2015 G12 PRELIM PAPER 1

1a (1) $3x^2 - 12x = x + 10$ ✓

$\therefore 3x^2 - 13x - 10 = 0$ ✓

$\therefore (3x + 2)(x - 5) = 0$ ✓

$\therefore x = \frac{-2}{3}$ or $x = 5$ ✓ (4)

(2) $4 - x^2 = 5x$ ✓

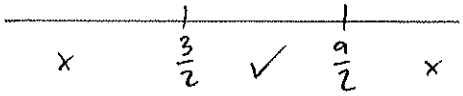
$x^2 + 5x - 4 = 0$ ✓

$x = \frac{-5 \pm \sqrt{25 - 4.(1).(-4)}}{2}$ ✓m

$x = \frac{-5 \pm \sqrt{41}}{2}$ ✓a or $x = 0{,}7$ $x = -5{,}7$ (4)

(3) $4x^2 - 24x + 27 \leq 0$ ✓

$(2x - 3)(2x - 9) \leq 0$ ✓



✓ ✓

$\frac{3}{2} \leq x \leq \frac{9}{2}$ ✓ (4)

(4) $3^{2x}(3^2 - 1) = 8\sqrt{3}$ ✓

$\therefore 3^{2x} = 3^{\frac{1}{2}}$ ✓

$\therefore 2x = \frac{1}{2}$ ✓

$x = \frac{1}{4}$ ✓ (4)

(5) $\sqrt{2x - 9} = x - 6$ ✓

$\therefore 2x - 9 = x^2 - 12x + 36$ ✓

$\therefore x^2 - 14x + 45 = 0$ ✓

$\therefore (x - 9)(x - 5) = 0$ ✓

$\therefore x = 9$ check

$x = 5$ (x no solution) ✓ (5)

b)

$2x^2 + 4xp + 2p^2 - 32 = 2x^2 + qx + 18$ ✓

$\therefore 4p = q$ ✓

And $2p^2 - 32 = 18$

$2p^2 = 50$ ✓

$p^2 = 25 \quad \therefore p = \pm 5$ ✓

$q = \pm 20$ ✓ (5)

c)

$$\sum_{i=1}^{7} \frac{3i + 1}{(i + 2)^2}$$ ✓ ✓ ✓ ✓ (4)

or

$$\sum_{j=0}^{6} \frac{3j + 4}{(j + 3)^2}$$ ✓ ✓ ✓ ✓

TOTAL [30]

Question 2

a) $a = -73$, $d = 22$ ✓ ✓ $\therefore T_{10} = a + 9d$ ✓

$T_{10} = -73 + 9(22)$

$T_{10} = 125$ → (3)

b) i) $T_1 = 280$, $d = 8$ ✓M $\therefore T_{91} = 280 + 90 \times 8$ ✓M

$T_{91} = 280 + 720$

$T_{91} = 1000$ → ✓a (3)

ii) $S_{91} = \frac{91}{2}[2.280 + 90.8]$ ✓M

$S_{91} = 58240$

Plus $1000 \times 12 = 12000$ ✓M

$S_{103} = 70240$ → ✓a (3)

c) i) $0,3 \times 0,3 = 0,09\,g$ ✓a (1)

ii) $0,3 \times 0,7 = 0,21\,g$ ✓a (1)

iii)

First process	$0,09 \times 450$	$= R40,5$	✓M
Second process	$0,063 \times 450$	$= R28,35$	✓M
Third ~	$0,0441 \times 450$	$= R19,85$	4 times ✓a
Forth ~	$0,03087 \times 450$	$= R13,89$	
Fifth ~	$0,02$	$= R9,72$ (not profitable)	✓M (4)

d) $\frac{a}{1-r} = 9$ ✓M $S_\infty = 9$ & $a = 1$

$\therefore 1 = 9(1-r)$ ✓M

$\therefore 9 - 9r = 1$ $\therefore 9r = 8$ $\therefore r = \frac{8}{9}$ → ✓a. (3)

