Name:

Teacher:

ST STITHIANS COLLEGE

Memo

GRADE 12	
MATHEMATICS PRELIM EXAM – PAPER 2	
DATE: 24 July 2015	**TIME: 180 minutes**
TOPICS: Paper 2	**TOTAL MARKS: 150**
EXAMINER: Cluster N143	**MODERATOR: Cluster N143**

INSTRUCTIONS:

1. This paper consists of 10 questions. Answer ALL of the questions.
2. This question paper consists of 22 pages.
3. Clearly show ALL calculations you have used to determine the answers.
4. An approved scientific calculator (non-programmable and non-graphical) may be used, unless otherwise specified.
5. If necessary, answers should be rounded off to TWO decimal digits, unless stated otherwise.
6. Diagrams are not necessarily drawn to scale.
7. It is in your own interest to write legibly and to present your work neatly.

QUESTION 1 **[22]**

1.1) $M(1;b)$ is the midpoint of the line segment joining $A(a;4)$ and $B(5;6)$.

Find the values of a and b. (3)

$1 = \frac{a+5}{2}$ ✓

$2 = a+5$

$\therefore a = -3$ ✓

$b = \frac{4+6}{2}$

$\therefore b = 5$ ✓

1.2) The points $C(1;-2)$, $D(5;1)$ and $E(c^2;c+1)$ are collinear.

Find the value(s) of c. (4)

$m_{CD} = \frac{-2-1}{1-5} = \frac{3}{4}$

$\therefore m_{DE} = \frac{1-(c+1)}{5-c^2} = \frac{-c}{5-c^2} = \frac{3}{4}$

$\therefore -4c = 3(5-c^2)$

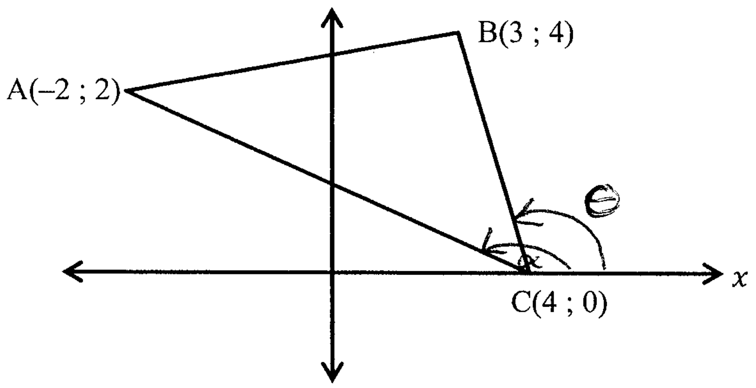
$-4c = 15-3c^2$

$3c^2-4c-15=0$

$(3c+5)(c-3)=0$

$c = -5/3$ or $c = 3$

1.3) Given points $A(-2;2)$, $B(3;4)$ and $C(4;0)$ on the Cartesian plane as sketched:



1.3.1) Calculate the size of $A\hat{C}B$, rounded off to **one** decimal digit. (6)

$m_{BC} = \frac{4-0}{3-4} = -4$

$m_{AC} = \frac{2-0}{-2-4} = -\frac{1}{3}$

$\therefore \tan\theta = -4 \quad \therefore \theta = 104{,}036\ldots$

$\tan\alpha = -1/3 \quad \therefore \alpha = 161{,}565\ldots$

$\therefore A\hat{C}B = 161{,}\ldots - 104{,}\ldots$
$= 57{,}5^\circ$

1.3.2) Show that the midpoint, M, of AC is $(1\,;1)$. (2)

$M(x;y) = \left(\frac{-2+4}{2}\,;\,\frac{2+0}{2}\right)$
$= (1;1)$

1.3.3) Determine the equation of the circle which has AC as a diameter.

Give your answer in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)

$(x-1)^2 + (y-1)^2 = r^2$

$(4-1)^2 + (0-1)^2 = r^2$

$\therefore\ r^2 = 10$

$\therefore\ (x-1)^2 + (y-1)^2 = 10$

1.3.4) Determine by calculation, whether point B lies inside or outside this circle.
Give a reason for your answer. (2)

$BM = \sqrt{(3-1)^2 + (4-1)^2}$

$= \sqrt{13}$

$BM = \sqrt{13}$ but radius $= \sqrt{10}$

$\therefore$ B lies autside the circle

1.3.5) Write down the value of the shortest distance from B to the circle.

