



# Parklands College of Education

## June Examinations - Autumn Quarter 2015

Subject : Mathematics Paper : 1  
Grade : 12 Marks : 150  
Examiners : F.A. du Preez ; B. Malan ; Time : 3 hours  
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Moderator : M. Naidoo

### **INSTRUCTIONS:**

- 1. This paper consists of 9 QUESTIONS and 9 PAGES, including an INFORMATION (FORMULA) SHEET and a DIAGRAM SHEET.**
- Each question (e.g. QUESTION 2) must be numbered in the middle of the page, and the sub-sections (e.g. 1.2), 1.3.2)) in the left hand margin.
- All the steps must be shown.*
- A right hand margin must be drawn on each page.
- A line must be drawn after QUESTION 1, QUESTION 2, but not after 1.1 and 1.2, etc.
- Unless stated otherwise, calculators (non-programmable) may be used, in which case answers must be correctly approximated to two decimals.**
- Where technology is used, all the steps must still be shown by the candidate.
- Assume all denominators to be non-zero, unless restrictions are required.
- The diagrams are not necessarily drawn to scale, unless stated otherwise.**
- The DIAGRAM SHEET for QUESTION 4.2 AND 8.2 must be detached from the main paper and stapled to your last page inside your ANSWER BOOK.**

**QUESTION 1**

1.1. Solve for  $x$ , showing all the steps:

1.1.1.  $\frac{10}{x} - 2x = 1$  (5)

1.1.2.  $x(4+x) \leq 12$  (4)

1.1.3.  $2x^2 - 15x + 5 = 0$  (4)

1.1.4.  $5^x + 5^x + 5^x = \frac{3}{25}$  (3)

1.2. Solve for  $x$  and  $y$  in the simultaneous equations:

$$x - 3y = -5 ; 3x^2 - 5xy - 2y^2 = 0 \quad (7)$$

1.3. Without solving for  $x$ , determine for which values of  $p$  the roots of the following equation will be real:

$$(2x + 3)(x + 1) = p \quad (5)$$

1.4 Show that the roots of the equation  $k^2x^2 = 4kx - 4$  are equal for all real values of  $k$ ,  $k \neq 0$ , without solving for  $x$ . (4)  
[32]

**QUESTION 2**

2.1. The *fourth* (4th) term of an Arithmetic sequence is 34, and the *twenty-sixth* (26th) term is  $-32$ .

2.1.1. Show that the constant difference is  $-3$ . (3)

2.1.2. Calculate the value of the *fortieth* (40th) term. (2)

2.1.3. Calculate the Sum of the Series to 200 terms. (3)

2.2. Calculate the value of  $18 + 12 + 8 + \dots + 1\frac{47}{81}$ . (7)

2.3. Calculate the value of  $x$  if  $\sum_{k=1}^{\infty} x \left(-\frac{2}{5}\right)^{k-1} = 10$ . (3)

2.4. If  $S_n = 5n - 3n^2$ , for a certain Series, determine the value of  $T_{12}$ . (3)  
[21]

**QUESTION 3**

The quadratic sequence  $x + 2y; 2x + y; 3x - 2y; 5x - 3y; \dots$  is given.

3.1. Prove that  $x = -4y$ . (4)

3.2. Hence, determine, determine the general ( $n$ -th) term if  $y = -1$ . (5)  
[9]

**QUESTION 4**

The functions  $f$  and  $g$  are defined as follows:

