ROEDEAN SCHOOL (SA)

INSPIRING A LIFE OF SIGNIFICANCE

MATRIC PRELIMINARY EXAMINATION
MATHEMATICS

SEPTEMBER 2015

PAPER II

Time: 3 Hours
Reading Time: 10 Min

Marks: 150
Examiner: M Brown
Moderator: R. Karam

EXAMINATION NUMBER

Memo

PLEASE READ THESE INSTRUCTIONS CAREFULLY:

1. The Habits of Mind that you should be making use of in this examination are: Questioning and Problem Solving, Applying Past Knowledge to new situations, Striving for Accuracy and Precision and Metacognition.
2. This question paper consists of 24 pages and an Information Sheet. Please check that your paper is complete.
3. Read the questions carefully.
4. Answer ALL the questions. **All the questions must be answered on the question paper.**
5. Number your answers as the questions are numbered.
6. All the necessary working details must be clearly shown. Answers only will not necessarily be awarded full marks.
7. Approved non-programmable and non-graphical calculators may be used unless otherwise stated.
8. Give answers correct to one decimal digit, where necessary.
9. It is in your own interest to write legibly and to present your work neatly.

PUPIL MARK

PUPIL %

SECTION A

Apply what you know about:
-properties of straight lines
-equations of straight lines

QUESTION 1

a) In each separate case, determine the numerical value of k if the line:

$4x + ky + 16 = 0$

1) passes through the point $(2\ ;\ 3)$. (2)

2) is parallel to the y-axis. (1)

3) is perpendicular to the line $3x - y + 7 = 0$ (3)

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a1) $4(2) + k(3) + 16 = 0$ ✓M

$3k = -24$

$k = -8$ ✓A

2) $k = 0$ ✓A

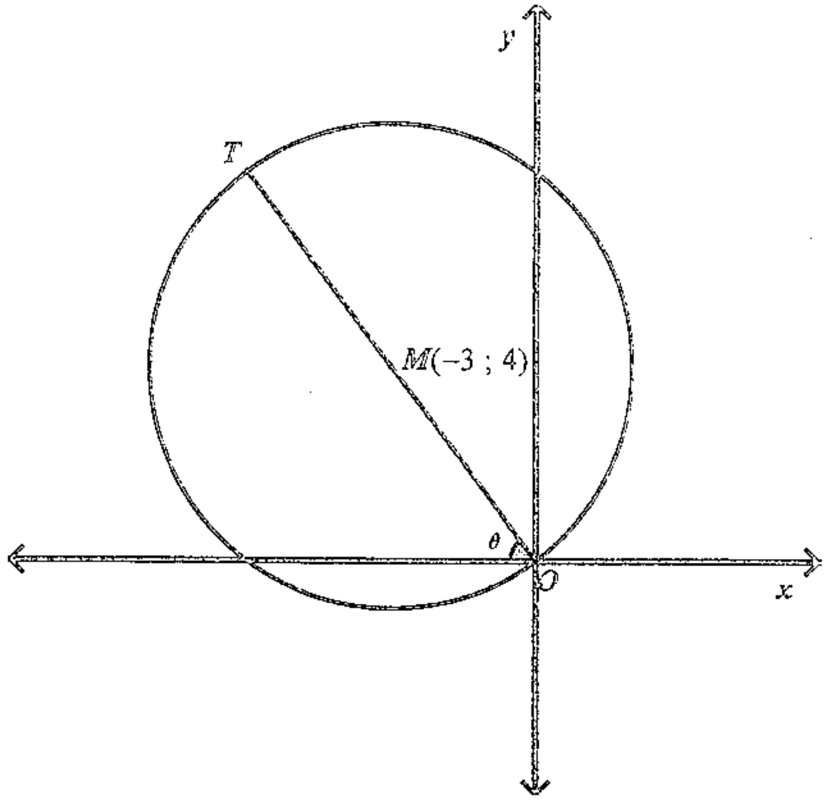
3) $ky = -4x - 16$ $\qquad\qquad y = 3x + 7$

$y = \frac{-4}{k}x - \frac{16}{k}$ ✓M

$\therefore \frac{-4}{k} = -\frac{1}{3}$ ✓A

$k = 12$ ✓A

b) In the diagram below, the circle has centre $M(-3\ ;\ 4)$ and passes through the origin.



1) Determine the length of the radius of the circle. (1)

$r = 5$ pythag ✓A

2) Write down the equation of the circle. (2)

$(x+3)^2 + (y-4)^2 = 25$ ✓

✓ ✓

3) Determine the co-ordinates of T, the end point of diameter OT. (2)

$T(-6\ ;\ 8)$ ✓A ✓A

4) Determine the value of θ, the angle that OT makes with the negative x-axis. (3)

$m_{OT} = \frac{-4}{3}$ ✓A

$\tan\theta = \frac{-4}{3}$ ✓m

$\theta = 53,1°$ ✓A

5) Determine the equation of the tangent to the circle at T. (4)

tangent : $m = \frac{3}{4}$ ✓cA

$y = \frac{3}{4}x + c$

$8 = \frac{3}{4}(-6) + c$ ✓m ✓A.

$y = 12\frac{1}{2}$

$y = \frac{3}{4}x + 12\frac{1}{2}$ ✓A.

