MEMO

St. Anne's Diocesan College

MEMO

Form 6 | Core Mathematics: Paper II | September 2015

Time: 3 hours | Marks: 150

Please read the following instructions carefully:

1. This question paper consists of 19 pages and an Information sheet. Please check that your question paper is complete.
2. Write your examination number in the space provided on this question paper.
3. **Answer all the questions on the question paper and hand this in at the end of the examination.**
4. Read the questions carefully.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. All necessary working details must be clearly shown.
7. Round off answers to **1 decimal digit** where necessary, unless otherwise stated.
8. Ensure that your calculator is in **DEGREE** mode.
9. Diagrams are not drawn to scale.
10. It is in your own interest to present your work neatly.
11. The last page can be used for additional working, if necessary. If this space is used, make sure that you indicate clearly which question is being answered.

EXAMINATION NUMBER:

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SECTION A

Question 1

(a) Two points have coordinates F(6; 2) and H(16, 14). If G is the midpoint of FH then determine the coordinates of G.

$G\left(\frac{6+16}{2};\ \frac{2+14}{2}\right)$

$\therefore G(11; 8)$ ✓A ✓A

(2)

(b) Points P (2; −3), Q (−3; −3) and R (5; y) are given.

1. Determine the lengths of PQ and RQ (in terms of y where necessary).

$PQ = \sqrt{(-3-2)^2+(-3+3)^2}$

$= \sqrt{25}$

$= 5$ units ✓A

$QR = \sqrt{(5+3)^2+(y+3)^2}$

$= \sqrt{64+y^2+6y+9}$

$= \sqrt{y^2+6y+73}$ units ✓A

(2)

2. Calculate the value (s) of y, if the distance from R to Q is twice the distance from P to Q.

$RQ = 2PQ$ ✓M double PQ

$RQ^2 = 4PQ^2$

$y^2+6y+73 = 4(25)$ ✓A

$y^2+6y-27=0$

$(y+9)(y-3)=0$ ✓M factorise

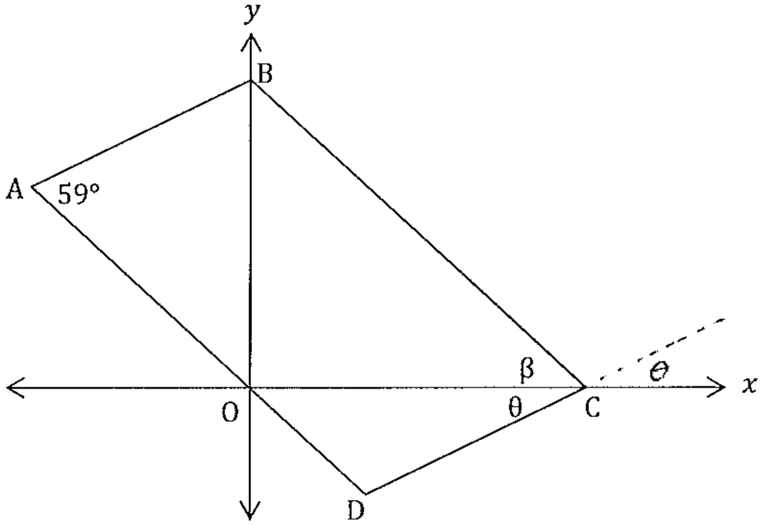
$\therefore y=-9$ or $y=3$ ✓A for both answers

(4)

[8]

Question 2

In the diagram below, ABCD is a parallelogram. The equation of the line passing through A and B is given by $y = \frac{1}{4}x + 6$. The line passing through A and D also passes through the origin.
$B\hat{A}D = 59°$, $B\hat{C}O = \beta$ and $D\hat{C}O = \theta$.



(a) Determine, correct to the nearest degree and with reasons, the size of:

1. θ

AB ∥ DC (opp. sides parm ABCD)

$m\ DC = \frac{1}{4}$ ✓A

✓A reason

$\tan\theta = \frac{1}{4}$ (vert. opp. ∠s) ✓M using tan

$\theta = 14{,}03624\ldots$

$\therefore \theta = 14°$ ✓A

(4)

2. β

$\hat{C} = 59°$ (opp. ∠s of a parm) ✓A reason

$\beta = 45°$ ✓A

(2)

(b) Determine the equation of AD.

$m\ AD = m\ BC$ (opp. sides of parm)

$m\ AD = \tan 135° = -1$ ✓A

$\therefore$ Equation AD: $y = -x$ ✓A

(2)

(c) Determine the equation of BC.