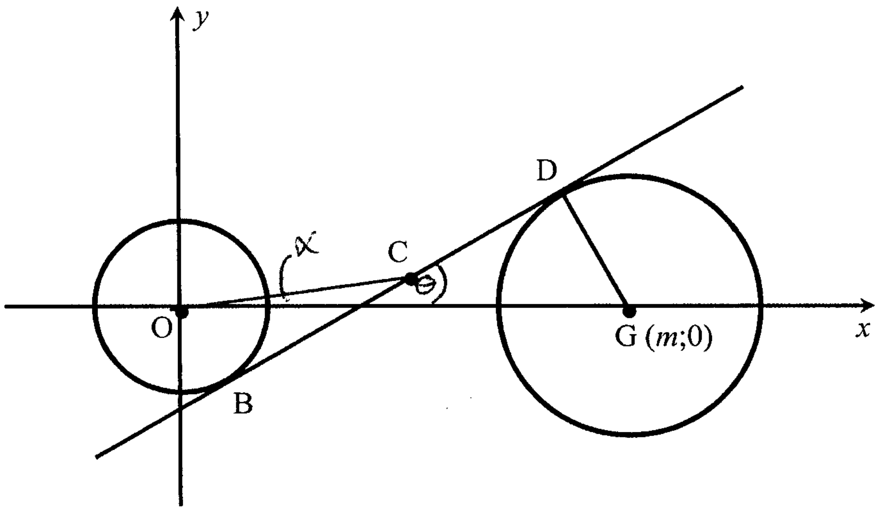
(Leave your answer in surd form) (2)

BM − radius $= \sqrt{13} - \sqrt{10}$

QUESTION 2 **[18]**

Refer to the diagram below. Given circle with centre O and equation $x^2 + y^2 = 20$.

$G(m;0)$ is the centre of the larger circle. A common tangent touches the circles at B and D respectively.



2.1) B $(t;-2)$ lies on the circumference of the small circle. Determine the value of t. (3)

$t^2 + (-2)^2 = 20$

$t^2 = 16$

$t = \pm 4$

$\therefore t = 4$

2.2) C $(6;2)$ is the midpoint of BD. Determine the coordinates of D. (2)

$6 = \frac{x+4}{2}$ $\qquad$ $2 = \frac{y-2}{2}$ ✓

$\therefore x = 8$ $\qquad$ $\therefore y = 6$ ✓

$\therefore D(8;6)$

2.3) Determine the gradient of DG. (3)

$$M_{BD} = \frac{6+2}{8-4} = 2$$

$$\therefore m_{DG} = -\frac{1}{2}$$

2.4) Show that $m = 20$ (3)

$$m_{DG} = \frac{6-0}{8-m} = -\frac{1}{2}$$

$$\therefore -8+m = 12$$

$$\therefore m = 20$$

2.5) Determine the equation of the circle with centre G. (3)

$$(x-20)^2 + (y-0)^2 = r^2$$

$$(8-20)^2 + (6-0)^2 = r^2$$

$$\therefore r^2 = 180$$

$$\therefore (x-20)^2 + y^2 = 180$$

2.6) Determine the size of angle $O\hat{C}B$. (4)

$m_{OC} = 1/3 \quad \therefore \tan\alpha = \frac{1}{3} \quad \therefore \alpha = 18,4°$

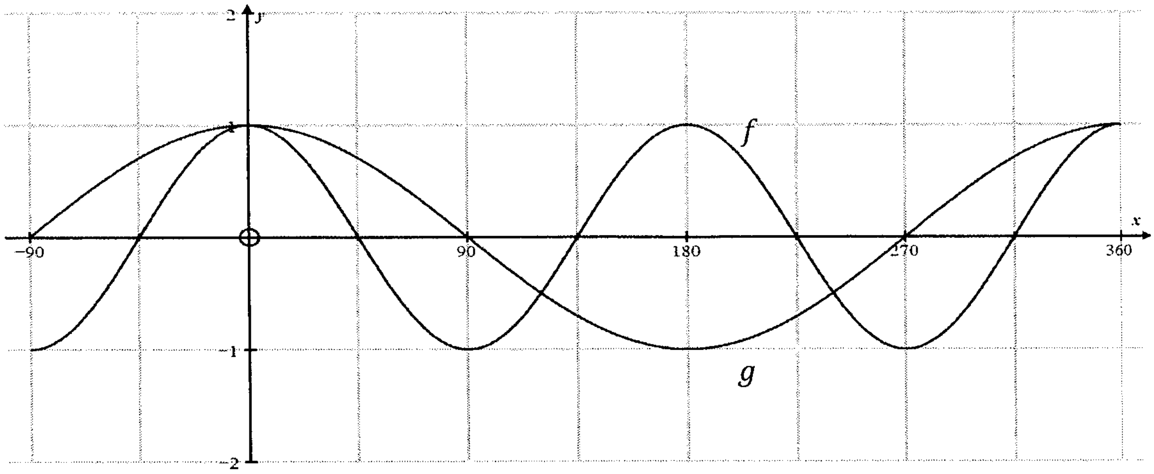
$m_{CB} = 2 \quad \therefore \tan\theta = 2 \quad \therefore \theta = 63,4°$

$\therefore O\hat{C}B = 63,4° - 18,4°$ ext $\angle\Delta$

$= 45°$

QUESTION 3 **[22]**

3.1) Trigonometric functions $f(x)$ and $g(x)$ are given below, with $x \in [-90°; 360°]$:



3.1.1) Write down the equations of f and g. (2)

$f(x) = \cos 2x$

$g(x) = \cos x$

3.1.2) Write down the period of f. (1)

$180°$

3.1.3) Write down the amplitude of g. (1)

1

3.1.4) Determine the values of x where $f(x).g(x) \geq 0$ for $x \in [90°\,; 270°]$ (4)

$x \in [90°; 135°] \cup [225°; 270°]$

3.2) If θ, 2θ and 3θ are the angles of a triangle, evaluate $cos^2\theta + cos^2 2\theta + cos^2 3\theta$ without the use of a calculator: (4)

$\theta + 2\theta + 3\theta = 180°$

$\therefore 6\theta = 180°$

$\therefore \theta = 30°$

$\therefore \cos^2\theta + \cos^2 2\theta + \cos^2 3\theta$

$= \cos^2 30° + \cos^2 60° + \cos^2 90°$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + 0^2$

$= 1$

3.3) Without the use of a calculator, solve $sin32°cosx + cos32°sinx = sin75°$ for x, where $-360° \leq x \leq 360°$: (5)

$\sin 32°.\cos x + \cos 32°.\sin x = \sin 75°$

$\therefore \sin(32° + x) = \sin 75°$

$\therefore 32° + x = 75° + k.360°$

$x = 43° + k.360°$

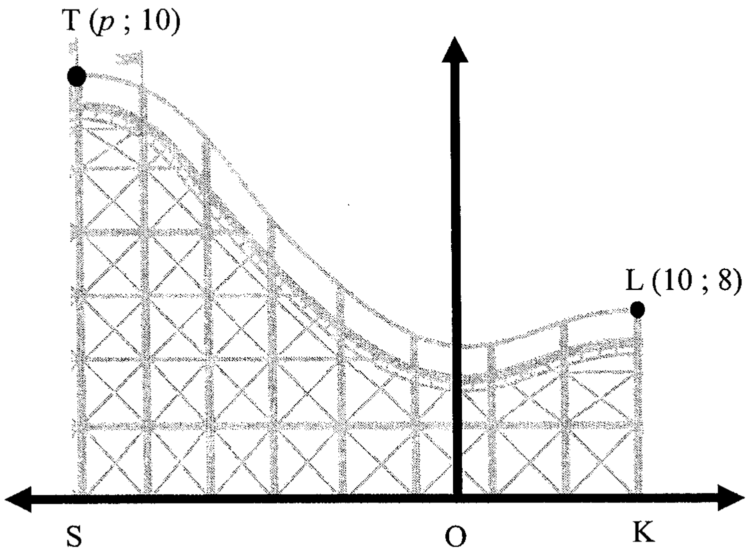
OR $32° + x = 105° + k.360°$

$\therefore x = 73° + k.360°$ $\quad k \in \mathbb{Z}$

$\therefore x = \{-317°; -287°; 43°; 73°\}$

3.4) You are riding the Colossus at Ratanga Junction and notice that consecutive peaks, T $(p;10)$ and L $(10;8)$, of the ride are in proportion to each other.

You also notice as you are riding, that $L\hat{O}K = S\hat{O}T = \beta$.



3.4.1) Determine the value of $\cos(90° + \beta)$.

(Leave your answer in surd form if necessary) (3)

$OL = 2\sqrt{41}$ Pythag

$\therefore \cos(90° + \beta)$

$= -\sin\beta$

$= \frac{-8}{2\sqrt{41}} = \frac{-4}{\sqrt{41}}$

3.4.2) Determine the value of p. (2)

$\frac{8}{10} = \frac{10}{p}$

$\therefore 8p = 100$

$p = -12,5$

QUESTION 4 [9]