[18]

Strive for accuracy:
-using trigonometric rules
-accurately applying CAST rule
-do not rush
-show all steps
-check yourself

QUESTION 2

a) Calculate the value of $M - T$, rounded to three decimal digits, if:

$M = \sin\dfrac{336°}{2}$ and $T = \dfrac{\sin 336°}{2}$ (2)

$M - T = 0{,}411$ ✓A ✓A.

b) Simplify the expression below to a single trigonometric ratio of A if $A \in (0°; 90°)$:

$\sqrt{1 - \sin A.\cos A.\tan A}$ (3)

$= \sqrt{1 - \sin^2 A}$ ✓M

$= \sqrt{\cos^2 A}$ ✓A

$= \cos A$ ✓A

c) Given: $5\tan A + 3 = 0$ and $0° \le A \le 270°$

Calculate the value of $\cos 2A$, without the use of a calculator, showing all working. (5)

alt memo

$\tan A = \dfrac{-3}{5}$

$x = -5$ ✓A

$y = 3$ ✓A.

$r = \sqrt{34}$ ✓A

$\cos 2A = \cos^2 A - \cos^2 A$ ✓A

$= \left(\dfrac{-5}{\sqrt{34}}\right)^2 - \left(\dfrac{3}{\sqrt{34}}\right)^2$ ✓CA

$= \dfrac{16}{34}$

$= \dfrac{8}{17}$ ✓A

d) Simplify the expression: $\dfrac{\sin(-180^\circ-\alpha).\tan(180^\circ-\alpha).\cos(360^\circ-\alpha)}{\sin^2(180^\circ+\alpha)+\sin^2(90^\circ+\alpha)}$ (6)

$= \dfrac{\left(-\sin(180+\alpha)\right)(-\tan\alpha)(\cos\alpha)}{(-\sin\alpha)^2+(\cos\alpha)^2}$ ✓M ✓A ✓A ✓M ✓A ✓A

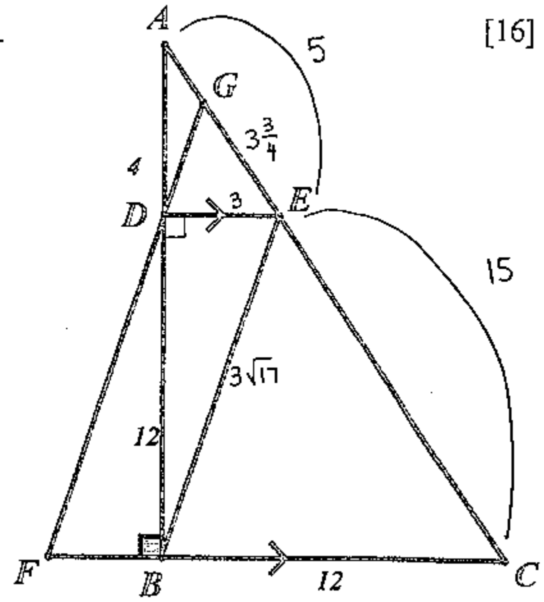
$= \dfrac{(+\sin\alpha)(-\tan\alpha)(\cos\alpha)}{1}$ ✓A

$= -\sin^2\alpha$ ✓A

$-\frac{1}{2}$ for incomplete reason

[16]

QUESTION 3



Refer to the diagram (not drawn to scale):

In the diagram ΔABC is a right-angled triangle.

The point D lies on AB and E lies on AC such that $DE//FC$.

$BC = 12$ units, $AD = 4$ units and $DB = 12$ units.

a) Show that $AC = 20$ units. (2)

$AC^2 = 12^2 + 16^2$ pythag ✓M

$AC^2 = 400$

$AC = 20$ units ✓A

b) Calculate, stating reasons, the size of:

1) AE 1½ for reason. (3)

2) EC (1)

1) DE // FC given ✓A.

∴ $\frac{AE}{AC} = \frac{AD}{AB}$ ~~prop. theorem~~ ✓M // side and prop.

$\frac{AE}{20} = \frac{4}{16}$

AE = 5 units ✓A

2) EC = 15 units ✓A

c) It is further given that: $GE = 3\frac{3}{4}$ units.

i) Determine the length of DE. (2)

ii) Hence, or otherwise, prove that DEBF is a parallelogram. (4)

ΔABC ||| ΔADE AAA

i) $\frac{DE}{BC} = \frac{AD}{AB}$ ~~prop theorem~~ ? ✓M ||| Δs sides in prop ΔABC ||| ADE.

$DE = 12 \times \frac{4}{16}$

= 3 units ✓A.

ii) $\frac{FC}{DE} = \frac{GC}{GE}$? ~~prop theorem~~ ✓M ||| Δs F sides in prop ΔADE ||| ΔGFC

$FC = 3 \times 18,75 \div 3,75$ $FC = \frac{18,75 \times 3}{3,75}$

= 15 units ✓A

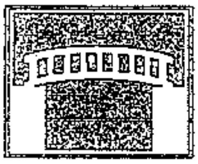
= 15

∴ FB = 3 units ✓A

∴ DEBF is a parm (one pair of opp = and //) ✓A.

