

JULY EXAMINATION 2015

MATHEMATICS GRADE 12

PAPER 1: LO 1, LO 2

Time: 3 hours

Total: 150

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 6 pages, graph paper, and a separate formula sheet. Please check that your paper is complete.
2. Read the questions carefully
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Answers must be rounded off to the first decimal place, unless otherwise stated.
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.

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SECTION A

QUESTION 1:

a) Solve for x for $x \in \mathfrak{R}$

1) $\sqrt{2x-5} = -1$ (2)

2) $2^{9-x^2} = 4^{x+3}$ (4)

b) Given $A(x) = \frac{2x^2 + 1}{x^2 - 2x}$

1) For which values of x is $A(x)$ is undefined. (3)

2) Calculate the values of k for which $A(x) = k$ have real roots. (5)

c) Solve for x and y if : $2x + y = 1$ and $2x^2 - xy + y^2 = 4$ (6)

[20]

QUESTION 2:

In the first stage of the soccer event at the Olympic Games, there are teams from four different countries in each group. Each country in a group must play every other country in the group once.

a) How many matches will be played in each group in the first stage of the event?(2)

b) How many matches would be played if there are 5 teams in each group? (2)

c) How many matches would be played if there are 6 teams in each group? (2)

d) Determine the general formula of the sequence. (4)

e) How many matches would be played if there are 28 teams in each group? (3)

[13]

QUESTION 3:

a) $\lim_{x \rightarrow 1} \frac{1-x^3}{1-x}$ (3)

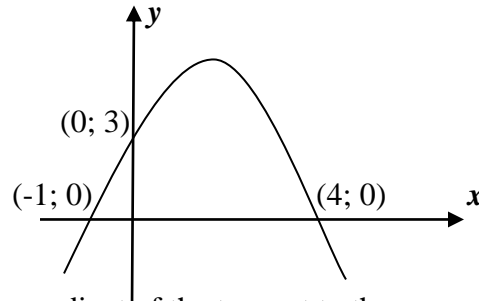
b) Determine $f'(x)$ from first principles if $f(x) = \frac{1}{2}x^2 - 2x$. (5)

c) Determine $\frac{dy}{dx}$ in each of the following:

1) $y = \frac{3}{2}x^4 - \frac{2}{3}x^3$ (2)

2) $y = \left(\frac{3}{x} + 1\right)\left(\frac{3}{x} - 1\right)$ (3)

- d) The function of $\frac{dy}{dx}$ is represented by the parabola in the sketch below.



- 1) What is the gradient of the tangent to the curve $y = f(x)$ at the point where $x = 0$ (1)
- 2) For what values of x will the gradient of the curve of $f(x)$ be positive? (2)
- 3) Give the x -coordinates of the turning points of the curve of $f(x)$, stating in each case whether the turning points at these x values are local maximum or minimum points. (4)

[20]

QUESTION 4:

a) Given: $h(x) = \frac{4}{x}$

- 1) Draw a sketch graph of h . (2)
- 2) Determine the values of m for which the graph of $y = mx$ intersects the graph of h . (3)

b) Given $h(x) = \left(\frac{1}{2}\right)^{x-1} - 2$.

- 1) Sketch the graph of $h(x)$ and $h^{-1}(x)$ on the same set of axes, clearly indicating the values of the intercepts with the axes and asymptotes. (5)
- 2) Calculate the equation $h^{-1}(x)$ (3)
- 3) Determine the values of x for which $h(x), h^{-1}(x) \leq 0$ (2)

[15]

QUESTION 5:

Nkosi takes both History and Additional Mathematics. Past student results indicate that the probability of passing Additional Mathematics is 0,75, that of failing History is 0,35 and that of passing at least one of the two subjects is 0,85. Find the probability that Nkosi will:

- a) pass History (1)
 - b) pass both subjects (3)
 - c) fail both subjects (2)
 - d) pass exactly one of the two subjects. (2)
- [8]**

SECTION B

QUESTION 6:

In how many ways can we rearrange the letters in “MATHS IS FUN?”

