TRIALS P1 2015 MEMO

SECTION A

Question 1

a) 1) $(x-4)(x+2) = 7$ ✓m(x out)

$x^2 - 2x - 8 - 7 = 0$

$x^2 - 2x - 15 = 0$ ✓m(=0)

$(x-5)(x+3) = 0$

$x = 5$ or $x = -3$ ✓A (3)

2) $8^{x-5} = 32^{10-2x}$

$2^{3x-15} = 2^{50-10x}$ ✓m(prime bases)

$3x - 15 = 50 - 10x$ ✓m equate exps

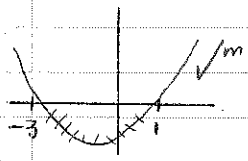
$13x = 65$

$x = 5$ ✓A (3)

3) $x(x+2) < 3$

$x^2 + 2x - 3 < 0$ ✓m

$(x+3)(x-1) < 0$ ✓f



✓m

$-3 < x < 1$ ✓A (4)

4) $\sqrt[3]{3x-23} = -2$

$3x - 23 = -8$ ✓m(cube)

$3x = 15$

$x = 5$ ✓A (2)

b) (1) $k = -2$ ✓A or $k \neq 2$ ✓A (2)

(2) $k = -3$ ✓A (1)

Question 2

(a) $\sqrt{3}.\sqrt{48} - \frac{4^{x+1}}{2^{2x}}$

$= \sqrt{3}.4\sqrt{3} - 2^{2x+2-2x}$ ✓m ✓A

$= 12 - 2^2$

$= 8$ ✓A (3)

(b) $2g(g(\sqrt{2}))$

$g(\sqrt{2}) = 5/4$ ✓A

$\therefore g(\frac{5}{4}) = 3$ ✓A

$\therefore 2.g(\frac{5}{4}) = 6$ ✓CA (2)

Question 3

(a) $\frac{3}{8}; \frac{3}{4}; \frac{3}{2}; \ldots\ldots$

$96 = (\frac{3}{8})(2)^{n-1}$ ✓m ✓A

$256 = (2)^{n-1}$

$2^8 = 2^{n-1}$

$\therefore n - 1 = 8$ ✓CA (3)

$n = 9$ ✓ (9^{th} term = 96)

(b) (i)(i) 3; 5; 7; 9. . . .

$T_n = 3 + (n-1)2$ ✓m

$= 2n + 1$ ✓A (2)

(ii) 1; 4; 9; 16 . . . .

$T_n = n^2$ ✓A (1)

(ii) 6; 15; 28; 45. . . .

turn over for 50n.

(iii) 6; 15; 28; 45

9 13 17 ✓m

4 4

$2a = 4$	$3a + b = 9$	$a + b + c = 6$
$a = 2$ ✓A	$\therefore b = 3$	$2 + 3 + c = 6$
	✓m	$c = 1$

$\therefore T_n = 2n^2 + 3n + 1$ ✓A (4)

(2) 5151 ✓m $= 2n^2 + 3n + 1$

$0 = 2n^2 + 3n - 5150$

$n = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(-5150)}}{2(2)}$ ✓f

$= \frac{-3 \pm \sqrt{41209}}{4}$

$n = \frac{-3 \pm 203}{4}$

$\therefore n = 50$ $\therefore T_{50} = 5151$ ✓A (3)

(c) $\sum_{n=4}^{123} (3n-1) = k$

(1) 11; 14; 17; 20 ✓A; 23 ✓A (2)

(2) $a = 11$ $d = 3$ $n = 120$ ✓A

$S_n = \frac{120}{2}[2(11) + 119(3)]$ ✓m

$= 22\,740$

$\therefore k = 22\,740$ ✓A (3)

(d) 108; 86,4; 69,12; . . .

