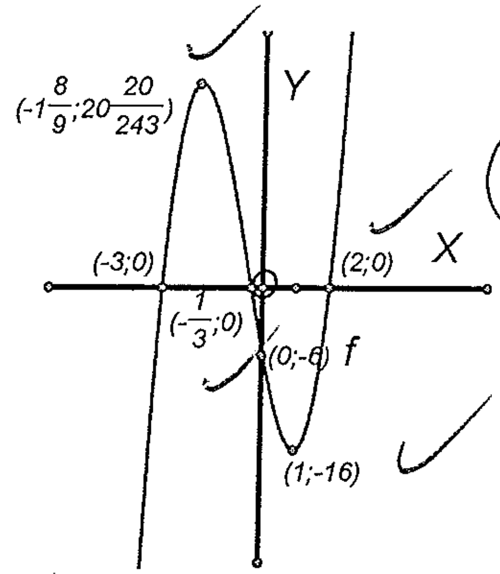
Therefore $x=-\frac{17}{9}$ or $x=1$; Therefore $y=f\left(-\frac{17}{9}\right)$ or $y=f(1)$

$y=3\left(-\frac{17}{9}\right)^3+4\left(-\frac{17}{9}\right)^2-17\left(-\frac{17}{9}\right)-6$ or $y=3(1)^3+4(1)^2-17(1)-6$

So $y=20\frac{40}{243}$ or $y=-16$

Therefore Turning points : $\left(-1\frac{8}{9};20\frac{40}{243}\right)$ and $(1;-16)$

8.2. Draw a neat sketch graph of f, showing all the intercepts with the axes and turning points. [The DIAGRAM SHEET must be used.] (4)



(all 3 x-intercepts)

8.3. For which values of x will $f'(x)>0$? (2)

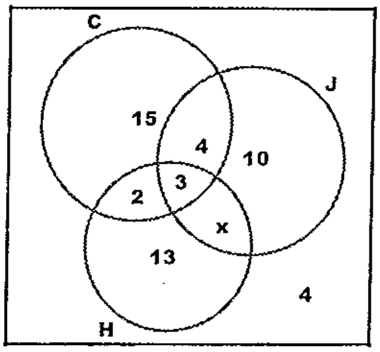
[18]

$x<-1\frac{8}{9}$ or $x>1$

QUESTION 9

9.1. The following Venn diagram illustrates the music preferences of 60 people.
C indicates the group of people listening to Classical music,
J indicates the Jazz lovers and
H the lovers of Heavy metal.

Calculate the following, leaving the answers in simplest fraction form:



9.1.1. The value of x (2)

$$x = 60 - (15+4+3+2+13+10+4)$$
$$= 9$$

9.1.2. The probability of someone listening to Classical music or Jazz (2)

$$= \frac{43}{60}$$

9.1.3. The probability of someone listening to Heavy Metal and Jazz, but not to Classical music (2)

$$= \frac{9}{60} = \frac{3}{20}$$

9.1.4. The probability of someone listening to Classical music only (2)

$$= \frac{15}{60} = \frac{1}{4}$$

9.1.5. The probability of someone not listening to any of these types of music (1)

$$= \frac{4}{60} = \frac{1}{15}$$

9.2. The sample space, S, is the set of the first 20 natural numbers. Each of these numbers is printed on a piece of paper and the pieces of paper must be selected at random. All the numbers have an equal chance of being selected.

A is the event that a Prime number is selected.
B is the event that a multiple of 3 is selected

Show whether the events A and B are independent. (3)

[12]

$$P(A).P(B)=\frac{8}{20}\times\frac{6}{20}=\frac{3}{25}$$

and $P(A \cap B)=\dfrac{1}{20} \neq P(A).P(B)$

Therefore not independent