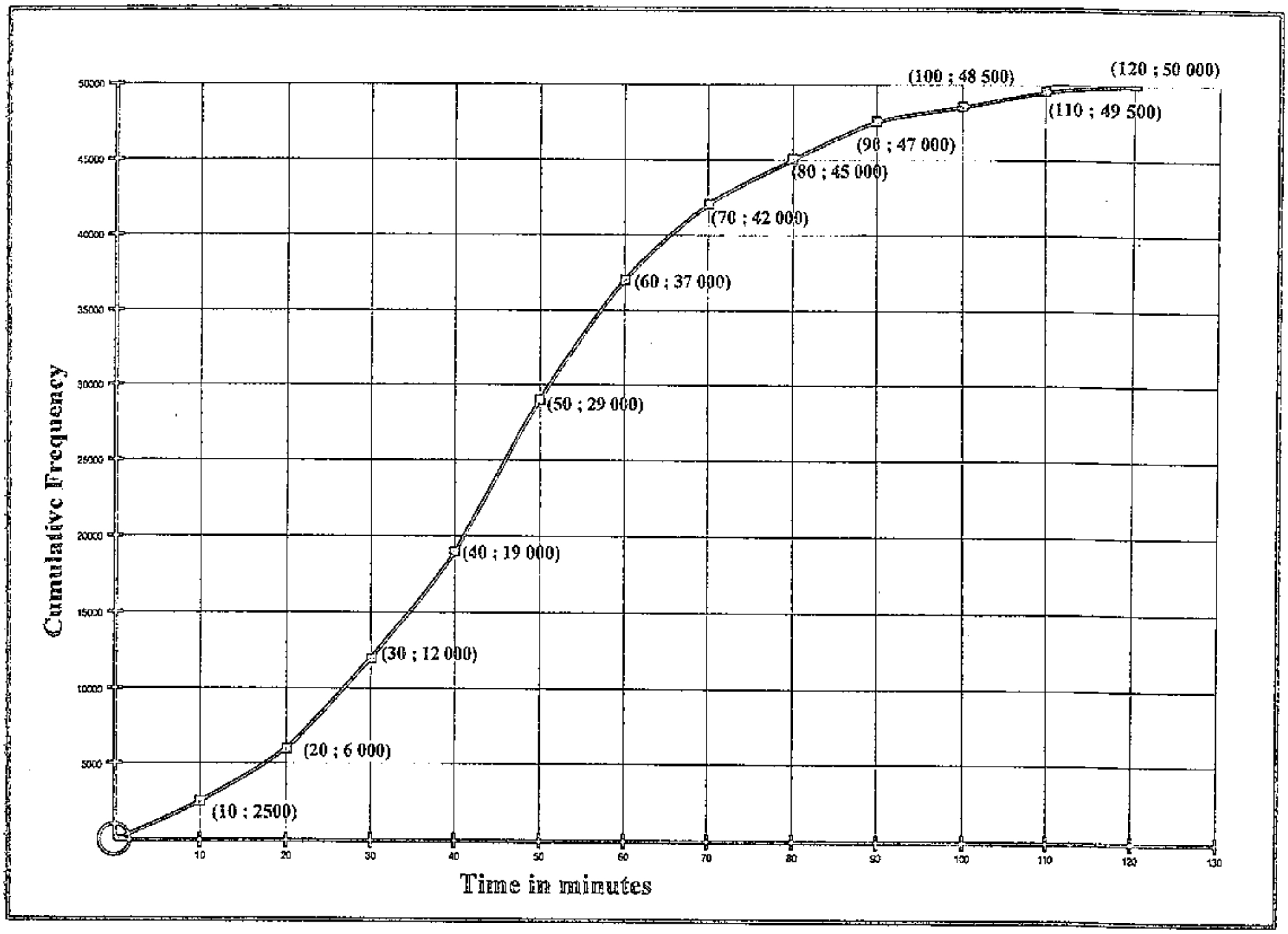
[12]

QUESTION 4



Apply what you know about:
-statistics
-ogives
-cumulative frequency
-box and whisker plots

a) Below is the cumulative frequency graph of the number of customers passing through the ticket booths at a theme park over a 2 hour period, starting at 14h00 and ending at 16h00.



1) Determine how many spectators passed through the ticket booths:

i) after the first hour. (1)

ii) between 14h30 and 14h40. (2)

iii) after 15h30. (2)

i) 37000 ✓A

ii) 7000 ✓M ✓A

iii) 3000 ✓M ✓A

2) Determine by what time:

i) 12% of the spectators had passed through the ticket booths. (2)

ii) 10% of the spectators still had to pass through the ticket booths. (2)

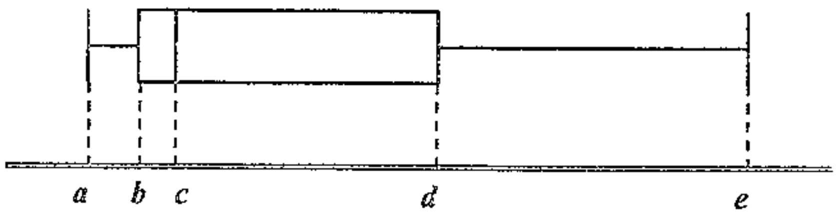
i) 14h20 ~~14h35~~ ✓M ✓A

ii) 15h20 ✓M ✓A

b) The stadiums that were built for the 2010 FIFA WORLD CUP together with their capacities are given in the table below:

Stadium	Capacity (Number of people)
Green point	70 000
Durban	70 000
Ellis Park – Johannesburg	61 000
Soccer City – Johannesburg	94 700
Free State – Bloemfontein	48 000
Port Elizabeth	48 000
Mbombela – Nelspruit	46 000
Peter Mokaba – Polokwane	46 000
Royal Bafokeng - Rustenburg	42 000
Loftus Versveld - Pretoria	50 000

1) The data in the table has been summarised by the box and whisker plot drawn below. Determine the values of a, b, c, d and e. (4)