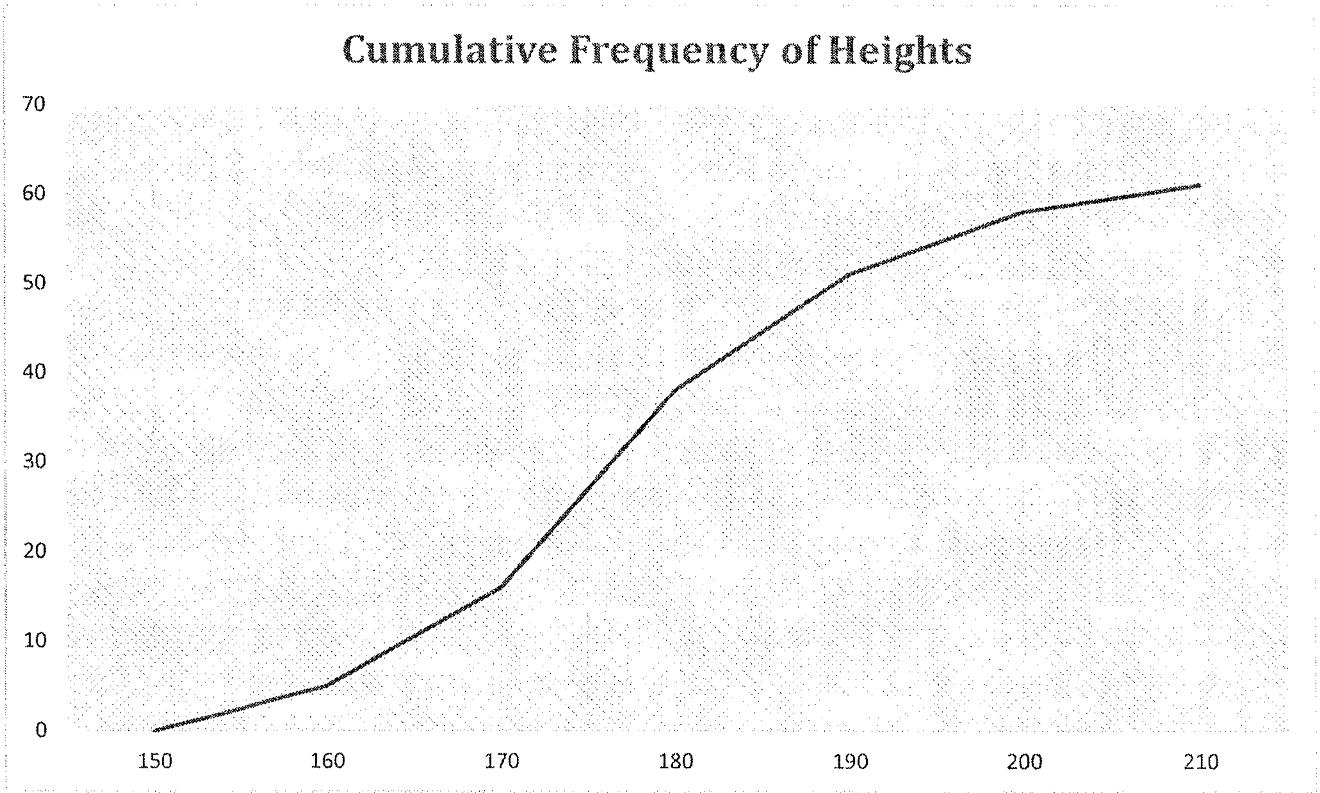
Mr Mears is curious to see the distribution of heights of all his History students.

The table below summarises the individual heights (in cm) of 61 History students.

4.1) Complete the table by filling in the unknown values for (a) and (b): (2)

Height Intervals in cm	Frequency	Cumulative Frequency
$140 \leq x < 150$	0	0
$150 \leq x < 160$	5	5
$160 \leq x < 170$	11	16
$170 \leq x < 180$	**(a)** 22	38
$180 \leq x < 190$	13	51
$190 \leq x < 200$	7	**(b)** 58
$200 \leq x < 210$	3	61

4.2) Below is an Ogive for the heights of the History students:



Use the Ogive to estimate the values of Q_1, Q_2 and Q_3, and show on the Ogive how you read off your answers.

4.2.1) Q_1 $\frac{1}{4}(n+1) = \frac{62}{4} = 15\frac{1}{2}^{th}$ value (1)

$\therefore Q_1 = 168$

4.2.2) Q_2 $\frac{1}{2}(n+1) = \frac{62}{2} = 31^{st}$ value (1)

$\therefore Q_2 = 177$

4.2.3) Q_3 $\frac{3}{4}(n+1) = \frac{186}{4} = 46\frac{1}{2}^{th}$ value (1)

$\therefore Q_3 = 186$

4.3) Which height interval(s) contain(s) heights from the 90th percentile. (2)

$190° \leq x < 200$

and $200 \leq x < 210$

4.4) Use the table of information to calculate an estimate for the mean of the History students' heights. (2)

$\bar{x} \approx \frac{10825}{61}$

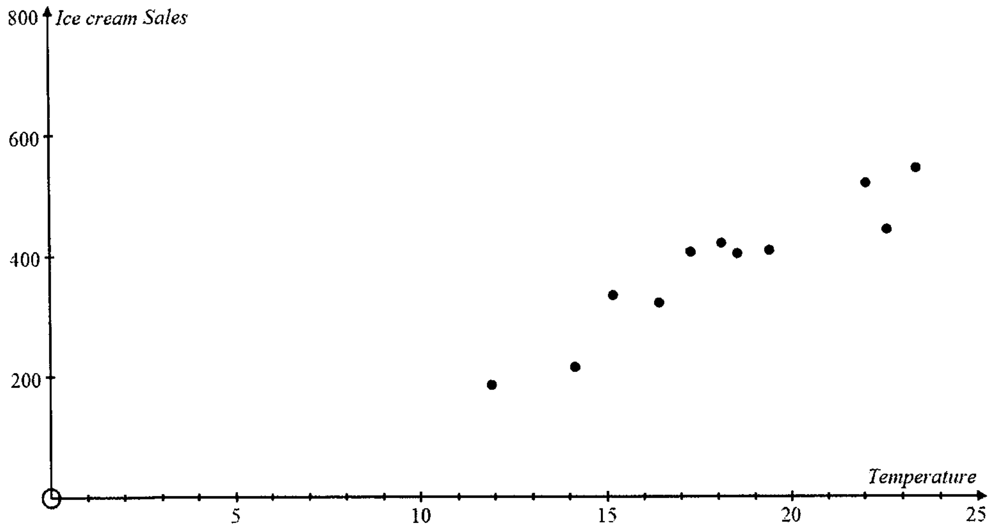
$\approx 177,46$ cm

QUESTION 5 [9]

