ST BENEDICT'S COLLEGE

SUBJECT	**Mathematics Paper 1**	**DATE**	15 July 2015
GRADE	12	**MARKS**	150
EXAMINER	Mrs Sillman	**MODERATORS**	Mr/s Povall, Benecke, Gill, Scott, Serafino
NAME		**DURATION**	3 hrs
CLASS		**TEACHER**	

QUESTION NO.	ASS STANDARD DESCRIPTION	TOTAL	ACTUAL
1	Algebra	23	
2	Finance	17	
3	Series and Sequences	21	
4	Functions and Inverses	29	
5	Calculus	28	
6	Probability	15	
7	Application – Functions and Calculus	17	
		150	

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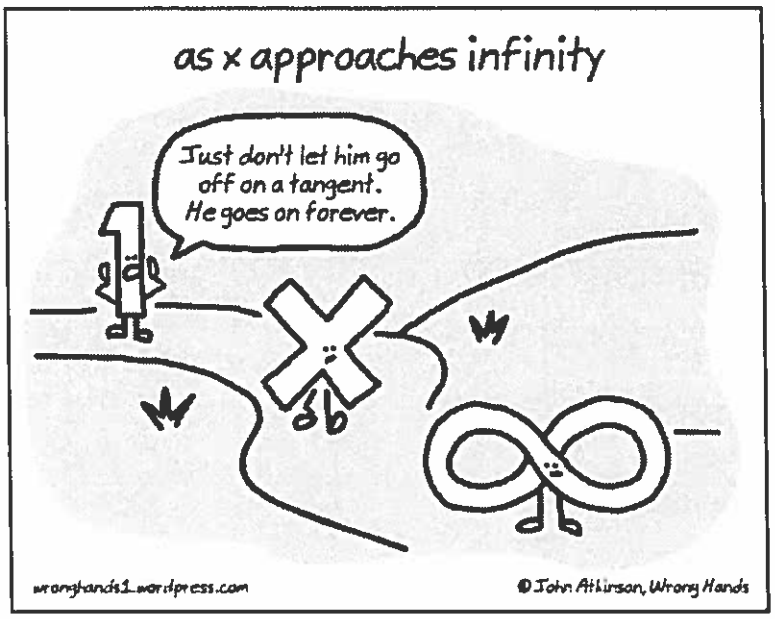
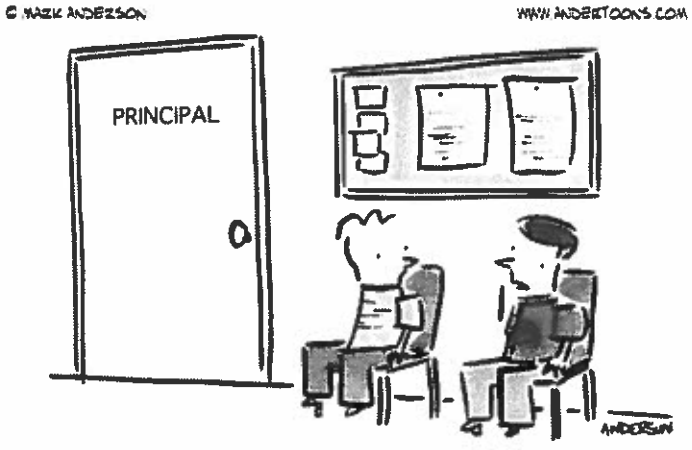
INSTRUCTIONS

1. This question paper consists of 7 questions and 7 pages. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
5. Round off your answers to one decimal digit where necessary.
6. All the necessary working details must be clearly shown. Answers only, without relevant calculations, may incur penalties.
7. It is in your own interest to write legibly and to present your work neatly.





"I wish we hadn't learned probability
'cause I don't think our odds are good."

QUESTION 1

a) Solve for x:

(i) $\dfrac{2x+3}{x-3} = x + \dfrac{9}{x-3}$ (4)

(ii) $4 - 3x - x^2 < 0$ (3)

(iii) $5 - \sqrt{4x+1} = x$ (5)

(iv) $3^x + 3.3^{-x} - 4 = 0$ (4)

(v) $2logx = \log 4 + \log(x-1)$ (4)

b) Describe the nature of the roots of the following equation:

$4x^2 + 49 = 28x$ (3)

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QUESTION 2

a) Greg deposits R9 000 into an account earning 8% pa compounded quarterly.