$a = 108$ $r = 0,8$ ✓A

$S_\infty = \frac{108}{1 - 0,8}$ ✓m(S∞)

$= 540\,cm^2$ ✓A (3)

Question 4

(4) (a) $f(x) = \frac{x^2}{4}$ ✓m

$f'(x) = \lim_{h \to 0} \frac{\frac{(x+h)^2}{4} - \frac{x^2}{4}}{h}$

$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{4h}$ ✓A

$= \lim_{h \to 0} \frac{h(2x+h)}{4h}$ ✓f

$= \frac{1}{2}x$ ✓A (4)

(b) (i) $P - 3v = 2v^3$

$P = 2v^3 + 3v$ ✓m

$\frac{dP}{dv} = 6v^2_{\checkmark A} + 3_{\checkmark A}$ (3)

(2) $D_x\left[\frac{x^2 - 4x + 3}{x}\right]$

$= D_x\left[\frac{x^2}{x} - \frac{4x}{x} + \frac{3}{x}\right]$ ✓m

$= D_x[x - 4 + 3x^{-1}]$ ✓A

$= 1 - 3x^{-2}$ OR ✓CA $1 - \frac{3}{x^2}$ (3)

(3) $g(x) = 2\sqrt{x} + \frac{1}{x^3} - \sqrt{2}.x$

$= 2x^{\frac{1}{2}} + x^{-3} - \sqrt{2}.x$ ✓m ✓A

$g'(x) = x^{-\frac{1}{2}} - \frac{3}{x^4} - \sqrt{2}$ ✓A

$g'(\sqrt{2}) = \frac{1}{\sqrt{2}} - \frac{3}{(\sqrt{2})^4} - \sqrt{2}$ ✓m(sub)

$= -1,3$ (4)

(c) $f(x) = x^3 - 3x^2 + kx + 8$

$f'(x) = 3x^2 - 6x + k$ ✓m

but $f'(1)$ ✓sub $= 0$ (Turning pt)

$\therefore f'(1) = 3 - 6 + k = 0$

$k = 3$ ✓A (3)

Question 5

a) (1) $y = 1$ ✓A (OR $x = 0$) (1)

(2) at A: $-x + 4 = \frac{-4}{x} + 1$

$-x^2 + 4x = -4 + x$ ✓m(equate)

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$ ✓f

$x = 4$ OR $x = -1$

A: $x = -1$ ✓A; $y = 5$ ✓cA

$A(-1;5)$ (4)

(3) $h(x) = \frac{-4}{x+2}$ ✓A $+ 1$ (1)

(4) $-1 \leq x < 0$ ✓ OR $x \geq 4$ ✓ (2)

(b)(i) at A: $0 = \log_{\frac{1}{3}} x$

$\left(\frac{1}{3}\right)^0 = x$

$\therefore x = 1$

A $(1;0)$ ✓A (1)

(ii) at B: $-3 = \log_{\frac{1}{3}} x$ ✓m

$\left(\frac{1}{3}\right)^{-3} = x$

$3^3 = x$

$x = 27$

B $(27;-3)$ ✓A (2)

(2) $\log_{\frac{1}{3}} x \geq -3$

$0 < x \leq 27$ ✓ ✓ (2)

(3) $f^{-1}(x) = ?$

$y = \log_{\frac{1}{3}} x$ $\therefore$ inv: $y = \left(\frac{1}{3}\right)^x$

$\left(\frac{1}{3}\right)^y = x$ ✓m(switch x and y) OR $f^{-1}(x) = \left(\frac{1}{3}\right)^x$ ✓A [OR 3^{-x}] (2)

(4) $f(x) = \log_{\frac{1}{3}} x$

reflect over x-axis $\rightarrow -y$

$\therefore y = -\log_{\frac{1}{3}} x$ ✓A (1)

(SECTION A: TOTAL = 74 marks)

SECTION B

Question 6

(a) $A = P(1-i)^n$

$x = 2x(1 - 0,10)^n$ ✓A(correct formula)