A tuck shop monitors how much ice cream it sells relative to the temperature on that day.

Here are their figures recorded for the past 12 days:

Temperature (°C)	14.2	16.4	11.9	15.2	18.5	22.1	19.4	25.1	23.4	18.1	22.6	17.2
Ice Cream Sales (R)	215	325	185	332	406	522	412	614	544	421	445	408



5.1) Discuss the trend in the data collected. (1)

The higher the temperature, the more icecreams sold.
(Strong positive correlation)

5.2) Determine, with the use of a calculator, the linear regression equation of the line of best fit and the correlation coefficient for the data. (3)

$y = 30,09x - 159,47$

$r = 0,96$

5.3) Use your equation from ~~5.2~~ 5.2 above to determine the ice cream sales when the temperature is 21°C. (2)

$y = 30{,}09(21) - 159{,}47$

$y = 472{,}42$ or 472/473

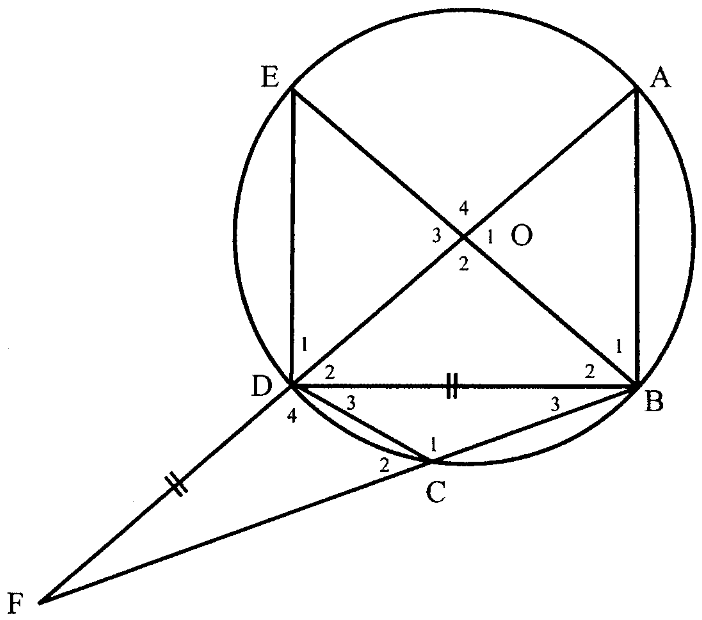
5.4) The weather bureau predicts a bit of a heatwave, with temperatures exceeding 30°C. Predict ice cream sales at these temperatures, and explain the validity of your answer. (3)

The icecream sales will be 743,23. or 743/744

The results could be unreliable as this is an example of extrapolation.

QUESTION 6 **[20]**

6.1) In the figure O is the centre of the circle and $DB = DF$.
AF, BE and BF are straight lines, and $\hat{F} = 20°$.



Find, with reasons, the magnitude of the following angles:

6.1.1) $\hat{D}_2$ $\hat{F} = \hat{B}_3 = 20°$ isos $\triangle$ (3)

$\therefore \hat{D}_2 = 40°$ ext $\angle$ $\triangle$

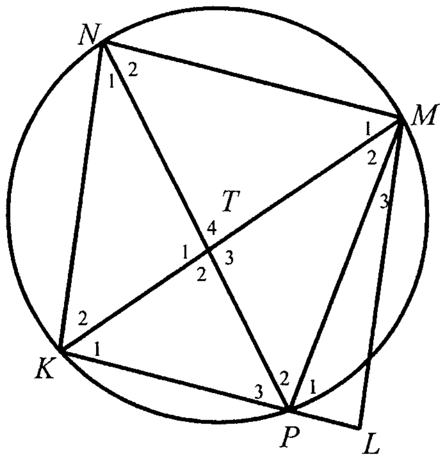
6.1.2) $\hat{A}$ $A\hat{B}D = 90°$ $\angle$ in semi-circle (3)

$\therefore \hat{A} = 50°$ $\angle^s$ in $\triangle$

6.1.2) $\hat{O}_2$ $\hat{O}_2 = 100$ $\angle^s$ in $\triangle$ OR $\angle$ @ centre $2 \times \angle$ at circ. (2)

6.1.3) $\hat{C}_1$ $\hat{C}_1 = 130°$ opp $\angle^s$ cyclic quad. (2)