$$f(x) = \frac{1}{2}x^2 - x - 4 \text{ and } g(x) = \frac{-4}{x+2} - 1.$$

4.1. Write  $f(x)$  in the form  $y = a(x - p)^2 + q$ , by completing the square. (4)

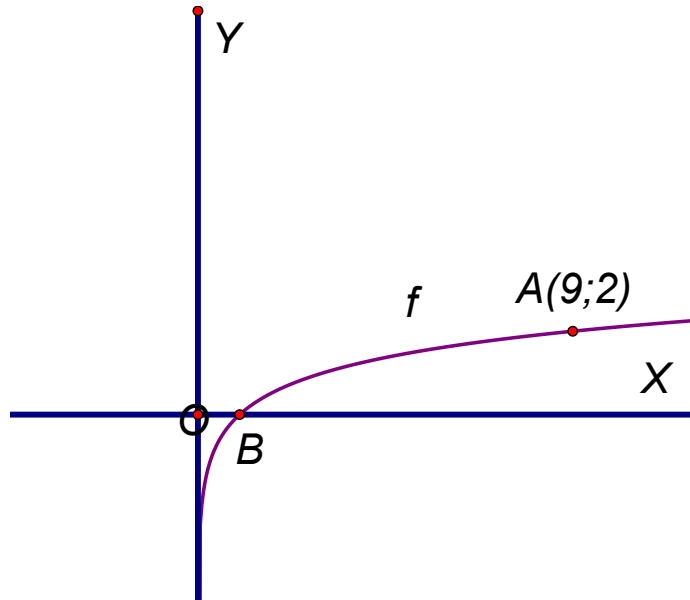
4.2. On the same set of axes, draw neat sketch graphs of  $f$  and  $g$ , showing all the necessary calculations, and indicate all the intercepts with the axes, axis of symmetry, asymptotes and turning point. [Sketch graphs must be drawn on the DIAGRAM SHEET.] (11)

4.3. How must the graph of  $f$  be shifted vertically so that  $f(0) - g(0) = 3$ ? (2)

4.4. Determine the new equation of graph  $g$ , if shifted by 2 units to the right and 2 units up. (2)  
[19]

### QUESTION 5

The function  $f(x) = \log_a x$  is represented in the diagram, with  $A(9; 2)$  a point on the graph.



- 5.1. Determine the value of  $a$ . (2)
  - 5.2. Write down the equation of the inverse of  $f^{-1}$  in the form  $y = \dots$  (2)
  - 5.3. Write down the *Range* of  $f^{-1}$ . (2)
  - 5.4. For which values of  $x$  will  $f(x) < -1$ ? (3)
- [9]

### QUESTION 6

- 6.1. The price of an article depreciates by a fixed rate, 12,95% p.a., according to the method of reducing balance. The original price was R 80 500, and after  $n$  years the price is R 40 500. Determine the value of  $n$ . (3)
- 6.2. If an investment is made at 9% p.a., interest compounded quarterly, calculate the *effective* annual interest rate. (4)

- 6.3. A loan of R 850 000 must be settled by paying interest at 11,5 % p.a., interest compounded monthly, and paying over a period of 20 years. Calculate the monthly payment if the first payment is made one month after the loan has been granted. (4)
- 6.4. The monthly payment on a sinking fund, paid over 6 years, is R 300, and the interest rate is 9,5 % p.a., interest compounded monthly. Calculate the final value of the sinking fund. (4)  
[15]

### QUESTION 7

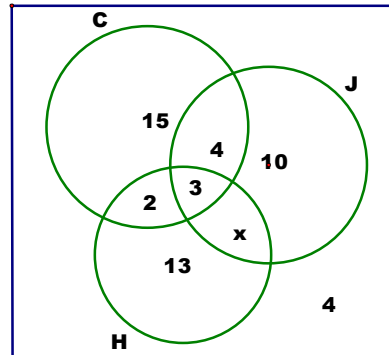
- 7.1.  $f(x) = 3 - 2x^2$ .  
Determine  $f'(x)$ , from *first principles*. (5)
- 7.2. Determine the following:
- 7.2.1.  $f'(x)$ , if  $y = f(x) = x^2 \left( x - \frac{3}{2} \right) + 2x - 7$  (3)
- 7.2.2. The value(s) of  $x$  for which the equation of the tangent to  $f$  is  $y = 8x - 17$  (4)
- 7.3. Determine  $\frac{d}{dx} \left[ \frac{3x^3 - 5x - 2}{x^2} \right]$ . (3)  
[15]

### QUESTION 8

- $f(x) = 3x^3 + 4x^2 - 17x - 6$ .
- 8.1. Determine the  $X$ -intercepts and the coordinates of the turning points of  $f$ . (12)
- 8.2. Draw a neat sketch graph of  $f$ , showing all the intercepts with the axes and turning points. [The DIAGRAM SHEET must be used.] (4)
- 8.3. For which values of  $x$  will  $f'(x) > 0$ ? (2)  
[18]

**QUESTION 9**

- 9.1. The following Venn diagram illustrates the music preferences of 60 people.  
 $C$  indicates the group of people listening to Classical music,  
 $J$  indicates the Jazz lovers and  
 $H$  the lovers of Heavy metal.



