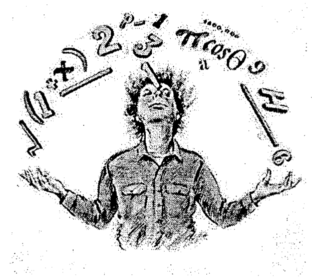
ST ANDREW'S COLLEGE   THE DIOCESAN SCHOOL FOR GIRLS   ST ANDREW'S PREPARATORY

Department of Mathematics



TIME: 3 Hours	DATE: 03 August 2015	Total marks: 150
Setter: DS	GRADE 12 PRELIM EXAMINATION MATHEMATICS: PAPER II	Moderator: AM

Name of student: MEMO

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

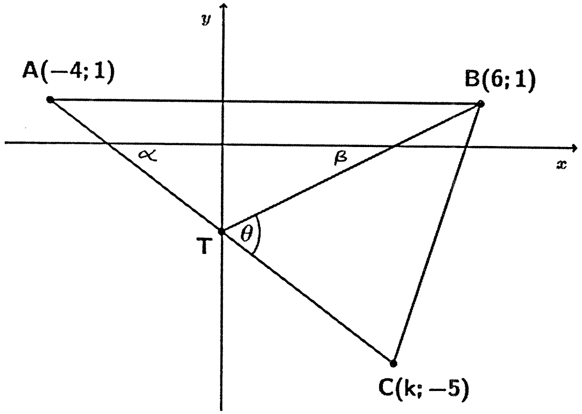
1. This question paper consists of 24 pages and an Information Sheet of 2 pages (i – ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. **Answer all the questions on the question paper and hand this in at the end of the examination. Remember to write your name on the paper.**
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. All necessary working details must be clearly shown.
7. Round off your answers to one decimal digit where necessary, unless otherwise stated.
8. Ensure that your calculator is in **DEGREE** mode.
9. It is in your own interest to write legibly and to present your work neatly.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	TOTAL
22	23	10	20	25	19	10	21	150

SECTION A

QUESTION 1

(a) In the diagram, triangle ABC is drawn with A(−4; 1), B(6; 1) and C(k; −5). T is a point on the y-axis. The line ATC has equation $y = -\frac{3}{4}x - 2$. Angle $B\hat{T}C = \theta$.



(1) Show that $k = 4$.

Sub $(k; -5)$ into $y = -\frac{3}{4}x - 2$ ✓M

$\therefore -5 = -\frac{3}{4}k - 2$ ✓a

$\therefore \frac{3}{4}k = 3$ ✓M

$\therefore 3k = 12 \quad \therefore k = 4$ ✓a

(4)

(2) Determine the equation of the line TB in the form $y = mx + c$.

$T(0; -2) \qquad B(6; 1)$

$\therefore m_{TB} = \frac{1+2}{6-0} = \frac{1}{2}$ ✓M ✓a

y-int is -2

$\therefore y = \frac{1}{2}x - 2$ ✓a

(3)

(3) Determine the area of triangle ABC.

$\text{Area} = \frac{1}{2} \times \text{base} \times \perp h$

$= \frac{1}{2}(AB) \times \perp h$ ✓M

$= \frac{1}{2}(10)(6)$ ✓a

$= 30 \text{ units}^2$ ✓a

(3)

(4) Determine, correct to one decimal digit, the size of θ.

$m_{ATC} = -\frac{3}{4} \qquad \tan^{-1}(-\frac{3}{4}) = -36{,}87°$ ✓M

$\therefore \alpha = 36{,}87°$ ✓a

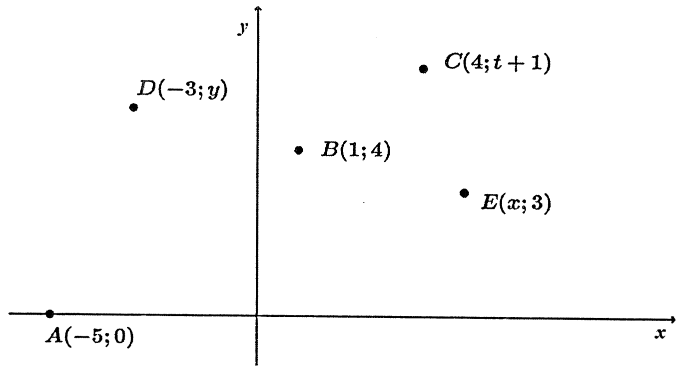
$m_{TB} = \frac{1}{2} \qquad \tan^{-1}(\frac{1}{2}) = 26{,}57°$ ✓M