- a) With no restrictions (3)
 - b) If the first and last letter must be vowels (4)
- [7]**

QUESTION 7:

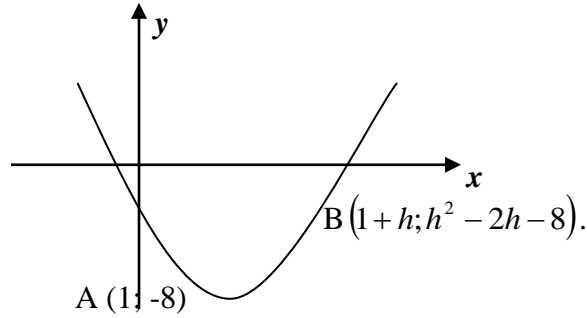
- a) The cost of a bus is R1,2 million. It is expected that the value of this bus will depreciate on a reducing balance per annum to R491 520 in 4 years time. The price of a new bus is expected to increase by 15% per annum. The bus will need to be replaced in 4 years time.
 - 1) Calculate the rate of depreciation of the bus. (4)
 - 2) Calculate the value of a sinking fund that needs to be set up to pay for the new bus, if the old bus is traded in. (4)
 - 3) Calculate the amount that must be invested monthly into a sinking fund to cover the replacement cost of the bus in 4 years time if the interest paid by the bank is 9% per annum compounded monthly. Payments commence at the beginning of the first month and end at the end of the last month.(5)
 - b) A father has three children whose present ages are 12, 15 and 17 years. He plans to give them R40 000 each on their first 21th birthday. How much money must he invest now if the interest rate is 14% p.a compounded monthly? (6)
- [19]**

QUESTION 8:

- a) 1) Calculate the value of a if $\sum_{k=1}^3 a \cdot 2^{1-k} = 28$ (4)
 - 2) Hence calculate $\sum_{k=1}^{\infty} T_n$ (3)
 - b) The first 2 terms of geometric series are $\tan 45^\circ$ and $\frac{1}{\sin 45^\circ}$. Find the sum of the series to 8 terms. (4)
 - c) An arithmetic progression has a first term of 12 and a fifth term of 18. Find the sum of the first 25 terms. (3)
- [14]**

QUESTION 9:

The sketch below of a parabola $f(x) = ax^2 + bx + c$ with points A (1; -8) and B(1+h; h² - 2h - 8).



- a) Determine the equation of the parabola. (5)
- b) Given that $f(x) = x^2 - 4x - 5$, calculate the equation of the tangent at the point where $x = 1$. (4)
- c) Write down the values k if $f(x) + k = 0$ has 2 positive real roots. (3)
- d) Write down the coordinates of the turning point of $h(x)$ if $h(x) = f\left(\frac{x}{a}\right) + 2$ in terms of a (3)

[15]

QUESTION 10:

- a) The graph of a cubic function $g(x)$ has turning points at $A(-1; p)$ and $B(2; q)$. The function $g(x)$ has the following properties:
 - $g'(x) > 0$ for $x < -1$ or $x > 2$
 - $g'(x) < 0$ for $-1 < x < 2$
 - $g(2) > 0$
 - 1) Draw a neat sketch of $g(x)$ and clearly label points A and B on the sketch. (it is not necessary to show the x and y intercepts) (4)
 - 2) If $g(x) = x^3 + bx^2 + cx + d$, calculate the values of b and c (6)
- b) In order to reduce temperature in a room from 28° C, a cooling system is allowed to operate for 10 minutes. The room temperature, T after t minutes is given in ° C by the formula: $T = 28 - 0.008t^3 - 0.16t$ where $t \in [0; 10]$
 - 1) At what rate is the temperature falling when $t = 4$ minutes? (4)
 - 2) Calculate the lowest temperature reached during the 10 minutes for which the cooling system operates. (2)
 - 3) Sketch the graph of T . (3)

[19]