$m_{AD} = m_{BC} = -1$ ✓A

$\therefore$ equation BC : $y = -x + 6$ ✓A ✓A

(2)

(d) Determine the **co-ordinates** of C. ✓m let $y=0$

let $y=0$: $0 = -x + 6$

$x = 6$

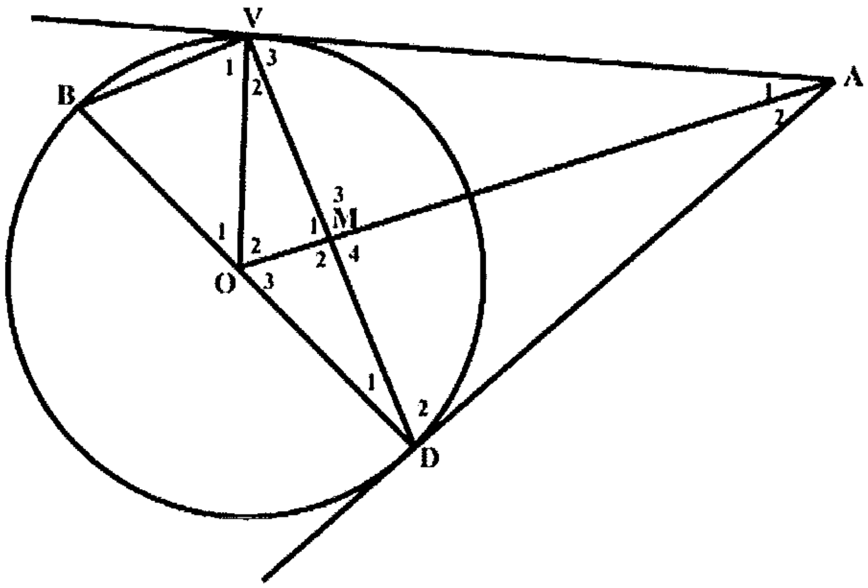
$\therefore C(6;0)$ ✓A

(2)

[12]

Question 3

From a point outside the circle, centre O, two tangents AD and AV are drawn. AO and VD meet in M. BOD is a diameter of the circle. BV and VO are drawn and $\hat{A}_1 + \hat{A}_2 = 40°$.



(a) Complete the following table by filling in the reasons:

	Statement	Reason
1.	$A\hat{D}O = 90°$	rad $\perp$ tangent ✓A
2.	$B\hat{V}D = 90°$	$\angle$ in semi-circle ✓A

(2)

(b) Calculate, with reasons, the size of:

1. $\hat{D}_1$

$AV = AD$ (tangents from same point) ✓A

$\hat{V}_3 = \hat{D}_2$ ($\angle$s opp eq. sides in ΔADV) ✓A

$\therefore \hat{V}_3 = \hat{D}_2 = 70°$ ($\angle$s in ΔADV)

but $\hat{D}_1 + \hat{D}_2 = 90°$ (rad. $\perp$ tang.) ✓A

$\therefore \hat{D}_1 = 20°$ ✓A (compl. adj. $\angle$s)

(4)

2. $\hat{O}_1$ $\hat{O}_1 = 40°$ ✓A ($\angle$ at centre thm) ✓A

(2)

(c) Prove, with reasons, that BV is parallel to OA.

$\hat{V}_2 + \hat{V}_3 = 90°$ (tang $\perp$ rad)

$\hat{V}_2 = 20°$ (compl. $\angle$s)

$\hat{V}_1 = 70°$ (compl. $\angle$s) ✓A

$\hat{O}_2 = 70°$ ($\angle$s in ΔVOA) ✓A

$\therefore BV \parallel OA$ (alt. $\angle$s are eq.) ✓A

(3)

[11]

Question 4

Consider the survey of cell phone calls received by subscribers during one week:

41 ; 5 ; 12 ; 46 ; 39 ; 52 ; 37 ; 38 ; 32 ; 43 ; 18 ; 30 ; 42 ; 41 ; 34 ; 41 ; 43 ; 35

5; 12; 18; 30; 32; 34; 35; 37; 38; 39; 41; 41; 41; 42; 43; 43; 46; 52

Q1 Q2 Q3

(a) Determine the mean, median and mode of the data.

Mean $= \frac{629}{18}$ ✓M $= 34,9$ ✓A

Mode $= 41$ ✓A

Median $= \frac{38+39}{2} = 38,5$ ✓A

(4)

(b) Write down the five-number summary for the data.

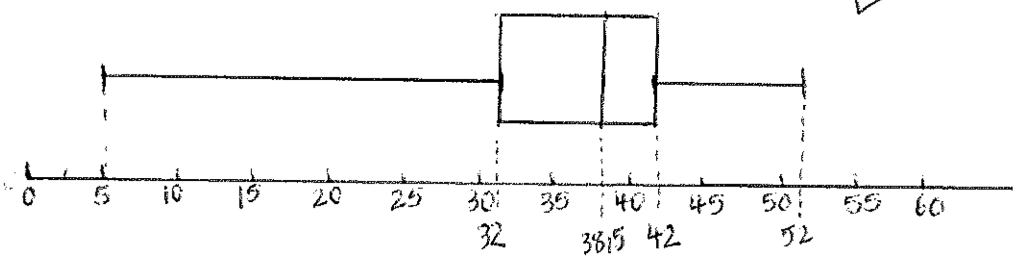
(5 ; 32 ; 38,5 ; 42 ; 52) ✓A Quartiles ✓A max and min

(2)

(c) Draw a box-and-whisker diagram for the data.