Question 3

a) 46 84 116 (any method) ✓m

38 32

−6

$2a = -6 \therefore a = -3$ ✓a

$3a + b = 38$

$b = 47$ ✓a

$a + b + c = 46 \therefore c = 2$ ✓a

$T_n = -3n^2 + 47n + 2$ ✓a →

(5)

b) $2a = 4 \therefore a = 2$ ✓m

$3a + b = -9 \therefore b = -15$ ✓m

$T_n = 2n^2 - 15n + c$ $T_{10} = 53$. ✓m

$\therefore 53 = 2(10)^2 - 15(10) + c$ ✓m

$\therefore c = 53 - 200 + 150$

$\therefore c = 3$ ✓a

(5)

→

QUESTION 4

a) $(-\infty; 6]$ or $y \leq 6$ $y \in \mathbb{R}$

b) $y = -(x-2)^2 + 6$

c) $x = 0$ $y = -5$

d) $h(x) = \frac{2}{x+2} + 5$ ✓ ✓ ✓

e) $g(x) = 6$ when $6 = \frac{-2}{x} - 5$ ✓m $\therefore \frac{2}{x} = -11 \therefore x = -\frac{2}{11}$ ✓a

$\therefore$ horizontal distance $= \frac{24}{11}$ units ✓a

f) $f(2) = 6$ ✓ $g(2) = -6$ ✓ $\therefore f(x) - g(x) = 12$ units ✓

(14)

SECTION B

QU5 a) $1 + Eff = \left(1 + \frac{0,12}{12}\right)^{12}$ ✓

$\therefore 1 + Eff = 1,1268\ldots$ $\therefore$ Eff Rate = 12,68% ✓ (2)

b) $1 + 0,12 = \left(1 + \frac{i^{(4)}}{4}\right)^{4}$ ✓

$\therefore 1 + \frac{i^{(4)}}{4} = (1,12)^{\frac{1}{4}}$ ✓

$\therefore \frac{i^{(4)}}{4} = 0,028737$ $\therefore i^{(4)} = 11,49\%$ ✓ (3)

c)(i) $650\,000$ ✓ $= \dfrac{x\left[1 - \left(1 + \frac{0,095}{12}\right)^{-240}\right]}{\frac{0,095}{12}}$ ✓ ✓

$\therefore x = \dfrac{650\,000}{107,281\ldots}$

$\therefore x = R6058,85$ ✓ (4)

(ii) METHOD 1

$650\,000\left(1 + \frac{0,095}{12}\right)^{120}$ ✓ $- 6058,85\left[\dfrac{\left(1 + \frac{0,095}{12}\right)^{120} - 1}{\frac{0,095}{12}}\right]$ ✓ ✓

$1674\,436,008 - 1206\,199,988 = R\,468\,236,02$ ✓ (5)

METHOD 2

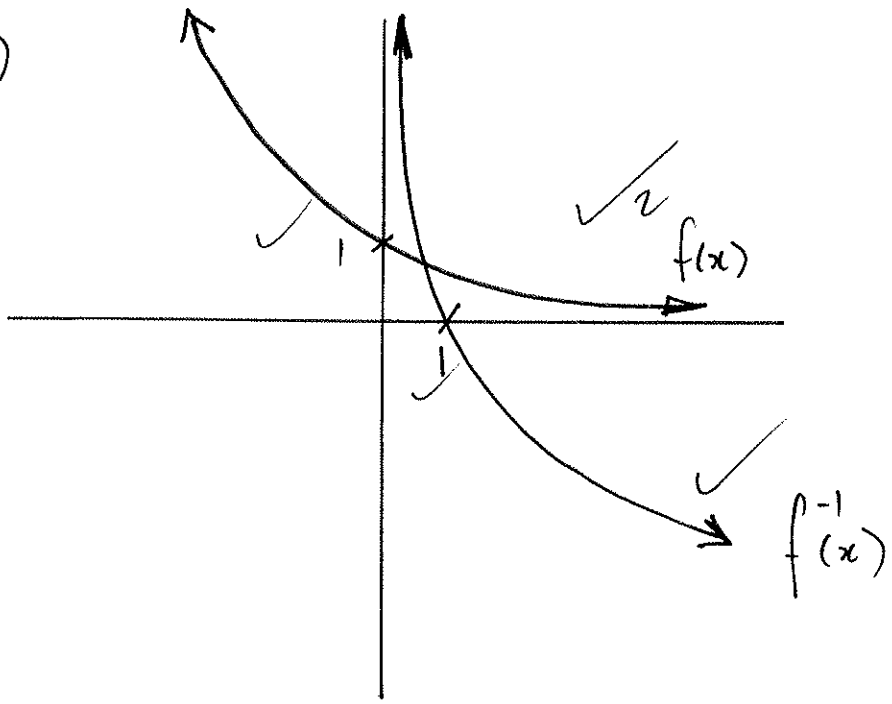
$6058,85\left[\dfrac{1 - \left(1 + \frac{0,095}{12}\right)^{-120}}{\frac{0,095}{12}}\right]$ ✓ ✓ ✓ ✓ $= R\,468\,235,27$ ✓