$a = 42000$ ✓A

$b = 46000$ ✓A

$c = 49000$ ✓A

$d = 70000$ ✓A

$e = 94700$ ✓A

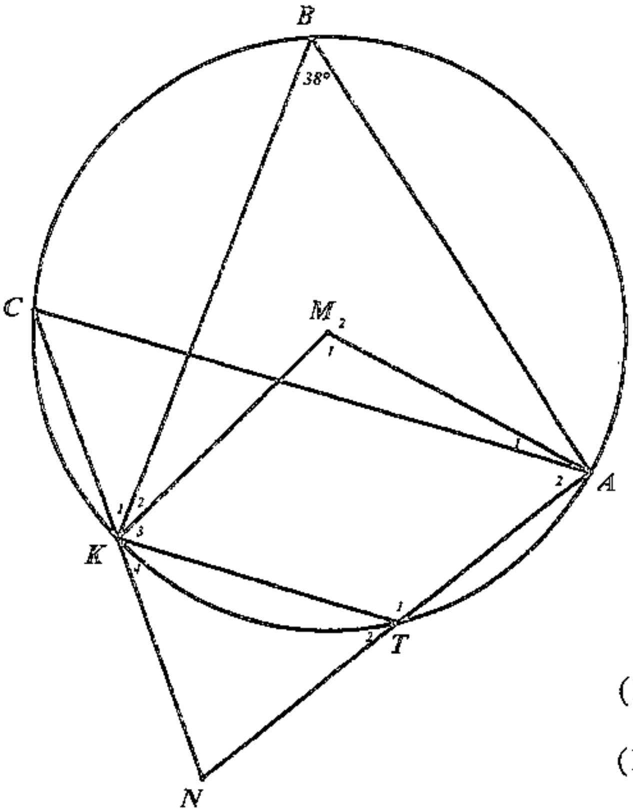
2) Discuss the skewness of the data. (2)

Skewed to the right ✓

There is a large difference in seating numbers between the lower 50% of the stadiums and the upper 50%. The lower 50% accommodate more similar numbers, while the upper 50% vary quite significantly in the grap sizes or capacity.

[15]

QUESTION 5



Refer to the diagram (not drawn to scale):
A, B, C, K and T lie on the circle.
AT produced and CK produced meet in N.
M is the centre of the circle.

$NA = NC$ and $\hat{B} = 38°$

a) Calculate, with reasons, the size of the following angles:

1) $\hat{M}_1$ (1)

2) $\hat{T}_2$ (1)

3) $\hat{C}$ (1)

4) $\hat{K}_4$ (3)

1) $\hat{M}_1 = 76°$ ∠ at centre ✓A

2) $\hat{T}_2 = 38°$ ext ∠ of cyclic quad ✓A

3) $\hat{C} = 38°$ ∠ in same seg ✓A

4) $\hat{C} = \hat{A}_2$ NA = NC isos Δ ✓M

$\hat{A}_2 = 38°$ isos Δ ✓CA

$\therefore \hat{K}_4 = 38°$ ext ∠ of cyclic quad ✓CA

b) Prove that $NK = NT$ (2)

$\hat{T}_2 = 38°$ proven

$\hat{K}_4 = 38°$ proven

$\therefore \hat{T}_2 = \hat{K}_4$ ✓CA

$\therefore NK = NT$ isos Δ ✓A

c) Prove that $KT // AC$ (2)

$\hat{K}_4 = \hat{C} = 38°$ proven ✓CA

$\therefore KT // AC$ corresp ∠'s are equal. ✓A.

d) Prove that AMKN is a cyclic quadrilateral. (2)

$\hat{N} = 104°$ ∠'s in a Δ

$\hat{M}_1 = 76°$ proven

$\therefore$ AMKN is cyclic opp ∠'s of quad are suppl.

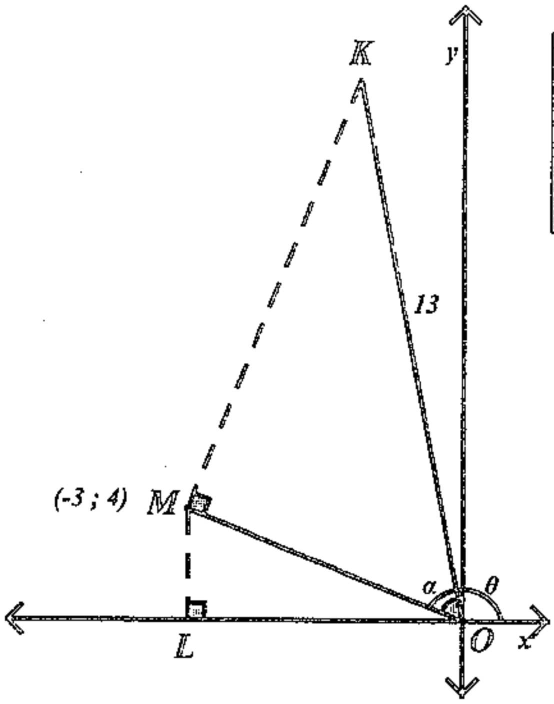
[12]

SECTION A TOTAL | 73 marks |

SECTION B

QUESTION 6

a) In the diagram (not drawn to scale), right-angled ΔOLM and ΔOMK have side lengths as shown. OM makes an angle of θ with the positive x-axis and $K\hat{O}M = \alpha$.



Strive for accuracy using:
- apply CAST rule accurately
-do not rush
-show all steps
-check yourself

1) Determine the value of $\sin\theta$ and $\sin\alpha$. (4)

2) Find, without using a calculator, the value of $\sin(\theta - \alpha)$. (3)

1) $OM = 5$ pythag ✓A.

$KM = 12$ pythag ✓A.