✓m box and whiskers

✓A



(2)

(d) Describe the skewness of the data. Show your working.

Mean − median

$= 34,9 - 38,5$ ✓m subtract mean and median

$= -3,6$

∴ data is negatively skewed ✓A

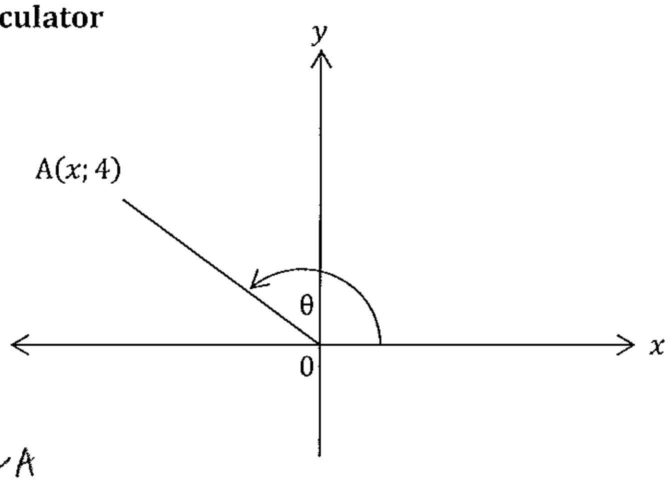
(2)

[10]

Question 5

Answer the following without the use of a calculator

(a) Consider the figure alongside.
A(x; 4) is a point on the Cartesian plane.
$X\hat{O}A = \theta$
$OA = 5$



Determine the value of:

1. x

$x = -3$ ✓A (Pythag thm) ✓A (2)

2. $\sin(180° + 2\theta)$

$= -\sin 2\theta$ ✓m reduction formula

$= -2.\sin\theta.\cos\theta$ ✓A double angle

$= -2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right)$

$= \frac{24}{25}$ ✓A (3)

(b) If $\sin 28° = m$, then determine each of the following in terms of m:

1. $\sin 152°$

$= \sin(180° - 28°)$ ✓m reduction rule

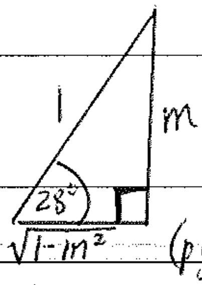
$= \sin 28°$

$= m$ ✓A

x (2)

2. $\cos 28°$

$= \sqrt{1-m^2}$ ✓A



✓m (pythag thm)

(2)

3. $\cos 56°$ $= \cos 2(28°)$ ✓m double angle

$= 2\cos^2(28°) - 1$ ✓A or $1 - 2\sin^2(28°)$ or $\cos^2(28°) - \sin^2(28°)$

$= 2(1-m^2) - 1$ $= 1 - 2m^2$ $= 1 - m^2 - m^2$

$= 1 - 2m^2$ ✓A $= 1 - 2m^2$ (3)

[12]

Question 6

(a) A circle is defined by the equation $x^2 + 6x + y^2 - 4y = 4$

1. Write the above equation in the form $(x - a)^2 + (y - b)^2 = r^2$ and hence write down the coordinates of P, the centre of of the circle.

✓m complete the square

$x^2 + 6x + (3)^2 + y^2 - 4y + (-2)^2 = 4 + 9 + 4$

$(x + 3)^2 + (y - 2)^2$ ✓A $= 17$ ✓A

$\therefore P(-3; 2)$ ✓CA

(4)

2. Determine the equation of the tangent to the circle at the point Q(−4; −2).

$m\ PQ = \frac{2+2}{-3+4} = 4$ ✓m

m tangent $= -\frac{1}{4}$ ✓A (rad ⊥ tang)

Equn of tangent: $y + 2 = -\frac{1}{4}(x + 4)$ ✓m ✓A

$y + 2 = -\frac{1}{4}x - 1$

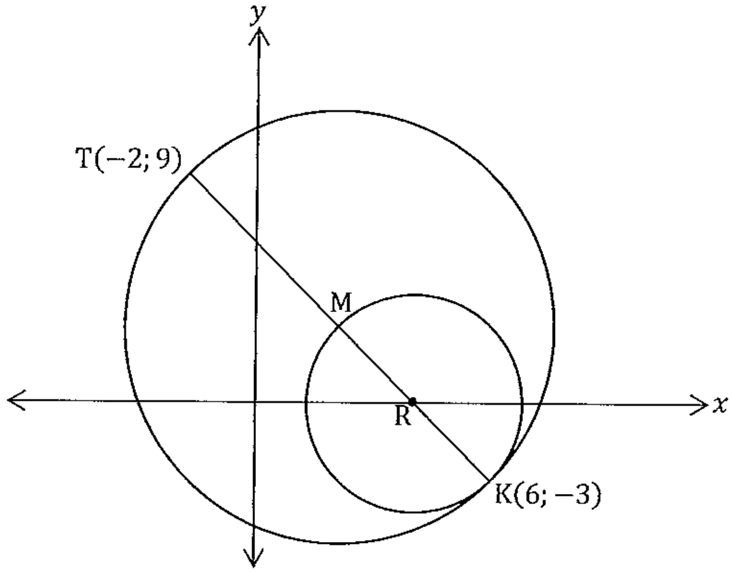
$y = -\frac{1}{4}x - 3$ ✓CA

(5)

[9]

Question 7

In the diagram below, two circles touch each other internally at K(6; −3). The smaller circle has its centre at R. M is the centre of the larger circle. T(−2; 9) and K(6; −3) are the end points of a diameter of the larger circle.