(3) $650\,000 = 7000\left[\dfrac{1 - \left(1 + \frac{0,095}{12}\right)^{-n}}{\frac{0,095}{12}}\right]$ $\therefore \left(1 + \frac{0,095}{12}\right)^{-n} = 0,2648809\ldots$

$\therefore -n = \dfrac{\log 0,2648809}{\log 1,0079166}$ $\therefore n = 168,47$ $n = 168$ months

Qu 6

a)



✓2

b) $f(x) = \left(\frac{1}{3}\right)^x$ ✓2

c) on sketch (show (0; 1))

d) Reflect across line $y = x$ ✓ ✓

e) $(-3; \infty)$ ✓ ✓

(10).

Qu 7

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ ✓m

$$= \lim_{h \to 0} \frac{(x+h)^2 - 5(x+h) - x^2 + 5x}{h}$$ ✓

$$= \lim_{h \to 0} \frac{2xh + h^2 - 5h}{h}$$ ✓

$$= \lim_{h \to 0} \frac{h(2x - 5 + h)}{h}$$ ✓

$$= 2x - 5$$ ✓

(5)

Qu 7 b) (i) $y = x^2 + 4 + \frac{4}{x^2}$ ✓

$y = x^2 + 4 + 4x^{-2}$ ✓

✓

$\therefore \frac{dy}{dx} = 2x - \frac{8}{x^3}$ ✓ (4)

(ii) $y = 3x^{-\frac{1}{2}} - \frac{3}{2}x^3$ ✓

✓ ✓

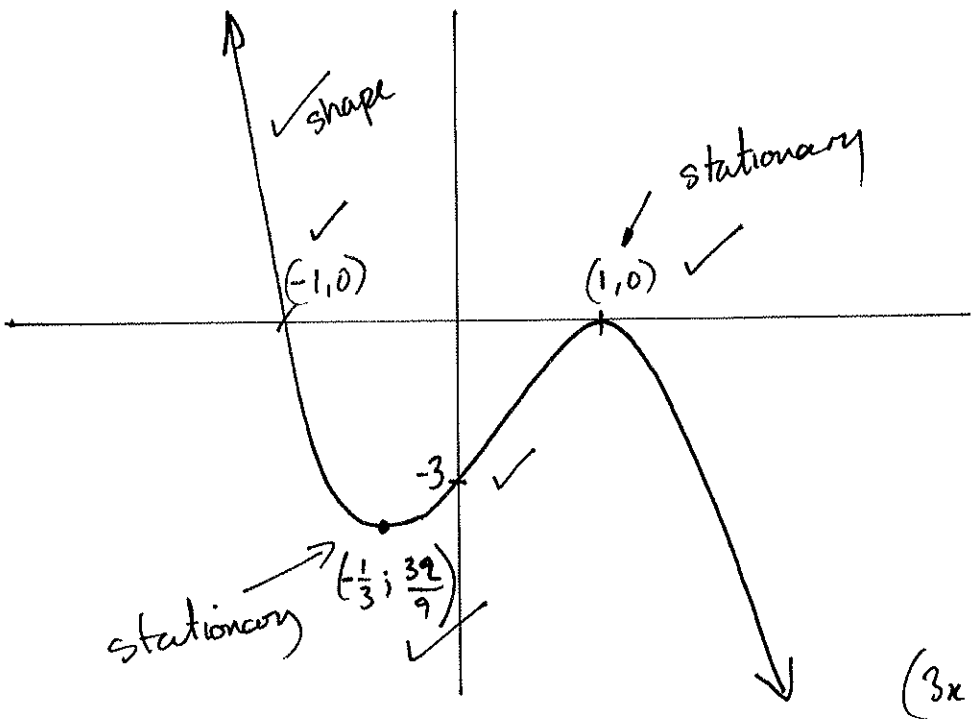
$\frac{dy}{dx} = -\frac{3}{2}x^{-\frac{3}{2}} - \frac{9}{2}x^2$