6.2) In the diagram below, parallelogram *KLMN* is given. $\hat{T}$ is **not** the centre of the circle. $\hat{L} = 66^o$ and $\hat{N}_1 = 24^o$. Determine the size of $\hat{M}_1$. (5)



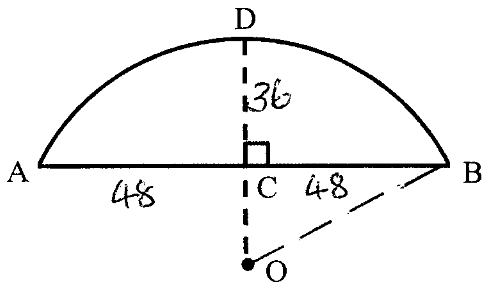
$\hat{N}_1 + \hat{N}_2 = \hat{L}$ opp ∠s ∥gram

$\therefore \hat{N}_2 = 42°$

$\hat{P}_3 = 42°$ alt ∠s NM∥KL

$\therefore \hat{M}_1 = 42°$ ∠s in same segment

6.3) An arch of a bridge is such that it is an arc of a circle and its height is 36m and its span is 96m. (i.e. $CD = 36\text{m}$ and $AB = 96\text{m}$).



Calculate with reasons the radius OD of the arch, i.e. calculate the length of OD. (Hint Let $OD = x$) (5)

$OD = x = OB$ radii

$\therefore OC = x - 36$

$AC = BC = 48$ line from centre $\perp$ chord

$\therefore OB^2 = 48^2 + (x-36)^2$

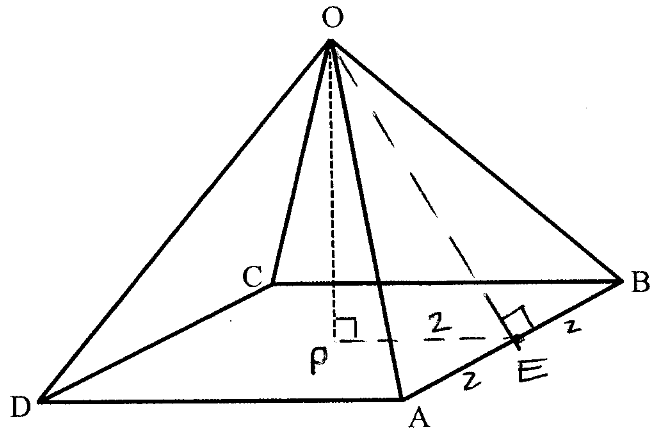
$x^2 = 2304 + x^2 - 72x + 1296$

$72x = 3600$

$x = 50 = OD.$

QUESTION 7 **[16]**

7.1) $OABCD$ is a right pyramid with a square base with sides of length 4cm as shown in the diagram below. $O\hat{A}B = 50°$ and $OA = OB$.



7.1.1) Determine the length of OA. (2)

$\cos 50° = \frac{2}{OA}$ OR $\frac{OA}{\sin 50°} = \frac{4}{\sin 80°}$

$\therefore OA = \frac{2}{\cos 50°}$

$\therefore OA = 3,11$ $\therefore OA = 3,11$

7.1.2) Determine the length OE, the slant height of triangle OAB, where E is the midpoint of AB. (3)

$OE^2 = 3,1^2 - 2^2$

$\therefore OE = 2,38$

7.1.3) Show that the perpendicular height is 1,29 (2)

$OP^2 = 2,38^2 - 2^2$

$\therefore OP = 1,29$

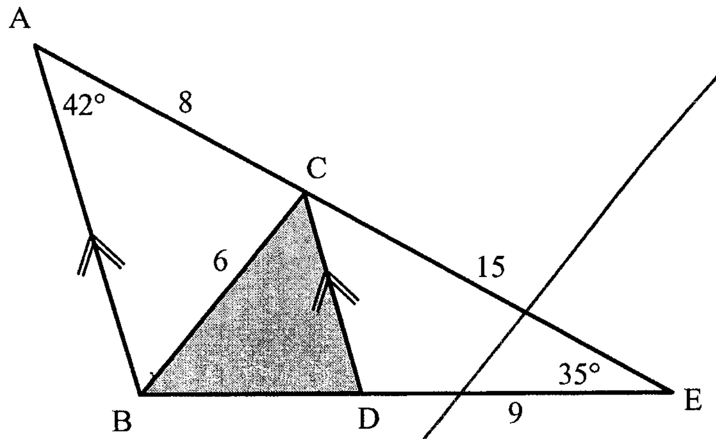
7.1.4) Hence, or otherwise, calculate the volume of the pyramid. (2)

$V = \frac{1}{3} \times \text{Area of base} \times h$

$= \frac{1}{3}(4)^2 \times 1,29$

$= 6,77\ cm^3$

7.2) Given AB // CD, $AC = \cancel{12}\ 8$, $CE = 15$, $DE = 9$, $BC = 6$, $\hat{A} = 42^o$ and $\hat{E} = 35^o$
Find the Area of the shaded ΔBCD. (7)