(a) Determine the coordinates of R.

$M\left(\frac{-2+6}{2}; \frac{9-3}{2}\right)$ ✓m midpoint

$\therefore M(2;3)$

$R\left(\frac{2+6}{2}; \frac{3-3}{2}\right)$ ✓m midpoint again

$\therefore R(4;0)$ ✓A

(3)

(b) Determine the equation of circle centre R.

$(x-4)^2 + (y-0)^2 = r^2$ ✓m subst. midpt.

$(6-4)^2 + (-3-0)^2 = r^2$ ✓m subst. coordinate

$4 + 9 = r^2$

$13 = r^2$

$\therefore$ Equn : $(x-4)^2 + y^2 = 13$ ✓A

(3)

[6]

SECTION B

Question 8

Answer the following without the use of a calculator

(a) Simplify, showing all working: $\dfrac{2-2\cos^2(360°-\theta)}{\cos^2(90°+\theta)}$

$= \left.\dfrac{2-2\cos^2\theta}{(-\sin\theta)^2}\right\}\checkmark^{A}$ reduction rules

common factor

$= \dfrac{2(1-\cos^2\theta)}{\sin^2\theta}\checkmark^{M} = \dfrac{2\sin^2\theta}{\sin^2\theta}\checkmark^{A} = 2 \checkmark^{CA}$

(4)

(b) Consider the following statement: $\dfrac{\sin 2x}{1+\cos 2x} = \tan x$

1. Is the statement true for $x = 60°$? Show all working to justify your answer.

$LHS = \dfrac{\sin 2(60°)}{1+\cos 2(60°)}\checkmark^{m}$ subst.

$RHS = \tan 60$

$= \sqrt{3}$

$= \dfrac{\sin 120°}{1+\cos 120°}$

$= \dfrac{\sin 60}{1-\cos 60}\checkmark^{A} = \dfrac{\frac{\sqrt{3}}{2}}{1-\frac{1}{2}} = \sqrt{3}\checkmark^{A}$

$\therefore LHS = RHS$ $\checkmark^{A}$

$\therefore$ Statement true

(4)

2. Prove the statement holds true for all values of x.

$LHS = \dfrac{2\sin x.\cos x}{1+\cos^2 x-\sin^2 x}$ $\checkmark$ A double angle, $\checkmark$ A double angle

$= \dfrac{2\sin x.\cos x}{\sin^2 x+\cos^2 x+\cos^2 x-\sin^2 x}$ $\checkmark^{A}$ square identity

$= \dfrac{2\sin x.\cos x}{2\cos^2 x}\checkmark^{m}$

$= \dfrac{\sin x}{\cos x}\checkmark^{A} = \tan x = RHS$

(5)

3. Hence or otherwise, determine the general solution to the equation $\frac{\sin 2x}{1+\cos 2x} = -1$

$\tan x = -1$ ✓A

Key Angle $= 45°$

Q2: $x = 180° - 45° + k \cdot 180°$, $k \in \mathbb{Z}$

$\underline{x = 135°}$ ✓A $\underline{+ k \cdot 180°}$ ✓A $\underline{, k \in \mathbb{Z}}$

(3)