$\therefore \beta = 26{,}57°$ ✓a

$\therefore \theta = 36{,}87° + 26{,}57° = 63{,}4°$ ✓a

(5)

(b) A$(-5;0)$, B$(1;4)$, C$(4;t+1)$, D$(-3;y)$ and E$(x;3)$ are points in the Cartesian plane.



(1) Determine x and y if B is the midpoint of DE.

$M\left(\frac{x_1+x_2}{2};\ \frac{y_1+y_2}{2}\right)$ ✓M

$x = 5$ ✓a

$y = 5$ ✓a

(3)

(2) Determine the value of t if A, B and C all lie on the same straight line.

$m_{AB} = m_{BC}$ ✓M

$\therefore \frac{4-0}{1+5} = \frac{t+1-4}{4-1}$ ✓a

$\frac{4}{6} = \frac{t-3}{3}$ ✓M

$\therefore t-3 = 2$

$\therefore t = 5$ ✓a

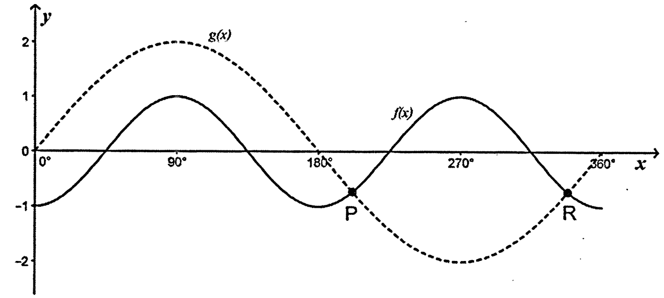
(4)

[22]

QUESTION 2

PLEASE ENSURE THAT YOUR CALCULATOR IS IN DEGREE MODE

(a) The diagram shows the graphs of $f(x) = a\cos bx$ and $g(x) = c\sin dx$ for $0° \leq x \leq 360°$. Points P and R are indicated.



(1) Write down the values of a, b, c and d.

$a = -1$ ✓a

$b = 2$ ✓a

$c = 2$ ✓a

$d = 1$ ✓a

(4)

(2) Write down the period of f.

$180°$ ✓a

(1)

(3) If the co-ordinates of P are $(200°; -1{,}75)$, write down the co-ordinates of R.

$R(340°; -1{,}75)$ ✓a

(1)

(4) Write down the value of k if $f(x) - k = g(x)$ has only one solution for $x \in [0°; 360°]$.

$k = 3$ ✓a ✓

(2)

(b) Simplify as far as possible: $\dfrac{\cos(90°-\theta)-\sin(180°+\theta)}{\sin 2\theta}$

$\dfrac{\sin\theta + \sin\theta}{2\sin\theta\cos\theta}$ ✓a ✓a ✓a

$= \dfrac{2\sin\theta}{2\sin\theta\cos\theta}$

$= \dfrac{1}{\cos\theta}$ ✓A

(4)

(c) Solve for θ if $\cos\theta = \sin 210°$ and

(1) $0° < \theta < 180°$

$\sin 210° = -\frac{1}{2}$

$\therefore \cos\theta = -\frac{1}{2}$ ✓M

$\cos^{-1}(-\frac{1}{2}) = 120°$

$\therefore \theta = 120°$ ✓a

(2)

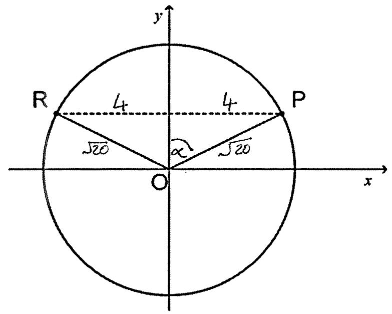
(2) $-180° < \theta < 0°$

$\theta = -120°$ ✓a ✓

[1 mark for $\theta = 240°$]

(2)

(d) In the diagram below, points P and R lie on the circle $x^2 + y^2 = 20$.
Point R is the reflection of P in the y-axis.



