

JULY EXAM - 2015

PAPER 1: MEMO

SECTION A

QUESTION 1

a) 1. $\sqrt{2x-5} = -1$
no solution (2)

2. $2^{9-x^2} = 4^{x+3}$
 $2^{9-x^2} = 2^{2x+6}$
 $9-x^2 = 2x+6$ (4)
 $-x^2 - 2x + 3 = 0$
 $x=1$ $x=-3$

b) 1. $x^2 - 2x = 0$
 $x=2$ $x=0$ (3)

2. $\frac{2x^2+1}{x^2-2x} = k$
 $2x^2+1 = kx^2 - 2kx$
 $(2-k)x^2 + 2kx + 1 = 0$
 $b^2 - 4ac \geq 0$ (5)
 $(2k)^2 - 4(2-k)(1) \geq 0$
 $4k^2 - 8 + 4k \geq 0$
 $k^2 + k - 2 \geq 0$
 $k \leq -2$ or $k \geq 1$

c) $y = 1 - 2x$
 $2x^2 - x(1-2x) + (1-2x)^2 = 4$
 $2x^2 - x + 2x^2 + 1 - 4x + 4x^2 = 4$
 $8x^2 - 5x - 3 = 0$
 $x=1$ or $x = -3/8$ (6)
 $y = -1$ $y = 7/4$

QUESTION 2

a) 6 (2)

b) 10 (2)

c) 15 (2)

d) 6 10 15

$3a+b = 4$
4 5

$2a = 1$ $3a+b = 4$

$2a = 1$ $b = 5/2$
 $a = 1/2$

$1/2 + 5/2 + c = 6$ (4)
 $c = 3$

$T_n = 1/2 n^2 + 5/2 n + 3$

e) $T_{25} = 378$ (2)
(78 terms)

QUESTION 3

a) $\lim_{x \rightarrow 1} \frac{1-x^3}{1-x}$
 $= \lim_{x \rightarrow 1} \frac{(-x)(1+x+x^2)}{1-x}$
 $= \underline{\underline{3}}$

b) $f(x) = \frac{1}{2}x^2 - 2x$
 $f(x+h) = \frac{1}{2}(x+h)^2 - 2(x+h)$
 $= \frac{1}{2}(x^2 + 2xh + h^2) - 2x - 2h$
 $= \frac{x^2}{2} + xh + \frac{h^2}{2} - 2x - 2h$

$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{x^2}{2} + xh + \frac{h^2}{2} - 2x - 2h - \frac{1}{2}x^2 + 2x}{h}$
 $= \lim_{h \rightarrow 0} \frac{h(x + \frac{h}{2} - 2)}{h}$
 $= \underline{\underline{x-2}}$

c) 1. $y = \frac{3}{2}x^4 - \frac{2}{3}x^3$

$\frac{dy}{dx} = \underline{\underline{6x^3 - 2x^2}}$

2. $y = \left(\frac{3}{x} + 1\right)\left(\frac{3}{2x} - 1\right)$

$= \frac{9}{x^2} - 1$

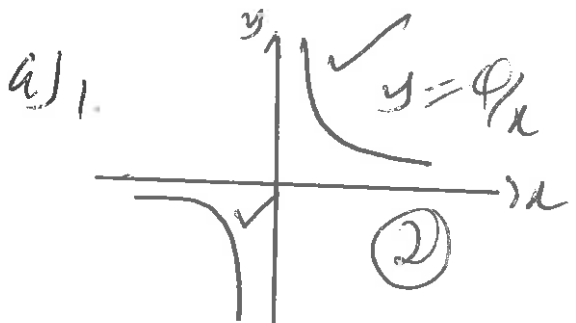
$\frac{dy}{dx} = \underline{\underline{-\frac{18}{x^3}}}$

d) 1. grad = 3 ✓ (1)

2. $-1 < x < 4$ ✓ (2)

3. $x = -1$ ✓ max
 $x = 4$ ✓ minimum
 (4)