(c) Determine the general solution of: $\sin^2 x + \cos 2x - \cos x = 0$

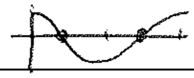
$\sin^2 x + 2\cos^2 x - 1 - \cos x = 0$

$1 - \cos^2 x$ ✓A $+ 2\cos^2 x - 1$ ✓A $- \cos x = 0$

$\cos^2 x - \cos x = 0$ ✓m simplify

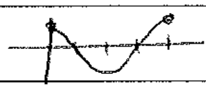
$\cos x (\cos x - 1) = 0$ ✓m factorise

$\cos x = 0$ or $\cos x - 1 = 0$



✓A

$\cos x = 1$



$\underline{x = 90° + k \cdot 180°, k \in \mathbb{Z}}$

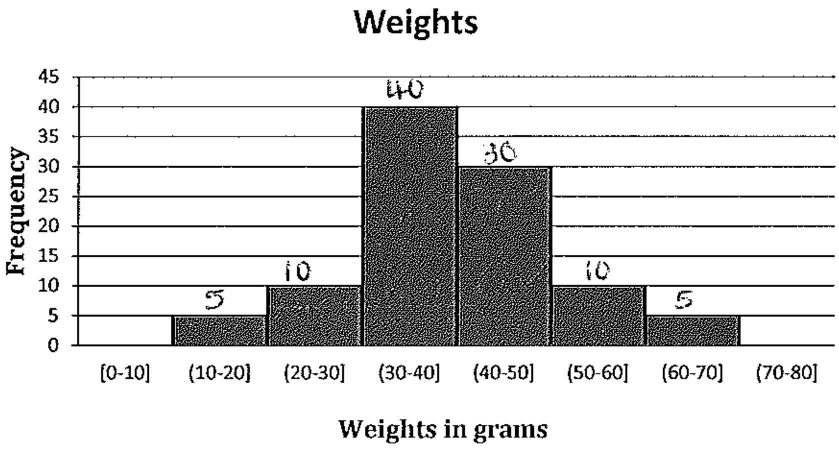
$x = 0° + k \cdot 360°$, $k \in \mathbb{Z}$ (6)

$\underline{x = k \cdot 360°, k \in \mathbb{Z}}$ ✓A

[22]

Question 9

The following histogram is of 100 weights in grams



(a) Determine the:

1. estimated mean

✓m (midpt x frequency)

Total $= (5 \times 15) + (10 \times 25) + (40 \times 35) + (30 \times 45) + (10 \times 55) + (5 \times 65)$

$\therefore \bar{X} = \frac{3950}{100}$ ✓A

$\bar{X} = 39,5$ ✓CA (3)

2. If the estimated standard deviation is 11,17, determine the estimated variance

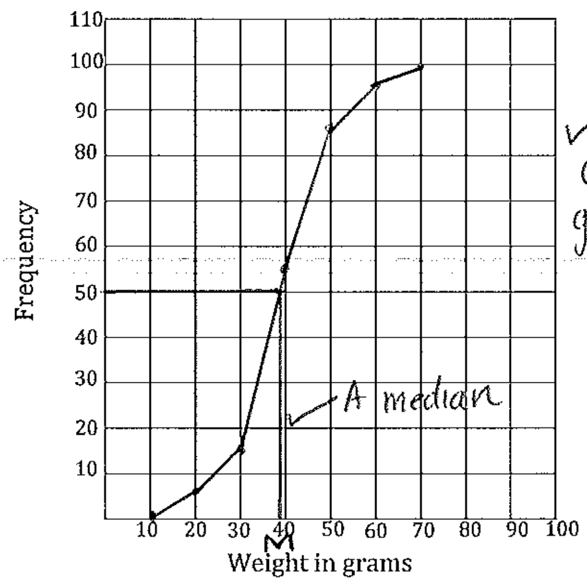
variance $= (11,17)^2 = 124,8$ ✓A (1)

3. If 5 weights in the fifties were raised by 10, how would this affect the:

Standard deviation? would increase ✓A (1)

Median? would be unchanged ✓A (1)

(b) Draw an ogive (cumulative frequency diagram) and then estimate the median.



✓A c.f. graph

(10;0)
(20;5)
(30;15)
(40;55)
(50;85)
(60;95)
(70;100)

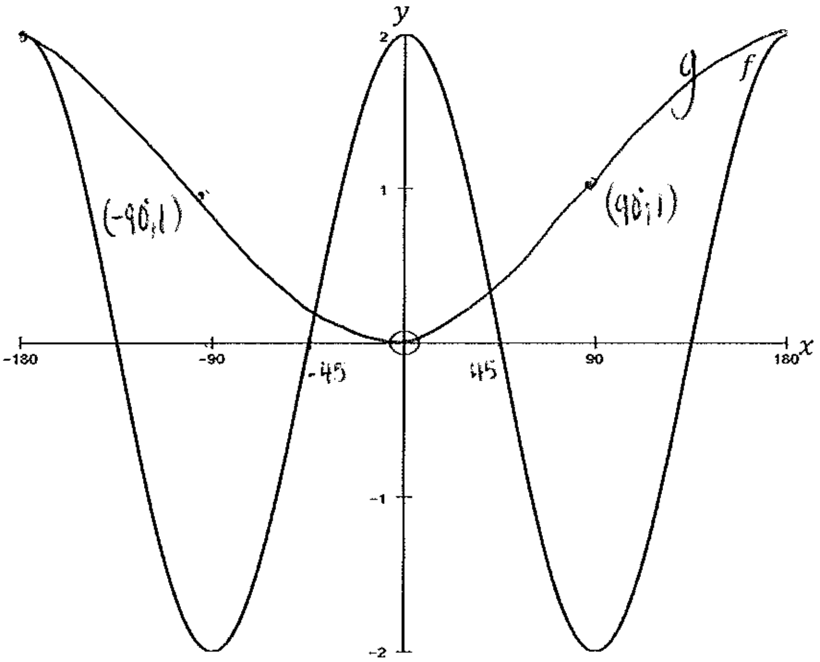
(2)

Estimated median: 38g ✓CA (read off graph) (1)

[9]

Question 10

The graph $f(x) = 2\cos 2x$ is drawn below where $x \in [-180°; 180°]$.