(1) If the horizontal distance PR is 8 units, determine the co-ordinates of R.

$x^2 + y^2 = 20$

$\therefore 4^2 + y^2 = 20$ ✓M

$\therefore y^2 = 20 - 16$

$y^2 = 4 \quad \therefore \quad y = \pm 2$ ✓a

$\therefore \quad R(-4\,;\,2)$ ✓a

(3)

(2) Determine, to the nearest degree, the size of angle $P\hat{O}R$.

$m_{OP} = \frac{1}{2} \qquad \tan^{-1}(\frac{1}{2}) = 26{,}57°$ ✓M ✓a

$\therefore P\hat{O}R = 180° - 2 \times 26{,}57°$ ✓M

$= 127°$ ✓a

or Using cosine rule: $8^2 = (\sqrt{20})^2 + (\sqrt{20})^2 - 2.\sqrt{20}.\sqrt{20}.\cos P\hat{O}R$

$\therefore 64 - 20 - 20 = -40\cos P\hat{O}R$

$\therefore \quad -\frac{3}{5} = \cos P\hat{O}R$

(4)

$\therefore \; P\hat{O}R = 127°$

[23]

or $\sin\alpha = \dfrac{4}{\sqrt{20}} \qquad \therefore \alpha = 63{,}43°$

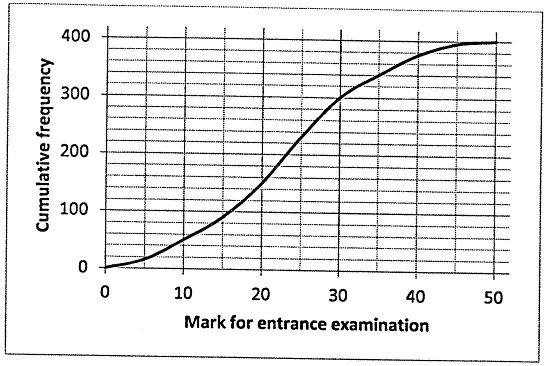
$\therefore \; P\hat{O}R = 2 \times 63{,}43°$

$= 127°$

QUESTION 3

A university entrance examination was written by 400 students. Scores ranged from 5 out of 50 to 50 out of 50. The data is represented below:

Score (x)	Cumulative Frequency
$x \leq 50$	400
$x \leq 45$	395
$x \leq 40$	375
$x \leq 35$	340
$x \leq 30$	300
$x \leq 25$	230
$x \leq 20$	150
$x \leq 15$	90
$x \leq 10$	50
$x \leq 5$	15



(a) How many students scored more than 35 out of 50 for the entrance examination?

60 ✓ a

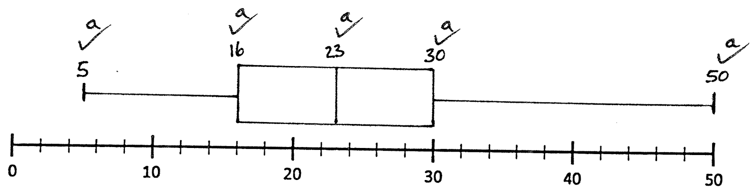
(1)

(b) How many students obtained marks in the interval $20 < x \leq 25$?

80 ✓ a

(1)

(c) Use the axis below to draw a box-and-whisker plot for the data. [Use the data provided to estimate values for the lower quartile, median and upper quartile.]



(5)

(d) Comment on the skewness of the distribution of the data.

Skewed to the right ✓ a

(positively skewed)

(1)

(e) One interpretation of the skewness of the data is that university entrance standards are dropping. Provide an alternative explanation that could account for the skewness of the data.

- This particular cohort of students was stronger than the norm.
- Students were better prepared

etc.