QUESTION 4



2) $\frac{c}{x} = mx$

$c = mx^2$ ✓

$mx^2 - c = 0$

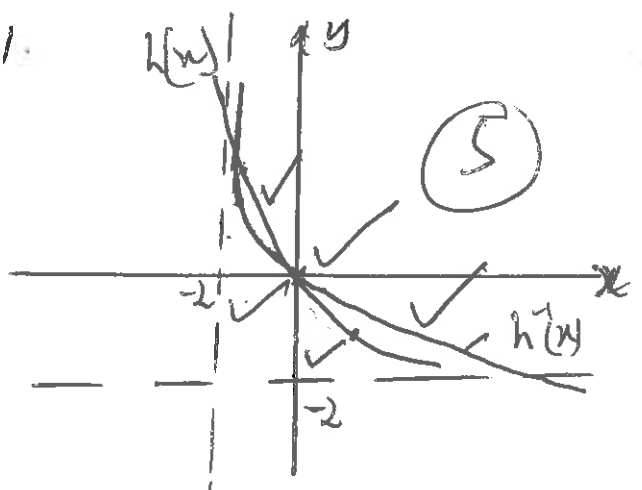
$b^2 - 4ac > 0$ (3)

$0 - 4(m)(-c) > 0$

$16m > 0$

$m > 0$ ✓

b) 1.



$$2. h(x) = \left(\frac{1}{2}\right)^{x-1} - 2$$

$$x = \left(\frac{1}{2}\right)^{y-1} - 2$$

$$x+2 = \left(\frac{1}{2}\right)^{y-1} \quad (3)$$

$$\log_{\frac{1}{2}}(x+2) + 1 = y = h^{-1}(x)$$

$$3. h(x) h^{-1}(x) \leq 0$$

$$\underline{x=0} \quad \checkmark \quad (2)$$

QUESTION 5

$$a) P(H) = 0,65 \quad \checkmark \quad \left(\frac{13}{20}\right) \quad (1)$$

$$b) P(H \cap A) = 0,75 + 0,65 - 0,85 \\ = 0,55 \quad \checkmark \quad \left(\frac{11}{20}\right) \quad (3)$$

$$c) P(F) = 1 - 0,85 \quad \checkmark \\ = 0,15 \quad \checkmark \quad (2)$$

$$d) P(\text{Pass exactly one of the two}) \\ = \underline{0,3} \quad \checkmark \quad (2)$$

SECTION B

$$a) \frac{10! \checkmark}{2! \checkmark} = \frac{18144 \checkmark}{400} \quad (3)$$

$$b) 3 \times 2! \cdot \left(\frac{8! \checkmark}{2! \checkmark}\right) \\ = 6 \left(\frac{8!}{2!}\right) \quad (4)$$

$$= \underline{120960} \quad \checkmark$$

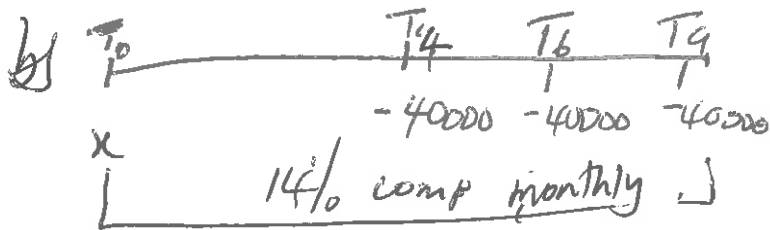
QUESTION 7:

$$a1. 491520 = 1200000(1-i)^4 \quad \checkmark \\ i = 20\% \quad \checkmark \quad (4)$$

$$2. A = 1200000(1+15\%)^4 \quad \checkmark \\ = 2098807,5 \quad \checkmark \quad (4)$$

$$\text{Sinking fund} = 2098807,5 \\ - 491520 \\ = \underline{1607287,5} \quad \checkmark$$

$$3. \quad \checkmark \\ 1607287,5 = x \left[\frac{(1+\frac{9\%}{12})^{49} - 1}{\frac{9\%}{12}} \right] \quad \checkmark \\ x = \underline{27264,29} \quad \checkmark \quad (5)$$



$$x \left(1 + \frac{14\%}{12}\right)^{9 \times 12} - 40000 \left(1 + \frac{14\%}{12}\right)^{8 \times 12} - 40000 \left(1 + \frac{14\%}{12}\right)^{6 \times 12} - 40000 = 0$$

