



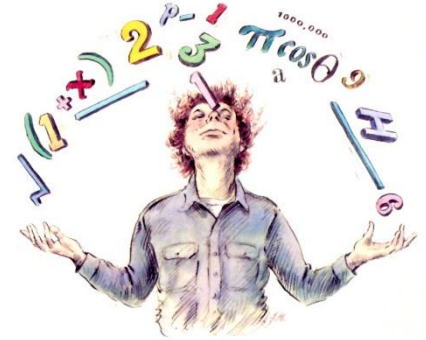
ST ANDREW'S COLLEGE



THE DIOCESAN SCHOOL FOR GIRLS



ST ANDREW'S PREPARATORY



# Department of Mathematics

<b>TIME:</b> <b>3 Hours</b>	<b>DATE: 27 July 2015</b>	<b>Total marks:</b> <b>150</b>
<b>Setter:</b> <b>AM</b>	<b>GRADE 12 PRELIM EXAMINATION</b> <b>MATHEMATICS: PAPER I</b>	<b>Moderator:</b> <b>JH</b>

<b>Name of student:</b>
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### PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 8 pages and an Information Sheet of 2 pages (i – ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. Answer all the questions on A4 lined paper. Remember to write your name on each sheet of paper.
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. All necessary working details must be clearly shown.
7. Round off your answers to one decimal digit where necessary, unless otherwise stated.
8. It is in your own interest to write legibly and to present your work neatly.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	TOTAL
JH	ME	ME	GP	MM	GP	SH	SH	AM	AM	MARKER
30	18	10	14	20	10	13	16	12	7	150

## SECTION A

### QUESTION 1

(a) Solve for  $x$ :

(1)  $3x(x - 4) = x + 10$  (4)

(2)  $\frac{4}{x} - x = 5$  (4)

(3)  $4x^2 \leq 24x - 27$  (4)

(4)  $9^{x+1} - 3^{2x} = 8\sqrt{3}$  (4)

(5)  $\sqrt{2x - 9} + 6 = x$  (5)

(b) Given  $2(x + p)^2 - 32 = 2x^2 + qx + 18$

Find the value(s) of  $p$  and  $q$ . (5)

(c) Write in sigma notation:  $\frac{4}{9} + \frac{7}{16} + \frac{2}{5} + \frac{13}{36} + \frac{16}{49} + \frac{19}{64} + \frac{22}{81}$  (4)

**[30]**

## QUESTION 2

- (a) Find the tenth term of the arithmetic progression  $-73; -51; -29; \dots$  (3)
- (b) A small company producing children's toys plans an increase in output. The number of toys produced is to be increased by 8 each week until the weekly number produced reaches 1000. In week 1, the number to be produced is 280; in week 2, the number is 288; etc.
- (1) Show that the weekly number produced will be 1000 in week 91. (3)
- (2) From week 91 onwards, the number produced each week is to remain at 1000. Find the total number of toys produced from week 1 until the end of the 103<sup>rd</sup> week. (3)
- (c) When the price of gold hit an all-time high in 2012 the SA gold mining industry started a process of recycling the mine dumps for residual gold. For every metric tonne of solid waste heaped in the dumps there is just 0,3 grams of gold. Every time the dump is processed the mining company manages to extract 30% of the available gold. Dumps can be recycled more than once if it remains profitable. Assuming a gold price of R450 per gram and that it costs R10 to process 1 metric tonne:
- (1) Calculate how many grams of gold would be extracted from 1 metric tonne after the first processing. (1)
- (2) Hence, determine how many grams of gold would be left behind after the first processing. (1)
- (3) Calculate how many times the same dump can be recycled before it becomes unprofitable. (4)
- (d) A geometric series has first term 1, common ratio  $r$  and the sum to infinity of this series is 9. Find the value of  $r$ . (3)

[18]

### QUESTION 3

- (a) The quadratic sequence 46; 84; 116; 142; 162; ... is given.

Find  $T_n$ . (5)

- (b) You are given a quadratic sequence,  $T_n = an^2 + bn + c$ , with the following information:

The tenth term of the quadratic sequence is 53,

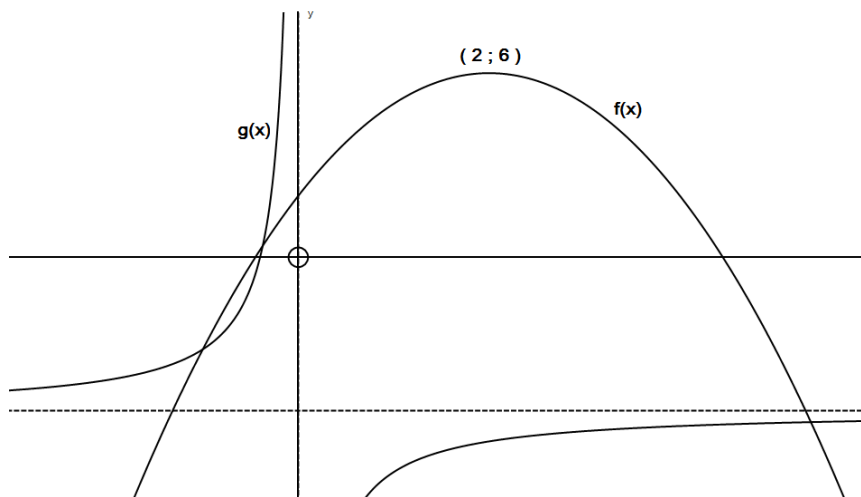
The first row of differences is  $-9; -5; -1; 3; 7; \dots$