✓✓ any reasonable interpretation

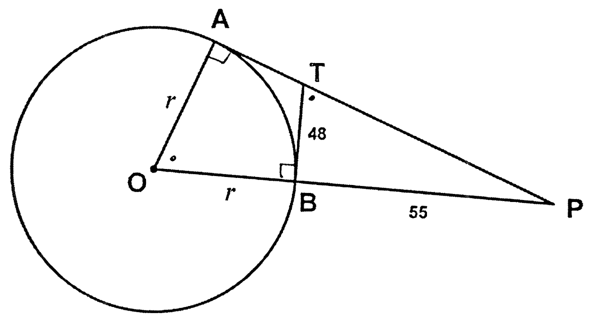
(2)

[one mark for partially correct alternative explanation.]

[10]

QUESTION 4

(a) In the diagram below, TA and TB are tangents to the circle with centre O. AT produced meets OB produced at P. TB = 48 cm, BP = 55 cm and OA = OB = r cm.



(1) Explain why BOAT is a cyclic quadrilateral.

$O\hat{A}T = O\hat{B}T = 90^\circ$ (tan ⊥ rad) ✓ a

∴ Opposite ∠s are supplementary ✓ a

∴ BOAT is cyclic (2)

(2) Explain why TA = 48 cm.

TA = TB (tangents from a common point) ✓ a (1)

(3) Prove that ΔPOA ||| ΔPTB.

(1) $O\hat{A}T = O\hat{B}T$ (both 90°; tan ⊥ rad) ✓ a

(2) $\hat{P}$ is common ✓ a

(3) $P\hat{T}B = B\hat{O}A$ ✓ a (∠s in Δ or ext ∠ of cyclic quad)

∴ ΔPOA ||| ΔPTB

(3)

(4) Hence, or otherwise, calculate the length of r.

$TP^2 = 48^2 + 55^2$ ∴ $TP = 73$ ✓ a

Since ΔPOA ||| ΔPTB : $\frac{PO}{PT} = \frac{PA}{PB} = \frac{OA}{TB}$

∴ $\frac{PO}{PT} = \frac{OA}{TB}$ ie. $\frac{r+55}{73} = \frac{r}{48}$ ✓ M ✓ a

∴ $48r + 2640 = 73r$

∴ $2640 = 25r$

∴ $r = 105{,}6$ cm ✓ a

(4)

or

$TP^2 = 48^2 + 55^2$ ∴ $TP = 73$

$OA^2 + AP^2 = OP^2$

∴ $r^2 + (48+73)^2 = (r+55)^2$

∴ $r^2 + 14641 = r^2 + 110r + 3025$

∴ $110r = 11616$

∴ $r = 105{,}6$ cm

or

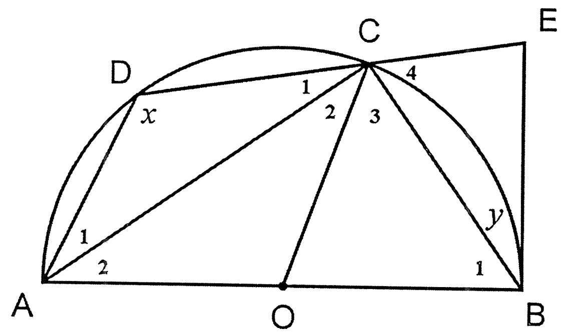
Since ΔPOA ||| ΔPTB

$\frac{OA}{AP} = \frac{TB}{BP}$

∴ $\frac{r}{48+73} = \frac{48}{55}$

∴ $r = \frac{48}{55} \times 121 = 105{,}6$ cm

(b) In the diagram below, EB is a tangent to the semi-circle with centre O passing through A, B, C and D. DC produced meets the tangent at E. $A\hat{D}C = x$ and $E\hat{B}C = y$.



(1) Name, giving reasons, two right angles in the figure.

$O\hat{B}E = 90°$ (tan ⊥ rad)

$A\hat{C}B = 90°$ (∠ in semi-circle) (4)

(2) Show, giving reasons, that $x - y = 90°$.