$\sin\theta = \frac{4}{5}$ ✓CA.

$\sin\alpha = \frac{12}{13}$ ✓CA.

2) $\sin\theta . \cos\alpha - \cos\theta . \sin\alpha$ ✓M

$= \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{-3}{5}\right)\left(\frac{12}{13}\right)$ ✓A

$= \frac{56}{65}$ ✓A.

b) 1) Complete the following: $\sin 3\alpha = \sin(2\alpha + \alpha) = \dots$ (1)

2) Hence, show that $\sin 3\alpha = \sin\alpha(4\cos^2\alpha - 1)$ (3)

3) Hence, prove that $\dfrac{\sin 3\alpha}{1 + 2\cos 2\alpha} = \sin\alpha$ (2)

1) $\sin 3\alpha = \sin(2\alpha + \alpha) = \sin 2\alpha \cdot \cos\alpha + \cos 2\alpha \cdot \sin\alpha$ ✓A.

2) $\sin 2\alpha \cdot \cos\alpha + \cos 2\alpha \cdot \sin\alpha$

$= 2\sin\alpha \cdot \cos\alpha \cdot \cos\alpha + (2\cos^2\alpha - 1)\sin\alpha$ ✓M

$= 2\sin\alpha \cdot \cos^2\alpha + 2\sin\alpha \cdot \cos^2\alpha - \sin\alpha$ ✓A.

$= 4\sin\alpha \cdot \cos^2\alpha - \sin\alpha$ ✓A.

$= \sin\alpha(4\cos^2\alpha - 1)$

3) LHS $= \dfrac{\sin 3\alpha}{1 + 2\cos 2\alpha}$

$= \dfrac{\sin\alpha(4\cos^2\alpha - 1)}{1 + 2(2\cos^2\alpha - 1)}$ ✓M ✓A.

$= \dfrac{\sin\alpha(4\cos^2\alpha - 1)}{4\cos^2\alpha - 1}$ ✓A.

$= \sin\alpha$

c) Given: $\tan 40° = \frac{1}{k}$.

Express $\frac{-4\cos^2 20° + 2}{2\sin 20° \cos 20°}$ in terms of k, without using a calculator. (5)

$$\frac{-2(2\cos^2 20° - 1)}{\sin 40°} \quad ✓A \quad ✓M$$

✓A

$$= \frac{-2\cos 40°}{\sin 40°} \quad ✓A$$

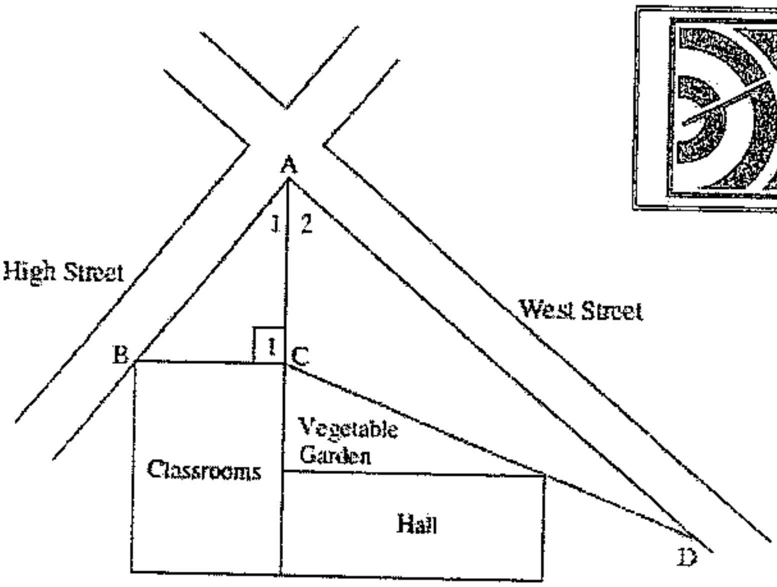
$$= -2k.$$

[18]

QUESTION 7

The local community was given a piece of ground at the intersection of High and West Streets on which to start a school. They have to find out how much fencing they need to buy, in order to enclose part of the property. $AB = 25$ meters, $\hat{A}_1 = \hat{A}_2 = 43°$, $\hat{D} = 21{,}3°$ and $\hat{C}_1 = 90°$.

The diagram is not drawn to scale.



Strive for accuracy using:
-do not rush
-show all steps
-check yourself
-have you answered the question that was asked?

a) Calculate, to one decimal digit the lengths of:

1) AC (2)

$\cos 43^\circ = \frac{AC}{AB}$

$AC = 25\cos 43^\circ$ ✓M

$= 18{,}3\,m$ ✓A

2) AD (4)

In $\triangle ACD$: $\hat{C} = 115{,}7^\circ$ $\angle$'s in a $\triangle$

$\frac{AC}{\sin 115{,}7^\circ} = \frac{18{,}3}{\sin 21{,}3^\circ}$

$AD = 45{,}4\ m$

b) For the proposed size of the school, they need at least $420m^2$ of available playing ground. They intend to use the areas designated by ΔABC and ΔACD for playing. Determine if these areas are sufficient to meet the departmental requirements. Show full working to justify your answer. (5)

$\triangle ABC \Rightarrow \text{Area} = \frac{1}{2} AB \times AC \sin 43^\circ$ ✓M

$= 156\ m^2$ ✓A

$\triangle ACD \Rightarrow \text{Area} = \frac{1}{2} AC \times AD \sin 43^\circ$ ✓M

$= \frac{1}{2}(18{,}3)(45{,}4)\sin 43^\circ$

$= 283{,}3\,m^2$

Total $= 439{,}3\,m^2$

$\therefore$ it is sufficient for departmental requirements.

