Matric Maths Paper 1
September 2015

QUESTION 1

a) $\frac{2^x \cdot 9^{x+1}}{3^x \cdot 6^{x-1}}$

$= \frac{2^x \cdot 3^{2x+2}}{3^x \cdot 2^{x-1} \cdot 3^{x-1}}$ ✓M ✓A

$= 2^{x-x+1} \cdot 3^{2x+2-x-x+1}$

$= 2^1 \cdot 3^3$

$= 2 \times 27$

$= 54$ ✓A ③

b1) $\log_{(x+1)} 8 = \frac{3}{2}$

$(x+1)^{\frac{3}{2}} = 8$ ✓M

$(x+1)^{\frac{3}{2}} = 2^3$

$x+1 = (2^3)^{\frac{2}{3}}$ ✓M

$x+1 = 2^2$

$x+1 = 4$

$x = 3$ ✓A ③

2) $9^x - 3^{x+1} = 54$

$3^{2x} - 3^x \cdot 3 - 54 = 0$ ✓M

Let $3^x = k$

$k^2 - 3k - 54 = 0$ ✓A

$(k-9)(k+6) = 0$ ✓M

$\therefore k = 9$ or $k = -6$

$3^x = 9$ $\quad 3^x = -6$

$x = 2$ ✓A not valid ④

c1) $px^2 + 6x - p = 0$

$x = \frac{-6 \pm \sqrt{(6)^2 - 4(p)(-p)}}{2p}$ ✓M

$= \frac{-6 \pm \sqrt{36 + 4p^2}}{2p}$ ✓A

$= \frac{-6 \pm 2\sqrt{9 + p^2}}{2p}$

$= \frac{-3 \pm \sqrt{9+p^2}}{p}$ ✓A ③

2) For real roots $\Delta \geq 0 \therefore p \in \mathbb{R}$ ✓✓A ② [15]

QUESTION 2

a) $\sum_{k=3}^{6} 2^k$

$= 2^3 + 2^4 + 2^5 + 2^6$ ✓M

$= 120$ ✓A ②

b) 3 ; 8 ; 15 ; 24

1st diff 5 ; 7 ; 9

2nd diff 2 ; 2 ✓M

$2a = 2$

$a = 1$ ✓A

$3a + b = 5$

$b = 5 - 3$

$b = 2$ ✓A

✓M

$a + b + c = 3$

$1 + 2 + c = 3$

$c = 0$

OR ✓M

$T_n = 3 + (n-1)5 + \frac{(n-1)(n-2)}{2} \cdot 2$

$= 3 + 5n - 5 + n^2 - 3n + 2$

$= n^2 + 2n$ ✓A ✓A

$\therefore T_n = n^2 + 2n$

$T_{50} = 50^2 + 2(50)$

$= 2600$ ✓A ⑤

c) $S_\infty = 5a$

$\frac{a}{1-r} = 5a$ ✓M ✓A

$a = 5a(1-r)$

$\frac{1}{5} = 1-r \quad (a \neq 0)$

$r = 1 - \frac{1}{5}$

$= \frac{4}{5}$ ✓A ③

d) $T_{12} - T_{11} = 5$

$\therefore d = 5$ ✓A

$S_{12} - S_{11} = 46$

i.e. $T_{12} = 46$ ✓A

$a + 11d = 46$

$a + 11(5) = 46$ ✓M

$a = 46 - 55$

$a = -9$

$\therefore T_1 = -9$ ✓A ④

e 1) $b ; a ; 9$

$\frac{T_2}{T_1} = \frac{T_3}{T_2}$ ✓M

$\frac{a}{b} = \frac{9}{a}$ ✓A

$a^2 = 9b$ ②

2) $a + b + 9 = 19$ ✓M

$a = 10 - b$ ✓A

$\therefore (10-b)^2 = 9b$ ✓M

$100 - 20b + b^2 - 9b = 0$

$b^2 - 29b + 100 = 0$ ✓A

$(b-25)(b-4) = 0$

$\therefore b = 25$ or $b = 4$

$\therefore$ Shortest side is 4 ✓A cm ⑤

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QUESTION 3

a) $\lim_{x \to 2} \frac{x^3 - 8}{2 - x}$

$= \lim_{x \to 2} \frac{(x-2)(x^2+2x+4)}{-(x-2)}$ ✓A ✓A ✓M

$= \lim_{x \to 2} -(x^2 + 2x + 4)$

$= -12$ ✓A ③

b 1) $3 - 4x^3$

$\frac{d}{dx} = -12x^2$ ✓A ①

2) $\frac{1}{\sqrt{x}} + \frac{3}{\sqrt{x^3}} - \frac{1}{4x^2}$

$= x^{-\frac{1}{2}} + 3x^{-\frac{3}{2}} - \frac{1}{4}x^{-2}$ ✓M

$\frac{d}{dx} = -\frac{1}{2}x^{-\frac{3}{2}} - \frac{9}{2}x^{-\frac{5}{2}} + \frac{1}{2}x^{-3}$