Calculate the following, leaving the answers in simplest fraction form:

- 9.1.1. The value of  $x$  (2)
- 9.1.2. The probability of someone listening to Classical music or Jazz (2)
- 9.1.3. The probability of someone listening to Heavy Metal and Jazz, but not to Classical music (2)
- 9.1.4. The probability of someone listening to Classical music only (2)
- 9.1.5. The probability of someone not listening to any of these types of music (1)
- 9.2. The sample space,  $S$ , is the set of the first 20 natural numbers. Each of these numbers is printed on a piece of paper and the pieces of paper must be selected at random. All the numbers have an equal chance of being selected.

$A$  is the event that a Prime number is selected.  
 $B$  is the event that a multiple of 3 is selected

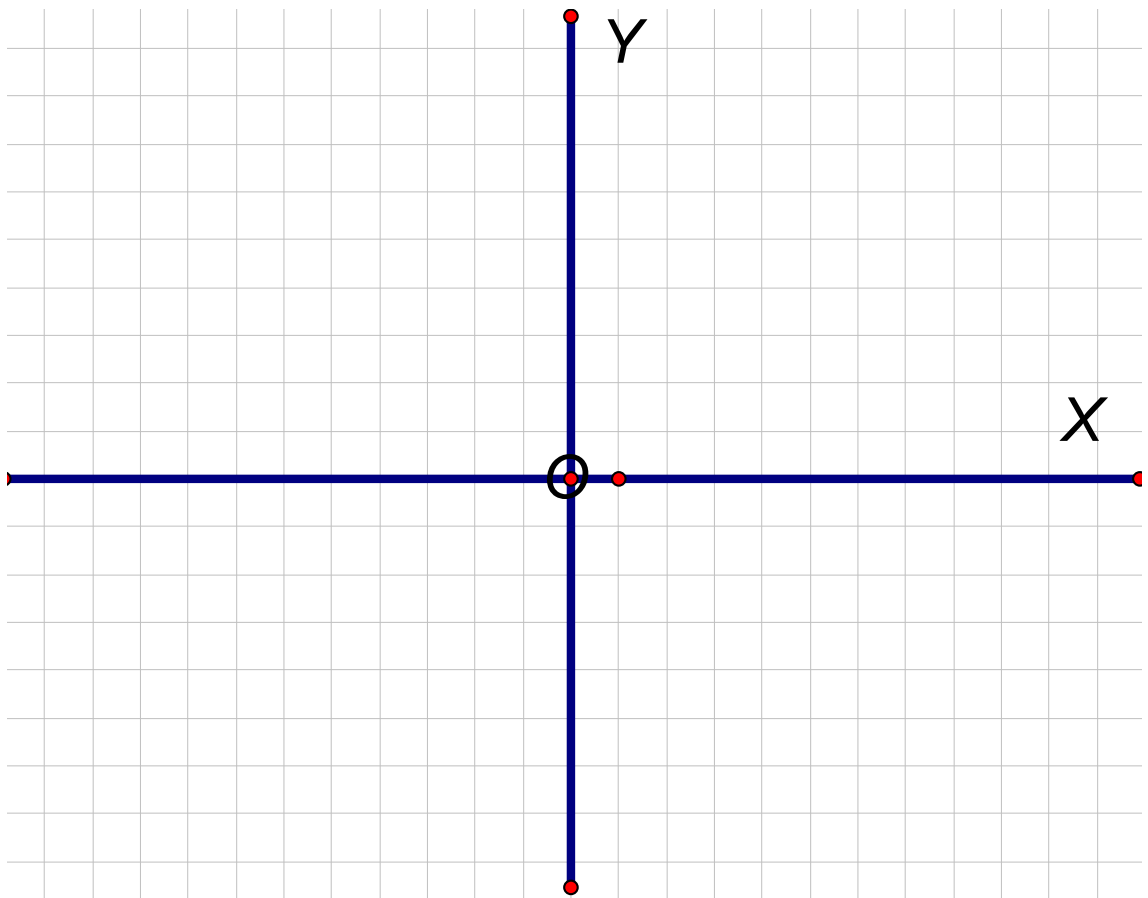
Show whether the events  $A$  and  $B$  are independent. (3)  
[12]

GRADE 12 MATHEMATICS  
JUNE 2015  
PAPER 1

**DIAGRAM SHEET**

**NAME:**

**QUESTION 4.2**



QUESTION 8.2