[11]

Questioning and Problem Posing:
- analyse the information given
- determine what information can this give you and how it can help you.
- plan a strategy before answering
- check your working
- What steps must you go through to get to the answer?

QUESTION 8

A grouped distribution of the running time in minutes for 96 DVDs is shown in the table below.

Playing Time x (min)		Frequency	
$40 \leq x < 45$	42.5	2	85
$45 \leq x < 50$	47.5	8	700
$50 \leq x < 55$	52.5	14	735
$55 \leq x < 60$	57.5	27	1552.5
$60 \leq x < 65$	62.5	21	1312.5
$65 \leq x < 70$	67.5	13	877.5
$70 \leq x < 75$	72.5	6	435
$75 \leq x < 80$	77.5	3	232.5
$80 \leq x < 85$	82.5	2	165

✓M

a) The DVD distributors claim that the average running time for a DVD is over an hour. By referring to both the estimated mean and the medial class, substantiate whether their claim is correct or not. (4)

Estimate Mean $= \frac{6095}{96} = 692$ min ✓A

Estimated median lies in class : $55 \leq x < 60$

If the estimated mean is used their claim is correct

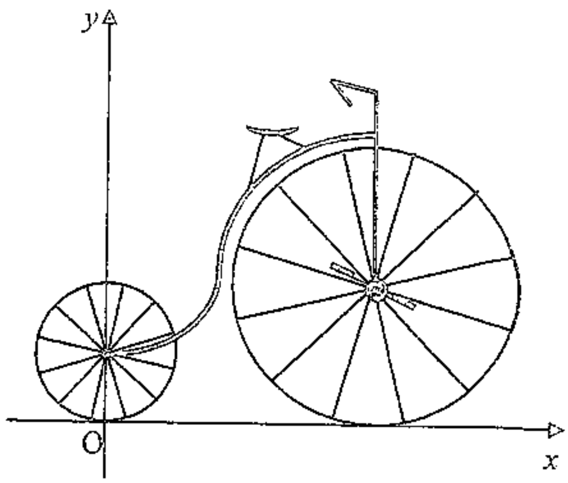
If the estimated median is used their claim is not correct

b) Estimate the standard deviation of the data given in the table. (1)

8,3

[5]

QUESTION 9



A penny-farthing bicycle is on display in a museum.

With coordinate axes as shown,
the equation of the rear wheel is $x^2 + y^2 - 6y = 0$
and the equation of the front wheel is
$x^2 + y^2 - 28x - 20y + 196 = 0$.
Distance is measured in centimetres.

a) Calculate the distance between the centres of the two wheels. (6)

small: $x^2 + y^2 - 6y + 9 = 9$

$x^2 + (y-3)^2 = 9$ ✓M

$\therefore r = 3$ centre $(0;3)$ ✓A

Big: $x^2 - 28x + 196 + y^2 - 20y + 100 = 100$

$(x-14)^2 + (y-10)^2 = 100$ ✓M

$r = 10$ centre $(14;10)$ ✓A

$D = \sqrt{(10-3)^2 + 14^2}$ ✓M

$= \sqrt{245}$

$= 15,7$ cm ✓A

b) Hence, calculate the clearance, i.e. the smallest gap, between the front and rear wheels. Give your answer to the nearest millimetre. (2)

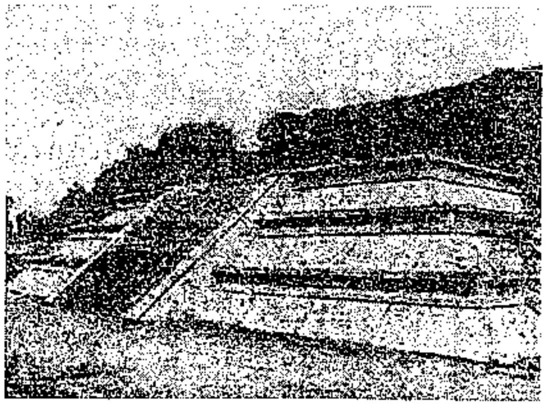
Smallest gap $= 15,7 - 10 - 3$ ✓M

$= 2,7$ cm ✓A

$= 27$ mm ✓A

[8]

QUESTION 10



The ancient Aztec pyramids consist of flat-topped pyramids placed on top of each other.

A side view of the Cholula pyramid (which consists of 2 layers) is shown in Figure 1.

An oblique view of a single layer is shown in Figure 2. The diagrams are not drawn to scale.

The base and top of each layer is a square.