✓ m inverted
✓ A

(a) On the **same set of axes**, sketch the graph $g(x) = 1 - \cos x$. Show clearly all the significant points.

(2)

(b) Determine the value of $f(0) - g(0)$

$= 2 - 0$

$= 2$ ✓ A

(1)

(c) Using your graphs, give the value(s) of x for which $f(x).g(x) \leq 0$

$-135° \leq x \leq -45°$ or $x = 0$ or $45° \leq x \leq 135°$

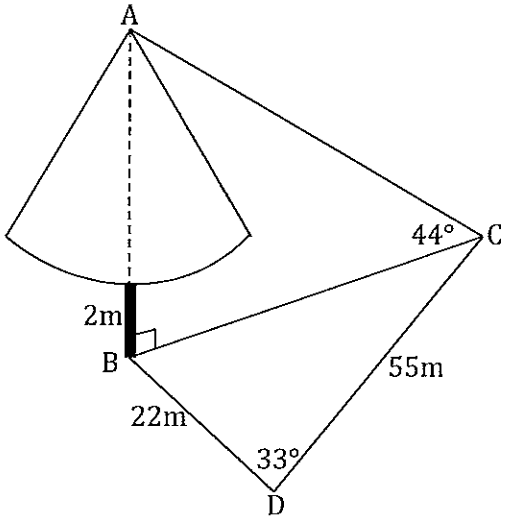
✓ A ✓ A ✓ A

(3)

[6]

Question 11

(a) The given diagram roughly illustrates a tall tree by using a right cone. The centre of the circular base of the cone is 2 metres above the point B, on the ground.



D and C are points on the ground so that BD = 22m, DC = 55m and $B\hat{D}C = 33°$. The angle of elevation of the top of the tree, A, from C is 44°.

1. Determine the height AB of the tree, correct to 2 decimal places.

In ΔBCD: $BC^2 = 22^2 + 55^2 - 2(22)(55)\cos 33°$ ✓m

$BC^2 = 1479,417\ldots$

$BC = 38,5m$ ✓A

In ΔABC: $\tan 44° = \frac{AB}{38,5}$ ✓m ✓ca

$38,5 . \tan 44° = AB$

$37,14m = AB$ ✓ca

(5)

2. If the diameter of the base of the cone is 6m, calculate the volume of the cone depicting the foliage of the tree in the diagram above, correct to 1 decimal place.

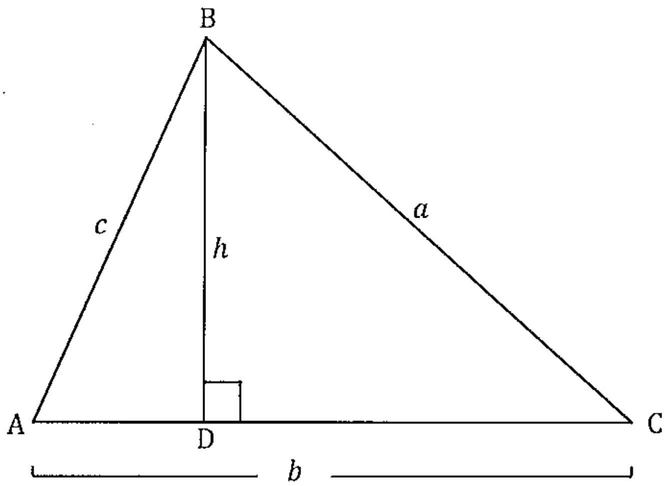
Note: Volume of a cone= $\frac{1}{3} \times$ Area of base$\times$ height.

$V = \frac{1}{3}(\pi . 3^2).(35,14)$ ✓A

$V = 331,2 m^3$ ✓cA

(2)

(b) In the given diagram, BD⊥ AC in ΔABC



Prove that $\frac{\sin\hat{A}}{a} = \frac{\sin\hat{B}}{b} = \frac{\sin\hat{C}}{c}$

Proof:

In ΔADB, $\sin\hat{A} = \frac{h}{c}$

$\therefore h = c.\sin\hat{A}$ .... (1) ✓A

In ΔCDB, $\sin\hat{C} = \frac{h}{a}$

$h = a.\sin\hat{C}$ .... (2) ✓A

Sub. (1) into (2) $c.\sin\hat{A} = a.\sin\hat{C}$ ✓M equalise

$\frac{\sin\hat{A}}{a} = \frac{\sin\hat{C}}{c}$ ✓A

Similarly it can be proved that:

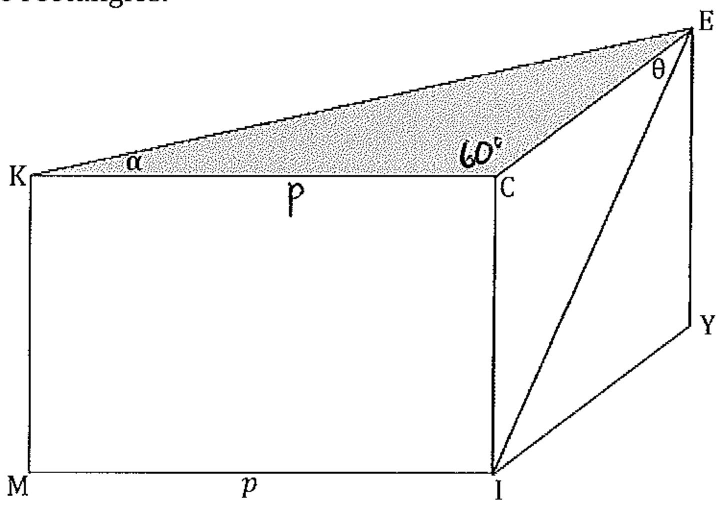
$\frac{\sin A}{a} = \frac{\sin C}{c} = \frac{\sin B}{b}$

(4)

[11]

Question 12

Shown alongside is a triangular prism. $E\hat{C}K = 60°$, $E\hat{K}C = \alpha$, $C\hat{E}I = \theta$, $MI = p$ units.
MICK and CEYI are rectangles.



Show that the height of the prism CI, is $CI = \dfrac{2p\sin\alpha\tan\theta}{\sqrt{3}\cos\alpha + \sin\alpha}$

In ΔKCE: $\dfrac{CE}{\sin\alpha} = \dfrac{p}{\sin(180 - (60+\alpha))}$ m(KÊC in terms of α) ✓m sin rule

$CE = \dfrac{p.\sin\alpha}{\sin(60+\alpha)}$ ✓a

In ΔCIE: $\tan\theta = \dfrac{CI}{CE}$ ✓m trig: tan ratio

$\therefore CI = CE.\tan\theta$

$CI = \dfrac{p.\sin\alpha.\tan\theta}{\sin(60+\alpha)}$ ✓ca

$CI = \dfrac{p.\sin\alpha.\tan\theta}{\sin 60.\cos\alpha + \cos 60.\sin\alpha}$ ✓CA expand

$CI = \dfrac{p.\sin\alpha.\tan\theta}{\frac{\sqrt{3}}{2}.\cos\alpha + \frac{1}{2}.\sin\alpha}$ ✓CA special angles.

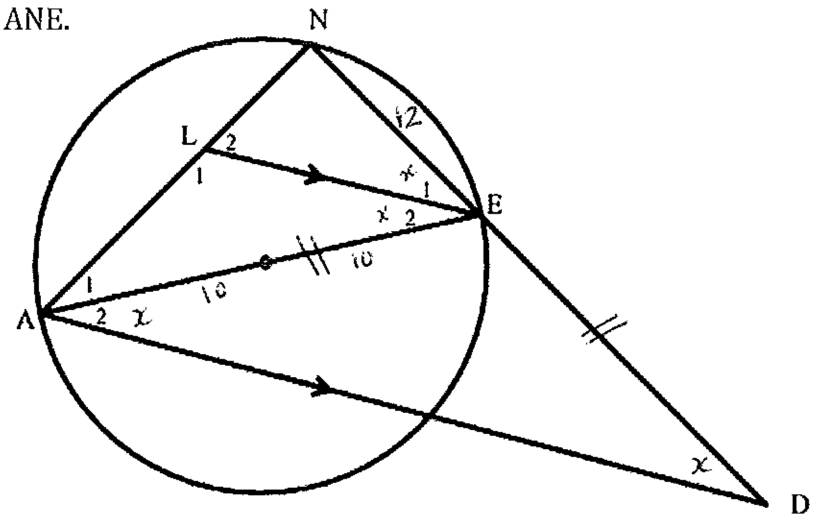
$CI = \dfrac{p.\sin\alpha.\tan\theta}{\frac{1}{2}(\sqrt{3}\cos\alpha + \sin\alpha)}$

$CI = \dfrac{2.p.\sin\alpha.\tan\theta}{\sqrt{3}.\cos\alpha + \sin\alpha}$

[7]

Question 13

In the figure, AE is a diameter of circle ANE.
L is a point on AN and LE bisects $A\hat{E}N$.
NE produced meets a line from A at D
so that LE||AD.
Let $\hat{E}_1 = \hat{E}_2 = x$



(a) Complete the following table by filling in the reasons:

	Statement	Reason
1.	$\hat{E}_1 = \hat{D}$	corresp. ∠s LE‖AD ✓A
2.	$\hat{E}_2 = \hat{A}_2$	alt. ∠s LE‖AD ✓A
3.	AE = ED	sides opp eq. ∠s ✓A

(3)

(b) If NE = 12 units and the diameter of the circle is 20 units, calculate giving reasons:

(a) AN

$\hat{N} = 90°$ (∠ in semi-circle) ✓A

$AN^2 = 20^2 - 12^2$ (Pythag. thm) ✓A

$AN^2 = 256$

$AN = 16u$ ✓A

(3)