$= -\frac{1}{2x^{\frac{3}{2}}}$ ✓A $- \frac{9}{2x^{\frac{5}{2}}}$ ✓A $+ \frac{1}{2x^3}$ ✓A

$= -\frac{1}{2\sqrt{x^3}} - \frac{9}{2\sqrt{x^5}} + \frac{1}{2x^3}$ ④

3) $\frac{2x^2-7x+3}{x-3}$

$= \frac{(2x-1)(x-3)}{(x-3)}$ ✓M

$= 2x-1$ ✓A

$\frac{d}{dx} = 2$ ✓A ③

c) $f(x) = \frac{16}{x}$

$= 16x^{-1}$

$f'(x) = -16x^{-2}$ ✓A

$f'(4x) - 4f'(x)$

$= -16(4x)^{-2} - 4(-16x^{-2})$ ✓M

$= -16\left(\frac{1}{16x^2}\right) + \frac{64}{x^2}$ ✓A

$= -\frac{1}{x^2} + \frac{64}{x^2}$

$= \frac{63}{x^2}$ ✓A ④

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QUESTION 4

$A = P\left(1+\frac{i^{(4)}}{4}\right)^{4n}$

✓A ✓A

$35000 = 5000\left(1+\frac{9,6\%}{4}\right)^{4n}$ ✓M

$7 = \left(1+\frac{9,6\%}{4}\right)^{4n}$

$4n = \log_{\left(1+\frac{9,6\%}{4}\right)} 7$ ✓M

$4n = 82,048...$

$n = 20,5121...$ ✓A

i.e. 20 years 7 months ✓CA

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QUESTION 5

a) $y = p^x$

$\frac{9}{4} = p^{-2}$ ✓M

$\frac{4}{9} = p^2$ ✓A

$\therefore p = \frac{2}{3}$ ✓A ③

b) $y = a(x-x_1)(x-x_2)$

$y = a(x-1)(x+3)$ ✓A

sub $(-2; \frac{9}{4})$

$\frac{9}{4} = a(-2-1)(-2+3)$ ✓M

$\frac{9}{4} = a(-3)(1)$

$\frac{9}{4} = -3a$

$a = -\frac{3}{4}$ ✓A

$y = -\frac{3}{4}(x-1)(x+3)$ ✓M

$= -\frac{3}{4}(x^2+2x-3)$

$= -\frac{3}{4}x^2 - \frac{3}{2}x + \frac{9}{4}$ ✓A ⑤

c) $f(x) = -\frac{3}{4}(x^2+2x-3)$

$= -\frac{3}{4}x^2 - \frac{3}{2}x + \frac{9}{4}$

$x = \frac{-b}{2a}$

$= \frac{-\left(-\frac{3}{2}\right)}{2\left(-\frac{3}{4}\right)}$

$= -1$ ✓A

$f(-1) = 3$ ✓CA

$Q(-1;3)$ ②

d) $S(0;\frac{9}{4})$ ✓A

$R(0;1)$ ✓A

$SR = \frac{9}{4} - 1$

$= \frac{5}{4}$ ✓CA ③

e1) $y = \left(\frac{2}{3}\right)^x$

$x = \left(\frac{2}{3}\right)^y$ ✓M

$y = \log_{\frac{2}{3}} x$

$g^{-1}(x) = \log_{\frac{2}{3}} x$ ✓A ②

2) Domain: $x > 0$ ✓A

Range: $y \in R$ ✓A ②

f) $y = -\frac{3x^2}{4} - \frac{3x}{2} + \frac{9}{4}$

Reflection about y-axis

$y = -\frac{3(-x)^2}{4} - \frac{3(-x)}{2} + \frac{9}{4}$ ✓M

h: $y = -\frac{3x^2}{4} + \frac{3x}{2} + \frac{9}{4}$ ✓A ②

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QUESTION 6

a)1) $A = P(1+i)^n$

$= 1\,200\,000(1 + 15\%)^4$ ✓M

$= R2\,098\,807,50$ ✓A ②

2) Percentage increase

$= \frac{2\,098\,807,50 - 1\,200\,000}{1\,200\,000}$ ✓M $\times 100$

$= 74,9\%$ ✓CA ②

b) $A = P(1-i)^n$ ✓A ✓A

$491\,520 = 1\,200\,000(1-i)^4$ ✓M

$\frac{256}{625} = (1-i)^4$

$\sqrt[4]{\frac{256}{625}} = 1-i$

$i = 1 - \sqrt[4]{\frac{256}{625}}$

$i = \frac{1}{5}$ ✓A

$i = 20\%$ p.a ✓CA ④

c) Shortfall $= 2\,098\,807,50 - 491\,520$

$= R1\,607\,287,50$ ✓CA ①

d) $x = \frac{1\,607\,287,50}{\left[\frac{\left(1 + \frac{9\%}{12}\right)^{12\times 4} - 1}{\frac{9\%}{12}}\right]}$ ✓A ✓A ✓M