(i) What is his investment worth at the end of six years? (3)

(ii) What is the effective annual interest rate he is given? (2)

b) An amount of R10 000 is invested at 15% pa compounded monthly. At the end of four years, half of the accumulated amount is taken out and invested into an account that earns 16% pa compounded quarterly. The other half is left in the original investment. What is the total value of both accounts after a further four years? (9)

c) Mark invests some money at 12% pa compounded monthly. Determine how long, to the nearest month, before his money doubles? (3)

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QUESTION 3

a) Given the arithmetic sequence:

$$w + 5\ ;\ 2w + 3\ ;\ w - 7\ \ldots\ldots$$

Determine the value of 'w' and hence the common difference. (3)

b) Calculate the 6th term of the geometric sequence: 2 ; 6 ; 18 .... (2)

c) Given the quadratic sequence:

$$4\ ;\ 5\ ;\ 8\ ;\ 13\ \ldots\ldots$$

(i) Determine the n^{th} term (4)

(ii) The sequence of first differences of the quadratic sequence forms a sequence of its own. Determine S_8, the sum of the first eight terms of this sequence. (3)

d) A converging geometric series is shown:

$$1 + 3p + 9p^2 + 27p^3 + \cdots$$

(i) For which values of p will the series have a sum to infinity? (3)

(ii) If $p = \frac{1}{4}$, find the sum to infinity. (2)

e) Showing working, determine 'n' if:

$$\sum_{i=1}^{n} (2i - 1) = 1089$$

(4)

QUESTION 4

a) The hyperbola $y = \frac{12}{x}$ is translated two units to the left and 5 units down.

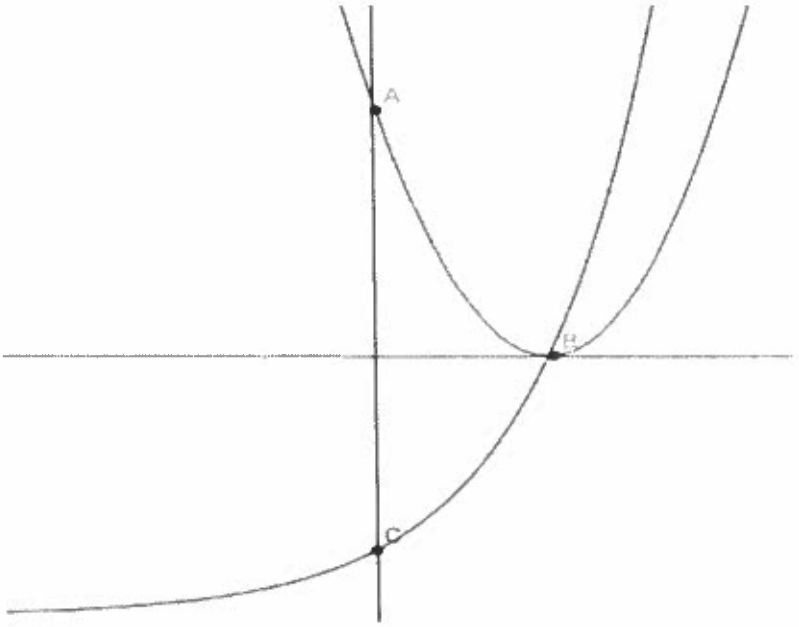
(i) Give the equation of the translated graph. (2)

(ii) Give the equation for ONE of the axes of symmetry of the translated graph. (3)

b) The diagram represents the graphs of

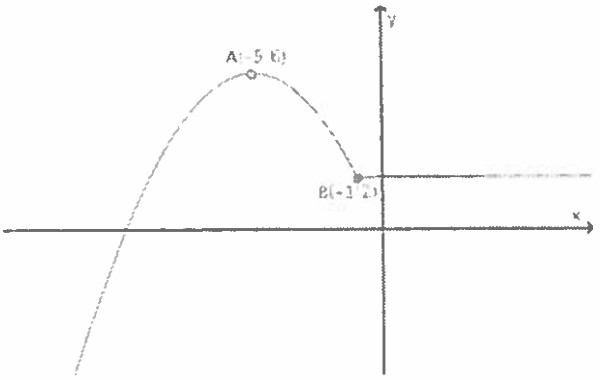
$$f(x) = (x - 2)^2 \text{ and } g(x) = 2^x - 4$$

Points A and C are the y-intercepts of f and g respectively, and B is their common x –intercept.



(i) Calculate the co-ordinates of A, B and C (3)
(ii) Write down the asymptote of g (1)
(iii) Determine the equation of $h(x) = f(2x) + 4$. Do not simplify your answer. (2)
(iv) Write down the equation of the relation $h^{-1}(x) =$ (3)

c) The sketch below shows $f(x)$, which is formed using part of a parabola and a horizontal line.