$$x = R 51724,35 \quad (6)$$

c) $a = 12$

$$a + 4d = 18$$

$$4d = 6$$

$$d = \frac{3}{2}$$

$$S_{25} = \frac{25}{2} (24 + 24 \cdot \frac{3}{2})$$

$$= 750 \quad (3)$$

QUESTION 8

a) $\sum_{k=1}^3 a \cdot 2^{k-1} = 28$

$$a + 2a + 4a = 28$$

$$7a = 28 \quad (4)$$

$$a = 4$$

2. $\sum_{k=1}^{\infty} r^k = \frac{a}{1-r}$

$$= \frac{16}{1 - \frac{1}{2}} = 32 \quad (3)$$

b. $\tan 45^\circ = \frac{1}{\sin 45^\circ}$

$$1 : \sqrt{2}$$

$$S_8 = \frac{1((\sqrt{2})^8 - 1)}{\sqrt{2} - 1}$$

$$= 15 + 15\sqrt{2} \quad (36,2)$$

QUESTION 9

$$f(x) = ax^2 + bx + c$$

a) $-8 = a + b + c$

$$h^2 - 2h - 8 = a(1+h)^2 + b(1+h) + c$$

$$h^2 - 2h - 8 = a + 2ah + ah^2 + b + bh + c$$

$$h^2 - 2h - 8 = ah^2 + h(2a+b) + a+b+c$$

$$a = 1 \quad 2a + b = -2$$

$$b = -4 \quad (5)$$

$$a + b + c = -8$$

$$c = -8 + 3 = -5$$

b) $x = 1$
 $y = -8$

$$f'(x) = 2x - 4$$

$$f'(1) = -2 \quad (4)$$

$$y = -2x + c$$

$$-8 = -2(1) + c$$

$$y = -2x - 6$$

9. c) $f(x) + k = 0$

$s \checkmark < k \checkmark < q \checkmark$ (3)

d) $h(x) = f\left(\frac{x}{a}\right) + 2$

TP: $\frac{x}{a} = 2$

$x = 2a$

$\therefore TP(2a, -7)$ (3)

b) 1. $T = 28 - 0,008t^3 - 0,16t$

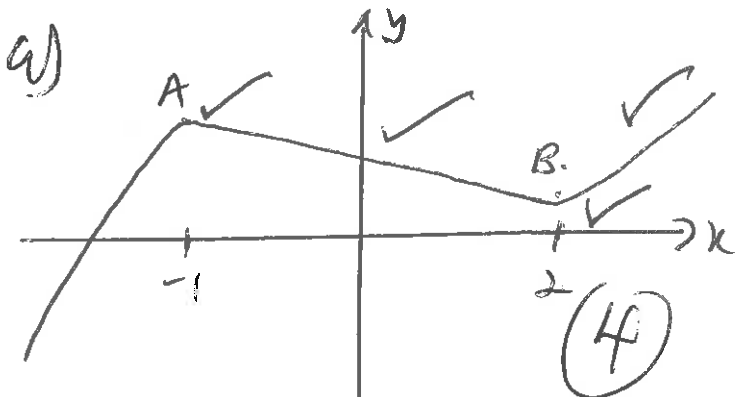
$T'(t) = -0,024t^2 - 0,16 \checkmark$

$T'(4) = -0,024(4)^2 - 0,16$

$= -0,544 \checkmark$ (4)

2)

QUESTION 10



2) $g(x) = x^3 + bx^2 + cx + d$

$g'(x) = 3x^2 + 2bx + c = 0$

$g'(-1) = 3 - 2b + c = 0 \checkmark$

$g'(2) = 12 + 4b + c = 0 \checkmark$

$9 + 6b = 0$

$6b = -9$

$b = -3/2 \checkmark$ (6)

$3 - 2(-3/2) + c = 0$

$c = -6 \checkmark$

lowest temp is at 10 m/s

$\therefore T = 28 - 0,008(10)^3 - 0,16(10)$
 $= 18,4 \checkmark$ (2)

3.