$\frac{1}{2} = (0,9)^n$

$\log\left(\frac{1}{2}\right) = \log(0,9)^n$ ✓m(log)

$\log\left(\frac{1}{2}\right) = n.\log(0,9)$

$\frac{\log\left(\frac{1}{2}\right)}{\log(0,9)} = n$

$n = 6,5788...$ ✓A

$\therefore$ 6 years and 7 months ✓cA (4)

(b) $A = 350\,000$ $P = 100\,000$ ✓A(all 3 values) $n = 10$ yrs

$350\,000 = 100\,000(1+i)^{10}$ ✓correct formula

$\sqrt[10]{\frac{35}{10}} = (1+i)^{10}$.

$\sqrt[10]{\frac{7}{2}} - 1 = i$

$i = 13,35\%$ ✓A p.a. (4)

(c) (1) R 980 000 − R150 000

= R 830 000 loan

$i = \frac{0,09}{12}$ $n = 180$ payments

$P_V = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$ ✓Pvalue

$830\,000 = \frac{x\left[1 - \left(1 + \frac{0,09}{12}\right)^{-180}\right]}{\frac{0,09}{12}}$ ✓A ✓A

$x = R8418,41$ ✓cA (4)

(2) No. of payments left $= 180 - 84$

$= 96$ ✓A

$0.B = \frac{8418,41\left[1 - \left(1 + \frac{0,09}{12}\right)^{-96}\right]}{\frac{0,09}{12}}$ ✓Pvalue ✓m

$= R\,574\,627,52$ ✓cA (4)

(3) $8418,41 \times 180$

$= R\,1\,515\,313,80$ ✓A (Amy's payments)

Amount borrowed = R 830 000

$\therefore$ Interest $= 1515\,313,80 - 830\,000$ ✓m

$= R\,685\,313,80$ ✓cA (3)

Question 7

(a) $y = a(x-2)^2 + 3$ ✓m

Sub $(0;0)$

$0 = a(-2)^2 + 3$

$a = -\frac{3}{4}$ ✓A

$\therefore y = -\frac{3}{4}(x^2 - 4x + 4) + 3$

$= -\frac{3}{4}x^2 + 3x$ ✓cA (3)

(b) $g(x) = p^x - 1$ ✓A

Sub $(2;3)$ ✓m

$3 = p^2 - 1$

$p^2 = 4$ $\therefore p = 2$

$\therefore$ Eqn. $g(x)$: $g(x) = 2^x - 1$ ✓cA (3)

(c) $y > -1$ ✓A (1)

(d) TPt $(-2; 8)$ ✓A ✓m (2)

Question 8

a) $f(x) = x^3 - 2x^2 - 4x + 8$

5 pts: $f'(x) = 0$

ie: $3x^2 - 4x - 4 = 0$ ✓A

$(3x+2)(x-2) = 0$ ✓f

$x = -\frac{2}{3}$ or $x = 2$ ✓

$y = \frac{256}{27}$ $\qquad y = 0$

(or 9,48) ✓

$\therefore E\left(-\frac{2}{3}; \frac{256}{27}\right)$ $\qquad B(2;0)$ ✓

x int: $0 = x^3 - 2x^2 - 4x + 8$

$0 = (x-2)(x^2 \text{ ✓m} - 4)$

$\therefore (x-2)(x-2)(x+2) = 0$

$x = -2$ or 2

$\therefore A(-2;0)$ ✓ (7)

(b) $f''(x) = 6x - 4$ ✓m (idea) ✓A $= 0$ at F

$\therefore x = {}^2/_3$ ✓A (3)