(i) Is $f(x)$ a function? Explain. (2)
(ii) Determine the range of f. (2)
(iii) Determine $f(3)$ (1)

d) (i) Sketch the graph $f(x) = log_{\frac{1}{3}}x$, clearly marking any intercepts with the axes. (2)
(ii) Write down the domain and range of f. (2)
(iii) Determine the equation of g, the reflection of f about the y-axis. Sketch this graph onto the same Cartesian plane marking any intercepts with the axes. (3)
(iv) Determine the equation of $h(x) = f^{-1}(x)$. Sketch this graph on the same Cartesian plane marking any intercepts with the axes. Ensure you have clearly labelled all three graphs on the Cartesian plane. (3)

QUESTION 5

a) Determine $\lim_{x\to 1} \frac{x^2+x-2}{1-x}$ (3)

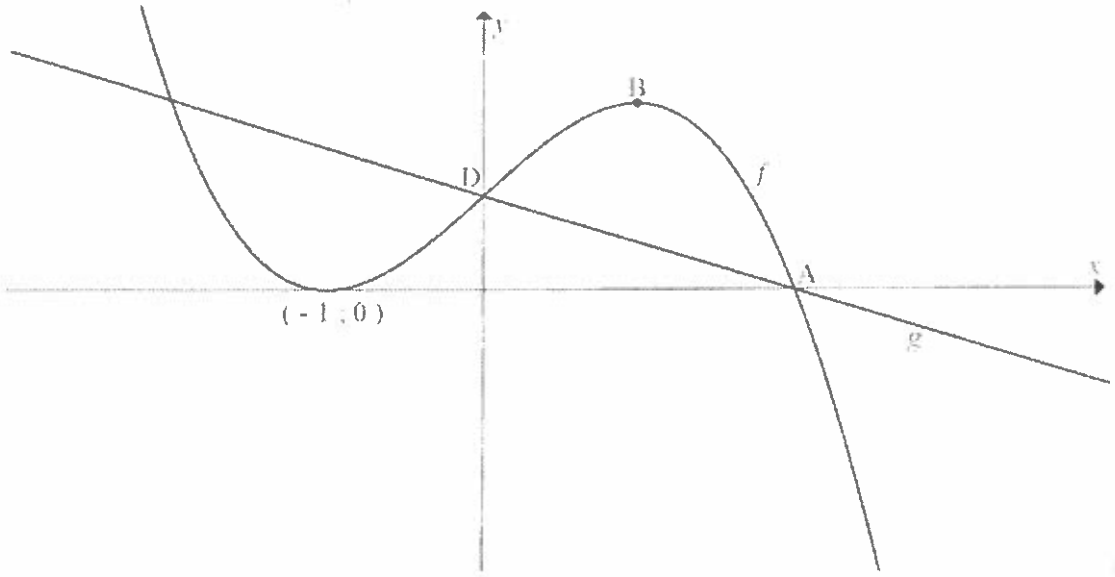
b) Determine $f'(x)$ from first principles if $f(x) = -2x^2 + 3x$ (5)

c) Calculate $D_x\left[\frac{x^{\frac{2}{3}}-3(\sqrt[3]{x})+1}{x^{\frac{1}{3}}}\right]$ (4)

d) The graphs represent the functions

$$f(x) = ax^3 + bx + 2 \text{ and } g(x) = -x + 2$$

Points A and D are the common intercepts of the two graphs. (-1 ; 0) and point B are the turning points of f

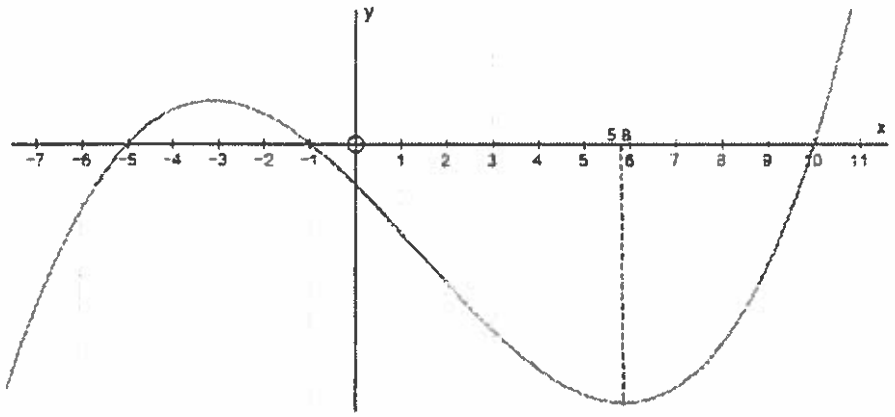


(i) Show, by means of calculation that a = -1 and b = 3 (5)

(ii) Determine the co-ordinates of B. (5)

(iii) For which values of x is $f'(x).g'(x) < 0$ (2)

e) The function $g(x)$ is sketched below:



Say whether each of the following is positive, negative or zero.