In ΔABC

$\frac{\sin B}{8} = \frac{\sin 42°}{6}$

$\therefore \sin B = 0,89...$

$\therefore \hat{B} = 63,15°$

In ΔCDE

$CD^2 = 15^2 + 9^2 - 2(15)(9)\cos 35°$

$CD^2 = 84,82...$

$\therefore CD = 9,21$

Area $\Delta BCD = \frac{1}{2}(6)(9,21)\sin 63,15°$

$= 24,65\ units^2$

QUESTION 8 [12]

8.1.1) Prove the identity: $\frac{\sin^2 2x}{\cos x} + 2\cos 2x.\cos x = 2\cos x$ (5)

LHS: $\frac{2\sin x\cos x \cdot 2\sin x \cdot \cos x}{\cos x} + 2\cos 2x \cdot \cos x$

$= 4\sin^2 x.\cos x + [2(2\cos^2 x - 1)]\cos x$

$= \cos x\,[4\sin^2 x + 4\cos^2 x - 2]$

$= \cos x\,[4 - 2]$

$= 2\cos x$

$=$ RHS.

OR $= 4\sin^2 x\cos x + [2(1 - 2\sin^2 x)]\cos x$

$= 4\sin^2 x\cos x + 2\cos x - 4\sin^2 x\cos x$

$= 2\cos x =$ RHS

8.1.2) Hence, determine the maximum value of $\frac{\sin^2 2x}{\cos x} + 2\cos 2x.\cos x$, and the value of x to give this maximum, where $-90^\circ \leq x \leq 90^\circ$ (2)

Max value $= 2$

$x = 0^\circ$

8.2) Determine the general solution of: $\cos(25^\circ - 2\theta) = \sin 4\theta$ (5)

$\cos(25^\circ - 2\theta) = \sin 4\theta$

$\cos(25^\circ - 2\theta) = \cos(90^\circ - 4\theta)$

$\therefore 25^\circ - 2\theta = \pm(90^\circ - 4\theta) + k.360^\circ$

$25^\circ - 2\theta = 90^\circ - 4\theta + k.360^\circ$

$\theta = 32{,}5^\circ + k.180^\circ$

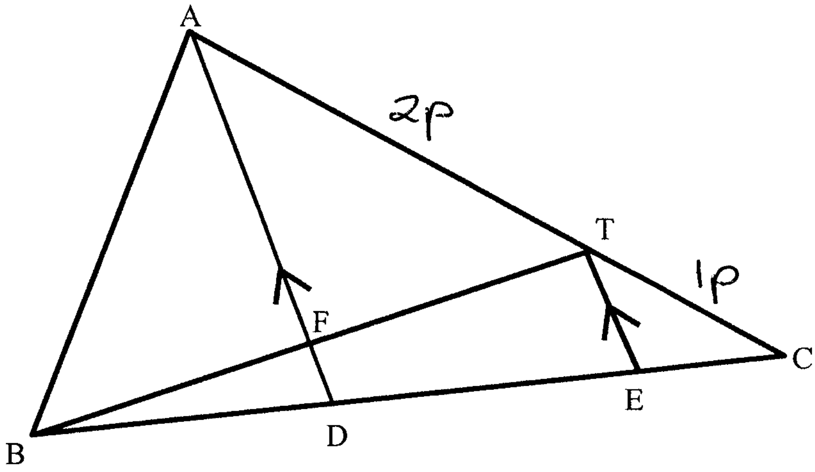
OR $25^\circ - 2\theta = -90^\circ + 4\theta + k.360^\circ$

$\therefore \theta = 19{,}17^\circ + k.60^\circ$

$k \in \mathbb{Z}$

QUESTION 9 **[9]**

In the figure below, ΔABC has D and E on BC, $BD = 6cm$ and $DC = 9cm$.
$AT : TC = 2 : 1$ and $AD \parallel TE$.



9.1) Write down the numerical value of $\frac{CE}{ED}$ (1)

$\frac{CE}{ED} = \frac{1p}{2p}$ line ∥ one side of Δ

$\therefore \frac{CE}{ED} = \frac{1}{2}$.

9.2) Show that D is the midpoint of BE. $DE = \frac{2}{3}(DC)$ (2)

$BD = 6$ given and $DE = 6 = 2/3\,(9cm)$

$\therefore$ D the midpoint of $BE = 6cm$.