(c) m of tangent $= 3$

$\therefore$ at D $\therefore$ m of $f(x) = 3$ ✓m (idea) = gradients

ie: $3x^2 - 4x - 4 = 3$

$3x^2 - 4x - 7 = 0$

$(3x - 7)(x + 1) = 0$ ✓f

$x = {}^7/_3$ or $x = -1$ ✓A

at D: $y = (-1)^3 - 2(-1)^2 - 4(-1) + 8$

$= 9$

$\therefore D(-1; 9)$

Eqn g: $y - 9 = 3(x+1)$ ✓m

$y = 3x + 12$ ✓A

at G: $0 = 3x + 12$ ✓A

$x = -4$ $\qquad G(-4;0)$ (6)

Question 9

(a) (1) $y = 1$ ✓A (1)

(2) $\frac{dy}{dx} = -\frac{1}{2}x + p$ ✓m (idea) $= -\frac{1}{2}$ ✓eq.

Sub $x = 4$ ✓m: $-\frac{1}{2}(4) + p = -\frac{1}{2}$

$p = {}^3/_2$ ✓A (4)

(3) $-3x^2 + 24x - 45 = 0$ ✓idea

$x^2 - 8x + 15 = 0$

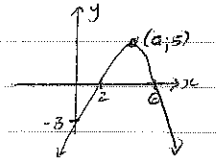
$(x-5)(x-3) = 0$ ✓f

$x = 5$ or $x = 3$

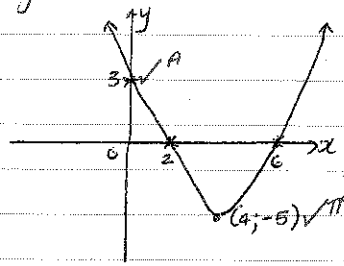
$\therefore x = 3$ ✓A (3)

(b) 1) f: y int: $(0; -3)$

x ints: $(2;0)$ $(6;0)$ TPt $(4;5)$



g: reflect f(x) over x-axis



✓shape ✓x ints ✓TPt (4)

2) $h'(x)$ must be +ve

$\therefore x < 2$ ✓A or $x > 6$ ✓A (2)

Question 10

a) $BP = 60 - 2x$ ✓A

$CQ = 40 - x$ ✓A (2)

b) Area ABCD $= 60 \times 40 = 2400$ ✓A m^2

Area $\Delta PCQ = \frac{1}{2}(2x)(40-x)$

$= 40x - x^2$

Area $\Delta AQD = \frac{1}{2}(x)(60)$

$= 30x$

Area $\Delta ABP = \frac{1}{2}(40)(60-2x)$

$= 1200 - 40x$

✓m ✓A ✓(all)

Δ Playground APQ $= 2400 - [40x - x^2 + 30x + 1200 - 40x]$ ✓CA/m

$= 2400 - [-x^2 + 30x + 1200]$

Area $= x^2 - 30x + 1200$ (5)

c) for minimum area $\frac{dA}{dx} = 0$ ✓idea

ie: $2x - 30 = 0$ ✓A

$x = 15$

$\therefore$ Min area $= (15)^2 - 30(15) + 1200$ ✓m (sub)

$= 975\ m^2$ ✓A (4)

Question 11

a) $f(x) = \frac{1}{x}$

$= x^{-1}$

$f'(x) = -\frac{1}{x^2}$

$\therefore f'(a) = -\frac{1}{a^2}$ ✓A

at $x = a$: $f(a) = \frac{1}{a}$ ✓A

$\therefore$ eqn. tangent at $x = a$

$y - \frac{1}{a}$ ✓m $= -\frac{1}{a^2}(x - a)$

$y = -\frac{1}{a^2}x + \frac{1}{a} + \frac{1}{a}$ ✓ x by a^2

$a^2 y = -x + a + a$

$x + a^2 y = 2a$ (4)

(b) y int: B $\qquad a^2 y = 2a$

$y = \frac{2}{a}$ ✓A

x int: C $\qquad x = 2a$ ✓A

$\therefore$ Area ΔOBC

$= \frac{1}{2}(2a)\left(\frac{2}{a}\right)$

$= 2$ units^2 ✓ (3)

Section B Total = 76

Total Marks: 150