$(i)\ g(-5)$ $(ii)\ g'(5{,}8)$ $(iii)\ g''(8)$ $(iv)\ g(1).g'(1)$

(4)

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QUESTION 6

a) The Bennies Big Bash is on a Friday evening and the organisers are concerned that it will be rained out.

They have been told that if it rains on a particular day, the probability that it will rain again the next day is 0,7. If it doesn't rain on a particular day, the probability that it will rain the next day is 0,25.
It rains the Wednesday before the event.
Represent this scenario using a tree diagram and hence determine the probability of it *raining* the night of the Big Bash. (6)

b) In Gauteng, the newest number plates have two letters of the alphabet (excluding the vowels a, e, i, o, u) followed by two digits followed by another two letters of the alphabet (no vowels). Numbers and digits may be repeated.

How many different number plates can be produced in Gauteng? (3)

c) There are five books on a shelf next to a desk. Three of the books are written by Pratchett, one by Larsson and one by Clarkson.

(i) How many different ways can the books be arranged with no restrictions? (2)

(ii) What is the probability that the three Pratchett books will be next to each other? (4)

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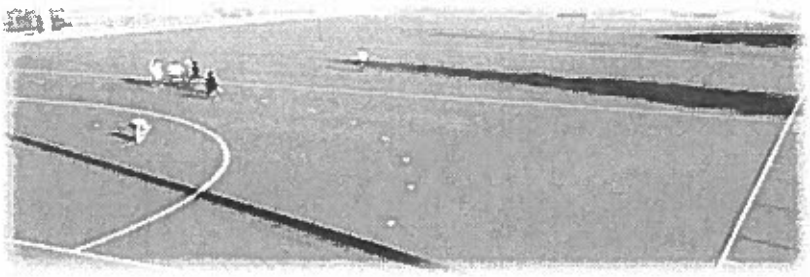
QUESTION 7

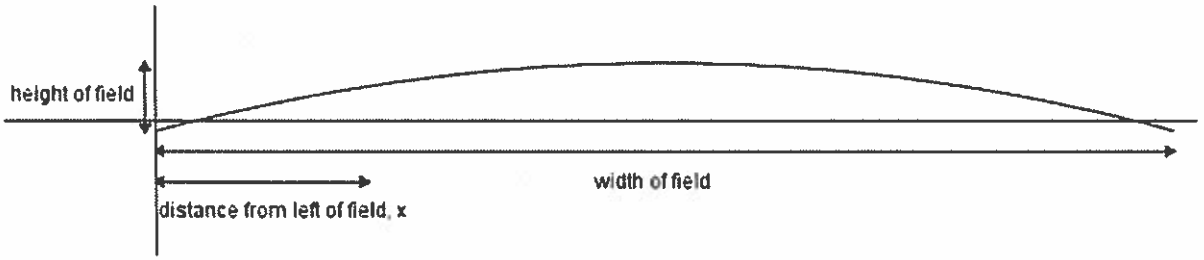
a)



Although a stadium field of synthetic turf appears to be flat, its surface is actually shaped like a parabola. This is so that rainwater can run off the sides. If we take a cross section of the turf, the surface would be modelled by:

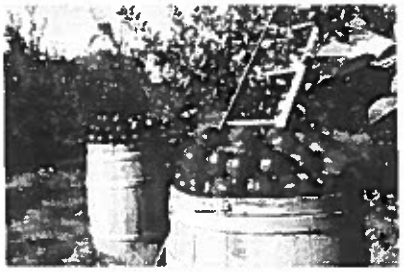
$$y = -0{,}000132x^2 + 0{,}01188x - 0{,}0173$$

where x is the distance from the left end of the field in the cross section and y is the height of the field in metres.



(i) Find the width of the field. (4)
(ii) Find the height of the field at the centre of the field. (3)
(iii) Find the gradient at the point where $x = 60m$ (2)

b) The manager of an apple farm must decide when the apples should be harvested.



If the apples are harvested immediately, the average harvest per tree is 50kg and the apples can be sold for R3 per kg. Experience has shown that the harvest per tree will increase by 5kg for every week that harvesting is delayed and that the price per kg will decrease by 10cents per kg.
Let x be the number of weeks that harvesting is delayed, therefore making the price per kg: $300 - 10x$.

(i) Construct an expression for the kilogram yield per tree. (2)
(ii) Construct an expression for the income per tree. (2)
(iii) Determine the value of x that will give a maximum income. (4)