$x + \hat{B}_1 = 180°$ (opp ∠s of cyclic quad)

but $\hat{B}_1 + y = 90°$ (tan ⊥ rad) reasons

$\therefore \hat{B}_1 = 90° - y$

$\therefore x + 90° - y = 180°$

$\therefore x - y = 90°$ (4)

(3) Determine, giving reasons, $C\hat{O}A$ in terms of y.

$C\hat{O}A = 2 \times C\hat{B}A$ (∠ at centre = 2 × ∠ at circ.)

$= 2 \times (90° - y)$

$= 180° - 2y$ (2)

[20]

75 marks

SECTION B

QUESTION 5

(a) (1) Prove the identity: $\dfrac{1-\cos 2x}{\sin 2x} = \tan x$

$LHS = \dfrac{1-(1-2\sin^2 x)}{2\sin x\cos x}$

$= \dfrac{2\sin^2 x}{2\sin x \cos x}$ A

$= \dfrac{\sin x}{\cos x}$

$= \tan x$ (4)

(2) Hence, or otherwise, determine a value for tan 15° without using a calculator.

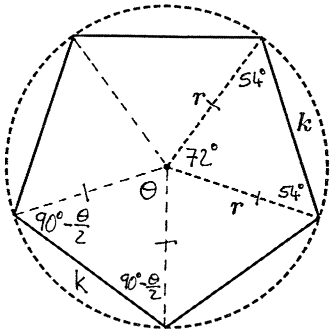
$\tan 15° = \dfrac{1-\cos 30°}{\sin 30°}$ M

$= \dfrac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$= 2 - \sqrt{3}$

or $\tan 15° = \dfrac{\sin(45°-30°)}{\cos(45°-30°)}$ etc. (3)

(b) The diagram shows a regular pentagon with side length k. The distance from the centre to each vertex is r, the radius of the circumscribed circle.



(1) Use the sine rule to determine k if $r = 10\ cm$.

$360° \div 5 = 72°$ ✓a

$\frac{k}{\sin 72°} = \frac{r}{\sin 54°}$ ✓M ✓a

$\therefore\ k = \frac{10 \cdot \sin 72°}{\sin 54°}$

$= 11{,}8\ cm$ ✓a

(4)

(2) If the angle between the two radii is θ, show that the ratio $\frac{k}{r}$ can be written as $2\sin\left(\frac{\theta}{2}\right)$.

Angle between radius & side (k) = $\frac{180° - \theta}{2} = 90° - \frac{\theta}{2}$ ✓a

$\frac{k}{\sin\theta} = \frac{r}{\sin(90 - \theta/2)}$ ✓a

$\therefore \frac{k}{r} = \frac{\sin\theta}{\cos(\theta/2)}$

$= \frac{2 \cdot \sin(\theta/2) \cdot \cos(\theta/2)}{\cos(\theta/2)}$ ✓M ✓a

$= 2\sin\left(\frac{\theta}{2}\right)$

(4)

(3) Hence determine how many sides a regular polygon has in which the ratio $\frac{k}{r}$ is 0,261.

$2\sin\left(\frac{\theta}{2}\right) = 0{,}261$

$\therefore \sin(\theta/2) = 0{,}1305$ ✓M

$\therefore\ \theta/2 = 7{,}4985$

$\therefore\ \theta \approx 15°$ ✓a

$\frac{360°}{n} = 15°$ ✓M

$\therefore\ n = \frac{360°}{15°}$

$= 24$ sides ✓a

(4)

(c) Determine the general solution to: $\frac{\sin\theta}{\sin 25°} + \frac{\cos\theta}{\cos 25°} = 2$