(b) AL

$\frac{AL}{AN} = \frac{DE}{ND}$ (prop. int. thm. LE‖AD) ✓A ✓M

$\frac{AL}{16} = \frac{20}{32}$ ✓A

$AL = 10u$ ✓A

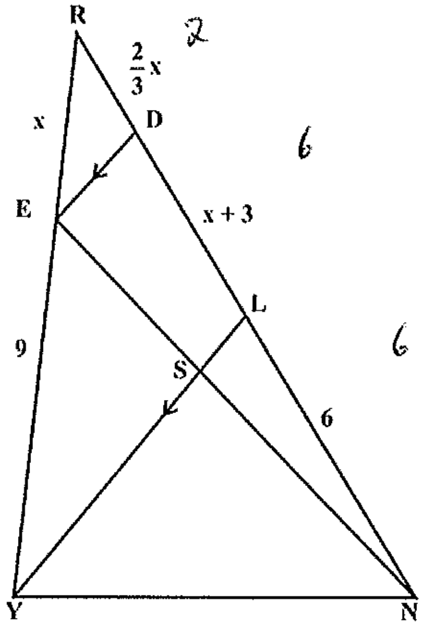
(4)

[10]

Question 14

NL = 6units ; RE = x units ; RD = $\frac{2}{3}$x units ; EY = 9units and DL = x + 3units.

S is a point on YL and ED||YL.



(a) Show that L is the midpoint of DN by first solving for x. Give reasons. ✓A

In ΔRYL $\quad \frac{x}{9} = \frac{\frac{2}{3}x}{x+3}$ ✓A (prop. int. thm DE // YL)

$x(x+3) = 6x$

$x^2 - 3x = 0$ ✓m factorise

$x(x-3) = 0$

$\therefore x = 3$ ✓A

$\therefore$ DL = 3 + 3 = 6u

LN = 6u

$\therefore$ L is midpt of DN (4)

(b) If SL = 1,4 units, write down the length of DE.

DE = 2,8u ✓A (1)

(c) If the area of ΔRED = 2,7units^2, determine the area of ΔREN. or ✓m determinant ✓A into A formula.

$\frac{\text{Area } \Delta RED}{\text{Area } \Delta REN} = \frac{\frac{1}{2}.2.h}{\frac{1}{2}.14.h}$ ✓m equalise ratio ✓A

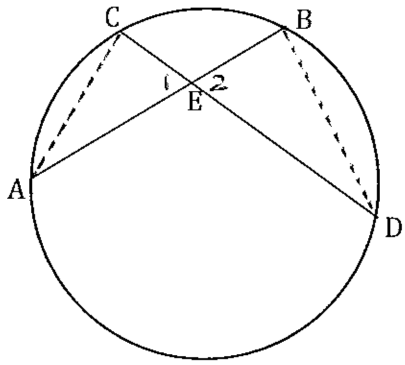
$\frac{2,7}{\text{Area } \Delta REN} = \frac{1}{7}$ $\quad \therefore$ Area ΔREN = 7 × 2,7

= 18,9 u^2 ✓A (3)

[8]

Question 15

A theorem you may never have seen before states:
If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



(a) Given: Chords AB and CD intersect at point E.
By constructing AC and BD, prove CE. ED = BE. EA

In ΔACE and ΔDBE

1. $\hat{C} = \hat{B}$ ($\angle$s in same segm.)
2. $\hat{A} = \hat{D}$ ($\angle$s in same segm.)
3. $\hat{E}_1 = \hat{E}_2$ (vert. opp $\angle$s)

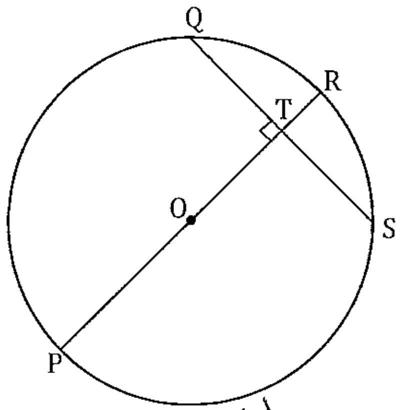
✓A ✓A

$\therefore \Delta ACE \mathbin{|||} \Delta DBE$ (AAA) ✓A

$\therefore \frac{CE}{EB} = \frac{AE}{ED}$ ($\mathbin{|||}$ Δs) ✓A

$\therefore CE . ED = AE . EB$

(4)



(b) Hence, if $QS = 6$ units, and $PT : TR = 4 : 1$, determine the length of the diameter of the circle.
P, Q, R and S are points on a circle, centre O, such that PR is perpendicular to QS.

$QT = TS = 3u$ ✓A (line from centre $\perp$ to chord) ✓A

let $PT = 4x$ and $TR = x$

$PT . TR = QT . TS$ ✓m (proven in (a))

$4x . x = 3 . 3$

$4x^2 = 9$

$x^2 = \frac{9}{4}$

$x = \frac{3}{2}$ ✓A

$\therefore$ diameter $= 5x = 5 . \left(\frac{3}{2}\right) = 7{,}5u$ ✓A

(5)

[9]