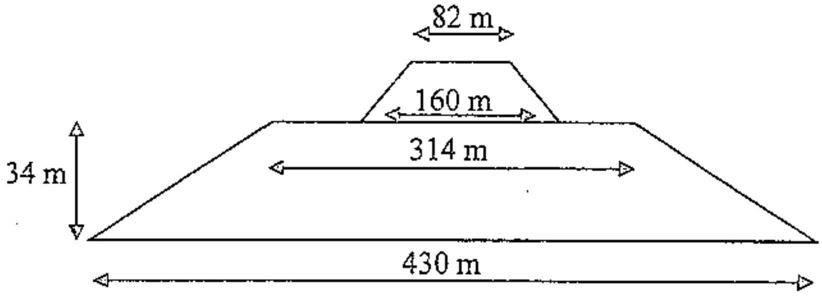


Figure 1: Side View

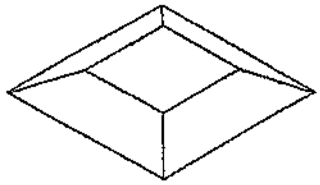
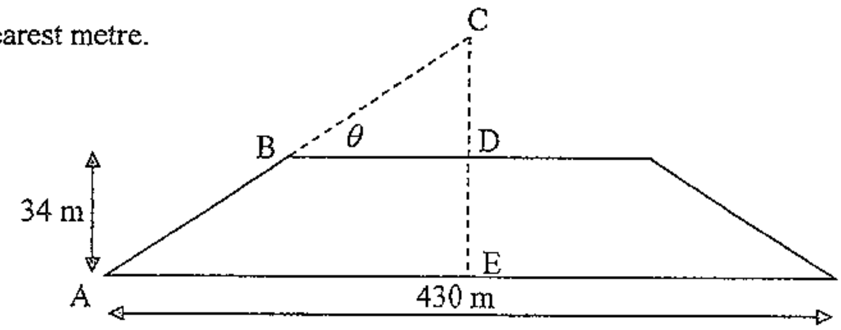


Figure 2: Oblique View

a) If the bottom layer of the Cholula pyramid had been built upwards to a point (like the Egyptian pyramids), determine how high it would be (i.e. determine CE), to the nearest metre. (5)



BD // AE

$\therefore \frac{BD}{AE} = \frac{CD}{CE}$ prop theorem ✓ M

AE = 215 m

BD = 157 m ✓ A let CD = x

Alt memo!

$\frac{157}{215} = \frac{x}{x+34}$ ✓✓ MA

$157(x + 34) = 215x$

$157x + 5338 = 215x$

$58x = 5338$

$x = 92$ ✓A

$\therefore$ CE $= 126$ m ✓A

b) Calculate the volume of the bottom layer of the Cholula pyramid to the nearest cubic metre, given that the height of the triangular pyramid (CE) is 126 metres.

The formula for the volume of a pyramid is:

$$\text{Volume} = \frac{1}{3} \times \text{base area} \times \text{height}$$ (4)

$A_{Base\ 1} = 430^2$ ✓A

$A_{Base\ 2} = 314^2$ ✓A

$V = \frac{1}{3}(430^2) \times 126 - \frac{1}{3}(314^2) \times 92$ ✓✓ MA

$= 474\ 2189\ m^3$ ✓A

[9]

Questioning and Problem Posing:
- analyse the information given
-determine what information can this give you and how it can help you.
-plan a strategy before answering
- check your working
- What steps must you go through to get to the answer?

QUESTION 11

a) Complete the statement:

A line parallel to one side of triangle …….. (2)

divides the other 2 sides of the triangle in proportia

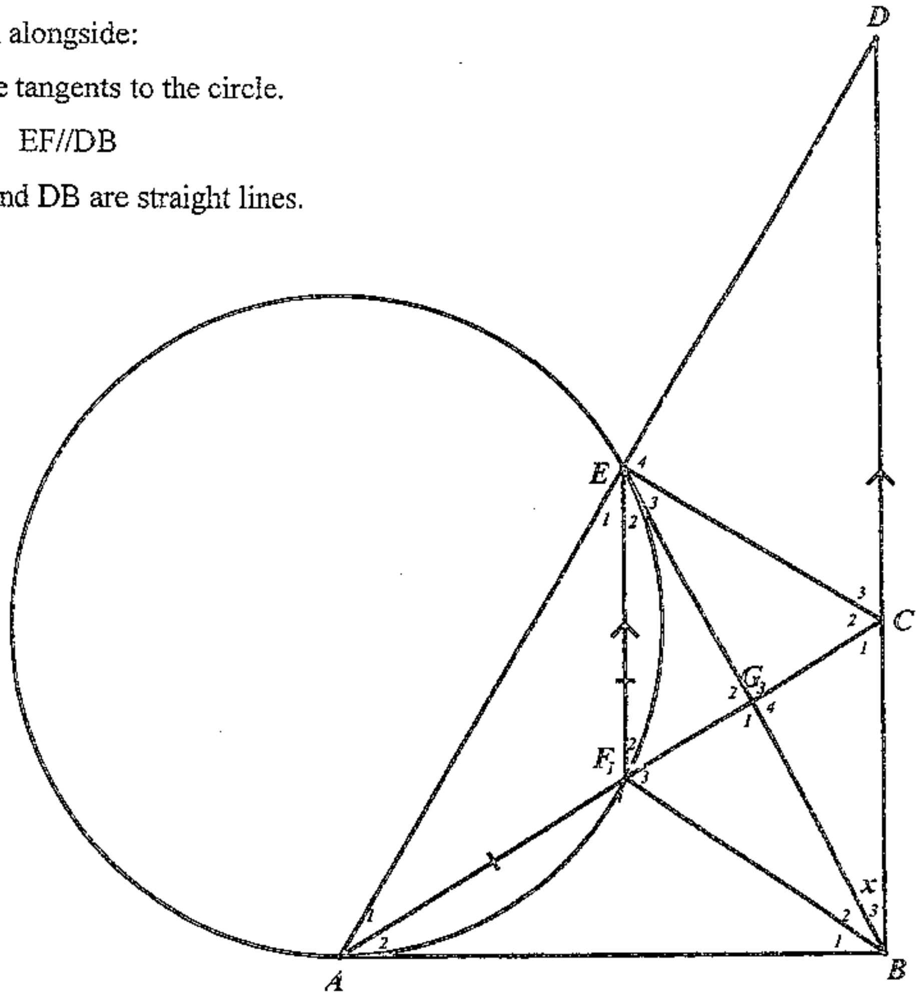
b) In the diagram alongside:

AB and BE are tangents to the circle.