$\frac{\sin\theta.\cos 25° + \cos\theta.\sin 25°}{\sin 25°.\cos 25°} = 2$ ✓M

$\therefore \sin\theta.\cos 25° + \cos\theta.\sin 25° = 2.\sin 25°.\cos 25°$

$\therefore \sin(\theta + 25°)$ ✓a $= \sin 50°$ ✓a

$\therefore \theta + 25° = 50° + k.360°$ or $\theta + 25° = 130° + k.360°$ ✓M

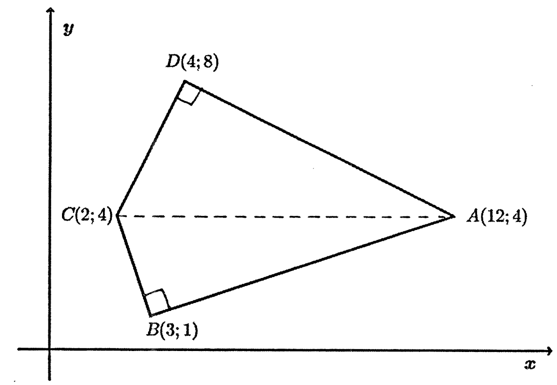
$\therefore \theta = 25° + k.360°$ ✓a $\quad \theta = 105° + k.360°$ ✓a

(6)

[25]

QUESTION 6

(a) The diagram shows quadrilateral ABCD with A(12; 4), B(3; 1), C(2,4) and D(4,8).



(1) Show that $A\hat{D}C = 90°$.

$m_{AD} = \frac{8-4}{4-12} = -\frac{1}{2}$ ✓a

$m_{CD} = \frac{8-4}{4-2} = 2$ ✓a

$m_{AD} \times m_{CD} = -\frac{1}{2} \times 2 = -1$ ✓M

✓a $\therefore AD \perp DC \quad \therefore \quad A\hat{D}C = 90°$

(4)

(2) If it is further given that $AB \perp BC$, find the equation of the circle passing through A, B, C and D in the form $(x-a)^2 + (y-b)^2 = r^2$

AC is diameter ∴ centre is midpoint

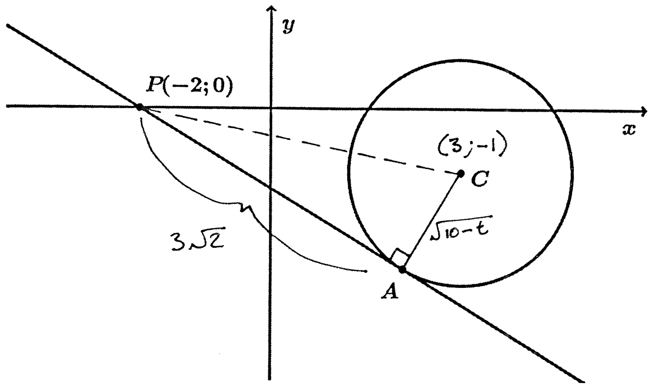
Midpoint of AC: $(7;4)$ ✓a

radius $= 5$ ✓a

$\therefore \underbrace{(x-7)^2 + (y-4)^2}_{\text{ca}} = 25$ ✓ca

(4)

(b) In the figure, P is the point P(−2; 0) and the equation of the circle with centre C is $x^2 + y^2 - 6x + 2y + t = 0$. The tangent PA touches the circle at A.



(1) Determine the co-ordinates of C, as well as the radius of the circle (in terms of t).

$x^2 - 6x + y^2 + 2y = -t$

$x^2 - 6x + 9 + y^2 + 2y + 1 = 10 - t$ ✓M

$(x-3)^2 + (y+1)^2 = 10 - t$

✓a ✓a

$\therefore\ C(3; -1)$ ✓CA

$r = \sqrt{10-t}$ ✓CA

(5)

(2) If the length of PA is $3\sqrt{2}$ units, determine the value of t.

$PC^2 = 5^2 + 1^2$ ✓M

$\therefore\ PC = \sqrt{26}$ ✓a

$AC^2 + PA^2 = PC^2$ ✓M

$(\sqrt{10-t})^2 + (3\sqrt{2})^2 = (\sqrt{26})^2$ ✓a ✓a

$10 - t + 18 = 26$

$\therefore\ t = 2$ ✓ca

(6)

[19]

QUESTION 7

(a) Consider the following data points:

$A(1;5)$ $B(2;8)$ $C(4;11)$ $D(5;17)$ $E(7;23)$ $F(10;30)$ $G(12;38)$

(1) Use a calculator to determine the equation of the least squares regression line, rounding to two decimal places.