Find the values of  $a$ ,  $b$  and  $c$ . (5)

**[10]**

#### QUESTION 4

Below is a sketch of the graphs of  $f(x) = -x^2 + 4x + 2$  and  $g(x) = -\frac{2}{x} - 5$ .  
 $f(x)$  has a turning point at  $(2; 6)$ .



- (a) Write down the range of  $f(x)$ . (1)
- (b) Write down the equation of  $f(x)$  in the form  $y = a(x - p)^2 + q$ . (2)
- (c) Write down the equation(s) of the asymptote(s) of  $g$ . (2)
- (d)  $g(x)$  is translated 2 units left and then reflected in the  $x$ -axis to form  $h(x)$ .  
Write down the equation of  $h(x)$ . (3)
- (e) Find the horizontal distance between  $f(x)$  and  $g(x)$  when  $y = 6$  (3)
- (f) Determine  $f(x) - g(x)$  when  $f'(x) = 0$ . (3)

[14]

END SECTION A - TOTAL [72]

## SECTION B

### QUESTION 5

Give your answers to 2 decimal places in this question.

- (a) Convert 12% per annum compounded monthly into an effective rate. (2)
- (b) If the effective interest rate is 12% per annum, determine the nominal rate per annum compounded quarterly. (3)
- (c) Mario takes out a bond of R650 000 to buy a house over 20 years. His monthly payments are calculated at an interest rate of 9,5% p.a. compounded monthly.
- (1) Show that his monthly payment, payable at the end of each month, is R6 058,85. (4)
- (2) Determine the balance outstanding on Mario's bond after 10 years. (5)
- (3) Isabella has a bond for the same amount as Mario attracting the same interest rate. She decides to make higher monthly repayments of R7 000,00. Determine, to the nearest month, how long it would take for Isabella to repay the loan. (6)

[20]

### QUESTION 6

Given  $f^{-1}(x) = \log_{\frac{1}{3}} x$

- (a) Draw the graph of  $f^{-1}(x)$  showing any intercept(s) with the axes. (2)
- (b) Write down the equation of  $f(x)$ . (2)
- (c) Draw, on the same set of axes as (a), the graph of  $f(x)$ . (2)
- (d) Describe in words how to transform  $f^{-1}(x)$  to  $f(x)$ . (2)
- (e) If  $h(x) = 2f(x) - 3$ , write down the range of  $h(x)$ . (2)

[10]

### QUESTION 7

(a) Determine  $f'(x)$  from first principles if  $f(x) = x^2 - 5x$ . (5)

(b) Determine  $\frac{dy}{dx}$  for each of the following, leaving answers with positive exponents:

(1)  $y = \left(x + \frac{2}{x}\right)^2$  (4)

(2)  $y = \frac{6\sqrt{x} - 3x^4}{2x}$  (4)

**[13]**

### QUESTION 8

Given:  $f(x) = 3(x - 1)(1 - x^2)$

(a) Draw the graph of  $f(x)$  clearly showing the co-ordinates of any intercepts with the axes as well as stationary points. Show all your calculations. (9)

(b) Find the equation of the tangent to  $f(x)$  at the point where  $f(x)$  crosses the y-axis. (3)

(c) Find the co-ordinates of the point where the tangent cuts  $f(x)$  again. (4)

**[16]**

### QUESTION 9

A perfectly cylindrical closed cola can has a capacity of 340ml.

[1ml = 1cm<sup>3</sup>]



(a) Find an expression for the height of the can in terms of the radius of its base,  $r$ . (2)

(b) Hence, or otherwise, show that the total surface area can be expressed as

$$2\pi r^2 + \frac{680}{r}. \quad (2)$$

(c) The cost of metal sheeting is R8,00 per  $m^2$ .

(1) Find an expression for the cost ( $C$ ) of the metal used to make the can. (2)

(2) Find the value of the radius which minimises the cost. (4)

(3) Determine the minimum cost of one can. (2)

[12]

### QUESTION 10

The first three terms of a geometric progression are equal to the first, second and sixth terms respectively of an arithmetic progression. The terms are not all equal.

Prove that the common ratio of the geometric progression is equal to 4.

[7]

**TOTAL MARKS: 150**