AF = FE and EF//DB

AD, AC, EB and DB are straight lines.

$\hat{B}_3 = x$



1) Complete: $\frac{AF}{FC} = \frac{AE}{ED}$ (1)

2) With reasons, write down 5 other angles equal to x. (5)

$\hat{E}_2 = x$ alt <'s EF // BD

$\hat{A}_1 = x$ tan - chord theorem

$\hat{E}_1 = x$ AF = FE isos Δ

$\hat{D} = x$ corresp <'s EF // BD

$\hat{A}_2 = x$ tan - chord theorem

3) Prove that AECB is a cyclic quadrilateral. (2)

$\hat{B}_3 = \hat{A}_1 = x$ proven

∴ AECB is a cyclic quad (equal <'s subtended by equal chord)

4) Prove that $\Delta ACB /// \Delta DAB$ (3)

$\hat{B}$ is common ✓

$\hat{A}_2 = \hat{D} = x$ proven ✓

∴ $\hat{C}_1 = \hat{A}_1 + \hat{A}_2$ <'s in a Δ ✓

∴ ΔACB /// ΔDAB (AAA) ✓

5) Hence, deduce that $AB^2 = DB.CB$ (2)

$\frac{AC}{DA} = \frac{BC}{AB} = \frac{AB}{DB}$ ✓ similar Δ's ✓

∴ $AB^2 = DB . BC$

6) Is EC a tangent to the circle passing through E, G and A? Give a reason. (3)

$\hat{E}_3 = \hat{A}_2$ <'s in same seg ✓

$\hat{A}_2 = \hat{A}_1 = x$ proven ✓

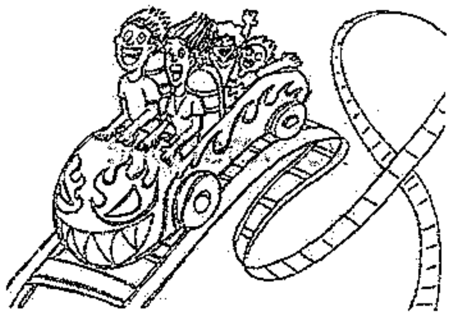
∴ $\hat{E}_3 = \hat{A}_1$ ✓

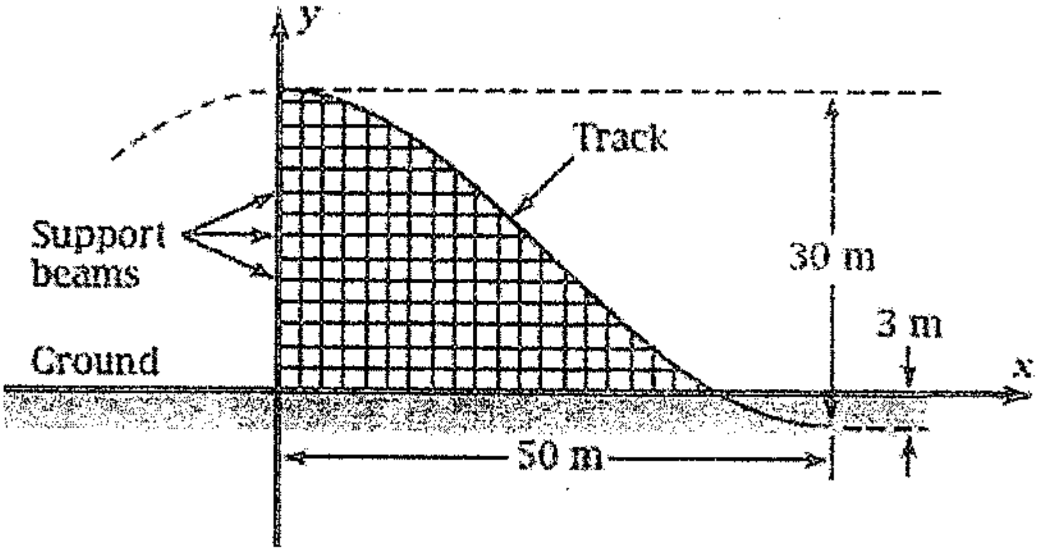
∴ EC is a tangent to EGA. (< btw line and chord = < in alt seg) ✓

[18]

QUESTION 12

A preliminary sketch of a portion of a roller coaster track which is being developed at a theme park is shown below. The track is in the shape of a cosine function. The highest and lowest points of the track are separated by 50 m horizontally and 30 m vertically.
The lowest point is 3 m below the ground.





The equation of the function is of the form: $f(x) = a\cos(px) + q$

a) Find the values of a and q respectively. (2)

$a = 15$ ✓A

$q = 12$ ✓A

b) Show that $p = 3{,}6$ (3)

$-3 = 15\cos(50p) + 12$ ✓M

$-15 = 15\cos(50p)$

$-1 = \cos(50p)$

$180° = 50°p$

$p = \frac{18}{5}$

c) Determine the horizontal distance from the y-axis where the rollercoaster goes below ground. (3)

$0 = 15 \cos \frac{18}{5}p + 12$ ✓M

$-\frac{12}{15} = \cos \frac{18}{5}p$

$\frac{18}{5}p = 14,31\ldots$ ✓A

$p = 39,8°$

$\therefore 39,8\text{ m}$ ✓A

[8]

SECTION B TOTAL 77 marks

Rough work

Total: 150 Marks