$y = A + Bx$

$A = 1{,}45$ $\quad B = 2{,}97$

$\therefore \quad y = 2{,}97x + 1{,}45$ ✓a ✓a

(4)

(2) The correlation coefficient between x and y for the data is 0,995, correct to three decimal places. If the x-coordinate of one point and the y-coordinate of a different point are changed slightly, the correlation coefficient equals 1. Determine which two points need to be changed, and write down the new co-ordinates of these points.

Gradient ≈ 3 ✓M ✓M

$\therefore$ C (3 ; 11) ✓a (or C(4;14))

F (10 ; 32) ✓a (or F($\frac{28}{3}$; 30))

(4)

(b) For a class test, scores ranged from 35 to 98, with a mean of 74. Which of the following is the most realistic value of the standard deviation: -10 ; 1 ; 12 ; 60 ? Explain your answer.

12 ✓a

Normal distributions tend to have ranges equal to ± 3 standard deviations

$\therefore$ s.d. $\approx \frac{98-35}{6} \approx 10\frac{1}{2}$ ✓M

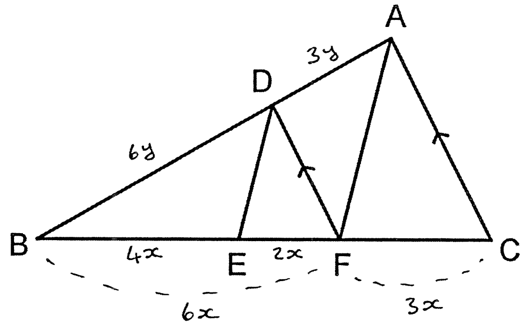
(2)

or s.d. can't be a negative value $\therefore$ not -10
1 is too small
60 is too big (almost same as data range)

[10]

QUESTION 8

(a) The diagram shows triangle ABC with DF//AC. E is a point on BC such that $\frac{CF}{FB} = \frac{1}{2}$ and $\frac{CF}{BE} = \frac{3}{4}$.



✓M

Prove that DE//AF.

$\frac{CF}{BE} = \frac{3}{4}$ (given) $\qquad$ Let $CF = 3x$ & $BE = 4x$ ✓M

$\frac{CF}{FB} = \frac{1}{2}$ (given) $\longrightarrow$ $\frac{CF}{FB} = \frac{3}{6}$ $\quad \therefore EF = 2x$

$\frac{AD}{DB} = \frac{CF}{FB} = \frac{1}{2}$ ✓a (Prop. int.)

But $\frac{FE}{EB} = \frac{2x}{4x} = \frac{1}{2}$ ✓a

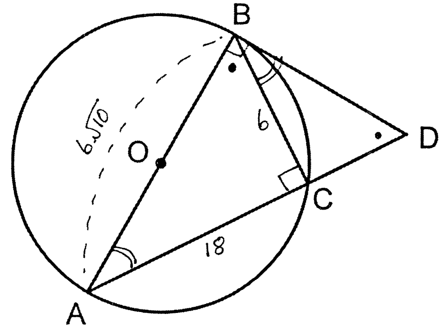
$\therefore \frac{AD}{DB} = \frac{FE}{EB}$ ✓a (both equal $\frac{1}{2}$)

$\therefore$ DE // AF (conv. prop. int) ✓a

(6)

(b) In the diagram below, a circle with centre O is drawn.

- DB is a tangent to the circle at B
- AB is a diameter of the circle
- AD cuts the circle at C
- BC = 6 cm and AC = 18 cm



Determine, giving reasons, the length of DC.

$AB^2 = 18^2 + 6^2 \quad \therefore \quad AB = 6\sqrt{10}$ a ✓

In $\triangle BAC$ & $\triangle DBC$

① $\hat{A} = D\hat{B}C$ (tan-chord th) a ✓

② $A\hat{C}B = B\hat{C}D = 90°$ ($\angle$ in semi-circle, $\angle$s on str line.) a ✓