✓

$\frac{dy}{dx} = \frac{-3}{2\sqrt{x^3}} - \frac{9x^2}{2}$ or $\left(\frac{-3}{2x^{3/2}} - \frac{9x^2}{2}\right)$ (4)

Qu 8

a)



$y = 3(x - x^3 - 1 + x^2)$

$y = -3x^3 + 3x^2 + 3x - 3$ ✓m

$\therefore \frac{dy}{dx} = -9x^2 + 6x + 3$ ✓m

$\frac{dy}{dx} = 0$ at Stationary Point

$3x^2 - 2x - 1 = 0$ ✓m

✓m

$(3x + 1)(x - 1) = 0$ (9)

Qu8 (b)

$f(x)$ crosses the y-axis at $(0,-3)$ i.e. $x=0$ ✓

$f'(x) = -9x^2 + 6x + 3$

$\therefore f'(0) = 3$ ✓ Equation of tangent $y = 3x + c$

$c = -3$

$\therefore y = 3x - 3$ ✓a (3)

(c) $3x - 3 = -3x^3 + 3x^2 + 3x - 3$ ✓M

$\therefore 3x^3 - 3x^2 = 0$

$3x^2(x-1) = 0$ ✓

$x = 0$ and $x = 1$ ✓

co-ordinates $(1, 0)$ ✓a (4)

Qu9

a) Volume = Area of base × height ✓

$340 = \pi r^2 \times h \quad \therefore \quad h = \frac{340}{\pi r^2}$ ✓ (2)

b) TSA $= 2\pi rh + 2\pi r^2$ ✓

$= 2\pi r\left(\frac{340}{\pi r^2}\right) + 2\pi r^2$

$= \frac{640}{r} + 2\pi r^2$ ✓ (2)

→

Qu 9 c)

1) R8 per m^2 = $\frac{8}{10000}$ per cm^2 ✓

$\therefore C = \frac{8}{10000}\left(2\pi r^2 + \frac{680}{r}\right)$ Rand ✓ (2)

2) $\frac{dC}{dr} = \frac{8}{10000}\left(4\pi r - \frac{680}{r^2}\right)$ ✓

Which minimises when $\frac{dC}{dr} = 0$ ✓

$\therefore 4\pi r^3 - 680 = 0$

$\therefore r^3 = \frac{680}{4\pi}$ ✓

$\therefore r = \sqrt[3]{\frac{170}{\pi}}$ ✓ $r = 3{,}78$ cm (4)

(3) $C = \frac{8}{10000}\left(2\pi r^2 + \frac{680}{r}\right) = 21{,}57$ cents

✓M or R0,22 ✓a (2)

Qu 10

$\frac{a+5d}{a+d} = \frac{a+d}{a}$ 2 ✓M

$\therefore a^2 + 5ad = a^2 + 2ad + d^2$ ✓1

$d^2 = 3ad \quad \therefore d(d-3a) = 0 \quad \therefore d = 3a$ ✓1

$r = \frac{a+d}{a} = \frac{4a}{a} = 4$ ✓ 2 ✓M

(7)