9.3) If $FD = 2cm$, calculate the length of TE. (2)

$TE = 4cm$ mdpt theorem

9.4) Calculate the numerical value of:

9.4.1) $\dfrac{\text{Area of } \Delta ADC}{\text{Area of } \Delta ABD}$ (1)

$$\frac{\text{Area } \Delta ADC}{\text{Area } \Delta ABD} = \frac{9}{6}$$

$$= \frac{\frac{1}{2} \times DC \times h}{\frac{1}{2} \times BD \times h} = \frac{9}{6} = \frac{3}{2}.$$

9.4.2) $\dfrac{\text{Area of } \Delta TEC}{\text{Area of } \Delta ABC}$ (3)

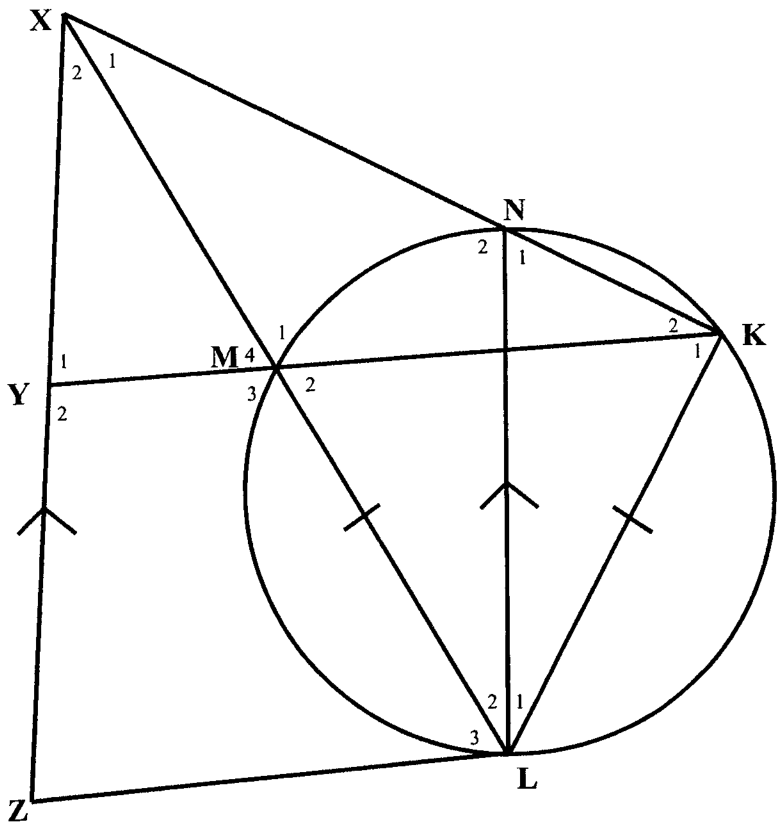
$$\frac{\text{Area } \Delta TEC}{\text{Area } \Delta ABC} = \frac{\frac{1}{2}.TC.CE.\sin C}{\frac{1}{2}.AC.BC.\sin C}$$

$$= \frac{x \times 3}{3x \times 15}$$

$$= \frac{1}{15}$$

QUESTION 10 [13]

Refer to the figure below. $LM = KL$, and LZ is a tangent to the circle at L. $XZ \parallel LN$ and KM produced meets XZ at Y. KNX is a straight line.



10.1) Prove that $YK \parallel ZL$. (4)

$\hat{L}_3 = \hat{K}_1$ tan chord

$\hat{K}_1 = \hat{M}_2$ isos $\triangle$

$\therefore \hat{L}_3 = \hat{M}_2$

$\therefore YK \parallel ZL$ $\angle^s$ in alt position

10.2) Prove that $\Delta XYM ||| \Delta KYX$. (4)

$\hat{L}_2 = \hat{X}_2$ alt $\angle^s$ $XZ \parallel LN$

$\therefore \hat{L}_2 = \hat{K}_2$ $\angle^s$ in same segment

In ΔXYM and ΔKYX

$\hat{X}_2 = \hat{K}_2$ proven above

$\hat{Y}$ is common

$\therefore \hat{M}_4 = Y\hat{X}K$ 3^{rd} $\angle$

$\therefore \Delta XYM ||| \Delta KYX$ $\angle\angle\angle$

10.3) Prove $XZ.XY = KY.LZ$ (5)

In ΔXLZ and ΔKYX

$\hat{X}_2 = \hat{K}_2$ proven above

$\hat{Z} = \hat{Y}_1$ corr $\angle^s$ $YK \parallel ZL$

$\therefore \hat{L}_3 = Y\hat{X}K$ 3^{rd} $\angle$

$\therefore \Delta XZL ||| \Delta KYX$ $\angle\angle\angle$

$\therefore \frac{XZ}{KY} = \frac{ZL}{XY} = \frac{XL}{KX}$

$\therefore XZ.XY = KY.LZ$

ST STITHIANS GIRLS' COLLEGE

GRADE 12

MATHEMATICS: PAPER 2

24 July 2015

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NAME OF LEARNER: ____________________

QUESTION	AIM 3 Trigonometry	AIM 4 Geometry	AIM 5 Statistics
1		/22	
2		/18	
3	/22		
4			/9
5			/9
6		/20	
7	/16		
8	/12		
9		/9	
10		/13	
TOTALS	/50	/79	/18
		/150	%