$= R27\,942,76$ ✓CA ④

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QUESTION 7

a 1) P(study all day)
$= \frac{1}{3} \times \frac{1}{5} \times \frac{1}{4}$ ✓M
$= \frac{1}{60}$ ✓A ②

2) P(TV all day)
$= 0$ ✓A ①

3) P(Shopping and Scrabble)
$= \frac{5}{60}$ ✓A
$= \frac{1}{12}$ ✓A ②

4) P(movies or reads)
$= P(\text{movies}) + P(\text{reads}) - P(\text{movies and reads})$ ✓M
$= \frac{20+15-5}{60}$ ✓A
$= \frac{1}{2}$ ✓A ③

b) Let r be radius of large semi-circle
Area of large semi-circle $= \frac{1}{2}\pi r^2$ ✓A
Area of 2 small semi ⊙s
$= 2 \times \frac{1}{2} \pi \left(\frac{r}{2}\right)^2$
$= \frac{\pi r^2}{4}$ ✓A

P(point in shaded region)
$= \frac{\pi r^2}{4} \div \frac{\pi r^2}{2}$ ✓M
$= \frac{1}{2}$ ✓A ④

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QUESTION 8

a 1) $f(x) = -x^3 + 5x^2 + 8x - 12$
$f'(x) = -3x^2 + 10x + 8$ ✓A
$f'(2) = -3(2)^2 + 10(2) + 8$ ✓M
$= 16$
$f(2) = 16$ ✓A
$y - y_1 = m(x - x_1)$
$y - 16 = 16(x-2)$
$y = 16x - 32 + 16$
$y = 16x - 16$ ✓A ④

2) $16x - 16 = -x^3 + 5x^2 + 8x - 12$ ✓M
$x^3 - 5x^2 + 8x - 4 = 0$ ✓A
$(x-2)(x-2)(\quad) = 0$
$(x^2 - 4x + 4)(x-1) = 0$

∴ The tangent cuts the graph again when $x = 1$ ✓A ③

b 1) $x \in [-2; 2)$ ✓A ✓A or $-2 \leq x < 2$ ②

2) $x \in \mathbb{R}$ ✓✓A ②

3) $x \in (-\infty; 0)$ or $x < 0$ ✓✓A ②

c 1) $x = 1$ ✓A ; $x = 3$ ✓A ②

2) $x = 2$ ✓✓A ②

3) $x < 1$ ✓A or $x > 3$ ✓A ②

4) 
✓✓A shape ②

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QUESTION 9

a) 3,2kg ✓A ①

b) $m(t) = \frac{t^3}{864} - \frac{t^2}{72} + 3,2$

$m'(t) = \frac{t^2}{288} - \frac{t}{36}$ ✓M

for minimum value $m'(t) = 0$

$\frac{t^2}{288} - \frac{t}{36} = 0$ ✓A

$t^2 - 8t = 0$ ✓M

$t(t-8) = 0$

$\therefore t = 0$ or $t = 8$ ✓A

i.e. After 8 days ✓CA ⑤

c) $\frac{t^3}{864} - \frac{t^2}{72} + 3,2 = 3,2$ ✓M

$t^3 - 12t^2 = 0$ ✓A

$t^2(t-12) = 0$ ✓A

$\therefore t = 0$ or $t = 12$

i.e. After 12 days ✓A ④

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QUESTION 10

a) Let the height be x
The length is $8x$
The width is $4x$ } ✓M

In Δ AHF:

$AF^2 = AH^2 + HF^2$

$= (8x)^2 + (4x)^2$ ✓A

$= 64x^2 + 16x^2$

$= 80x^2$ ✓A

In Δ AFE:

$AE^2 = AF^2 + EF^2$

$36^2 = 80x^2 + x^2$ ✓M

$1296 = 81x^2$

$x^2 = 16$

$x = 4$ ✓A

∴ Volume $= l \times b \times h$

$= 8(4) \times 4(4) \times 4$ ✓M

$= 2048\,cm^3$ ✓CA ⑦

b) Let the two whole numbers be x and y

$x^2 + y^2$ ···· ①

$(x+y)^2$ ··· ②

$(x+y)^2 = x^2 + y^2 + 72$ ✓M✓A

$x^2 + 2xy + y^2 = x^2 + y^2 + 72$

$2xy = 72$

$xy = 36$ ✓A

∴ Possibilities are:

1 and 36 ✓A

2 and 18 ✓A

3 and 12 ✓A

4 and 9 ✓A

6 and 6 ✓A ⑤

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QUESTION 11

a) $F_9 = 34$ ✓A ①

b) $L_7 = 29$ ✓A ①

c) $\phi = 1,61803398...$

$\phi = 1,61803$ ✓A ①

d) 1) $F_n \times L_n$

$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]\left[\left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$ ✓M

$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{2n} - \left(\frac{1-\sqrt{5}}{2}\right)^{2n}\right]$ ✓A ②

2) F_{2n} ✓✓A ②

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