③ $A\hat{B}C = \hat{D}$ ($\angle$s in $\triangle$) a ✓

$\therefore \triangle BAC \,|||\, \triangle DBC$ (AAA) a ✓

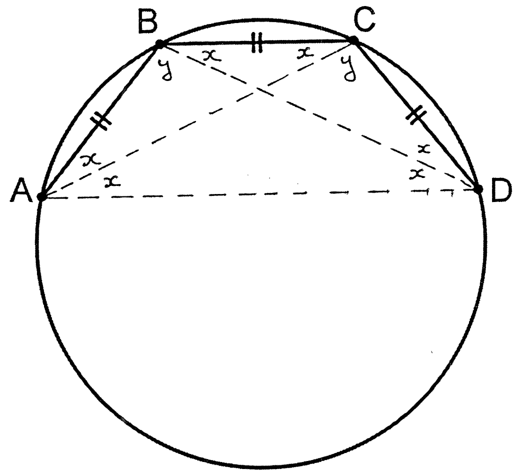
$\therefore \frac{\cancel{BA}}{\cancel{DB}} = \frac{BC}{DC} = \frac{AC}{BC}$ M ✓

$\therefore BC^2 = AC \cdot DC$

$\therefore DC = \frac{BC^2}{AC} = \frac{6^2}{18} = 2$ a ✓

(7)

(c) (1) The diagram shows a circle with three equal chords, AB, BC and CD.



Prove that $A\hat{B}C = B\hat{C}D$.

Join AC, BD and AD

Let $C\hat{A}D = x$

$\therefore C\hat{A}D = C\hat{B}D = B\hat{D}C = B\hat{A}C = B\hat{C}A = B\hat{D}A = x$ a ✓

(Equal chords subtend equal angles) M ✓

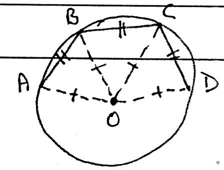
Let $A\hat{B}D = y$

then $A\hat{C}D = y$ ($\angle$s in same seg.) a ✓

$\therefore A\hat{B}C = x + y$ & $B\hat{C}D = x + y$ a ✓

$\therefore A\hat{B}C = B\hat{C}D$

or



(4)

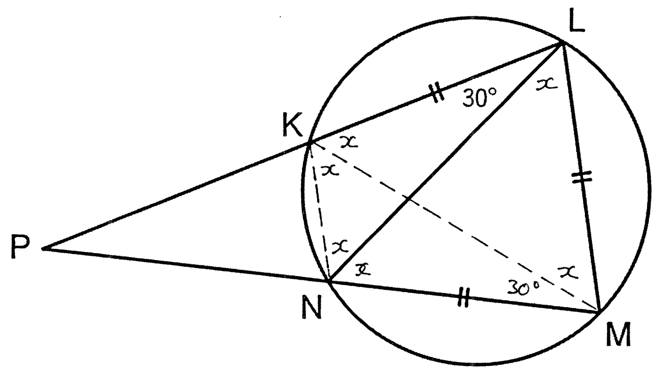
Draw radii OA, OB, OC, OD

$\triangle OAB \equiv \triangle OBC \equiv \triangle OCD$ (SSS)

$\therefore O\hat{A}B = O\hat{B}A = O\hat{B}C = O\hat{C}B = O\hat{C}D = O\hat{D}C$ (isos $\triangle$s)

$\therefore A\hat{B}C = B\hat{C}D$

(2) In the diagram, chords KL, LM and MN are equal in length. PKL and PNM are straight lines. N and L are joined, and $P\hat{L}N = 30°$.



Determine, giving reasons, the size of $\hat{P}$.

Join KN and KM

$K\hat{M}N = 30°$ ($\angle$s in same seg.) M

Let $L\hat{N}M = x$

Then: $L\hat{K}M = M\hat{K}N = K\hat{N}L = L\hat{N}M = N\hat{L}M = L\hat{M}K = x$ a

(Equal chords subtend equal angles)

Now, $K\hat{N}M + K\hat{L}M = 180°$ (opp $\angle$s of cyclic quad)

ie $3x + 30° = 180°$ M

$\therefore \; x = 50°$

$\hat{P} = 180° - 2(x + 30°)$

$= 180° - 2 \times 80°$

$= 20°$ a

(4)

[21]

75 marks

Total: 150 marks