St John's College

Preliminary Examinations
July 2015
Mathematics Paper 2

Examiner: G Evans
Moderator: W Young

Time: 3 hrs
Marks: 150

Name: Memo

Teacher: **GE** **WY** **KJ** **DG** **DC** **BT**

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 22 pages. An **Information Sheet is provided separately**. Please check that your paper is complete.
2. Read the questions carefully.
3. Answer ALL the questions on the question paper. Note that there is space for additional working at the end of the paper.
4. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
5. Round off your answers to one decimal digit where necessary.
6. All the necessary working details must be clearly shown. Equations may not be solved solely with a calculator.
7. It is essential that you present your work neatly and logically.

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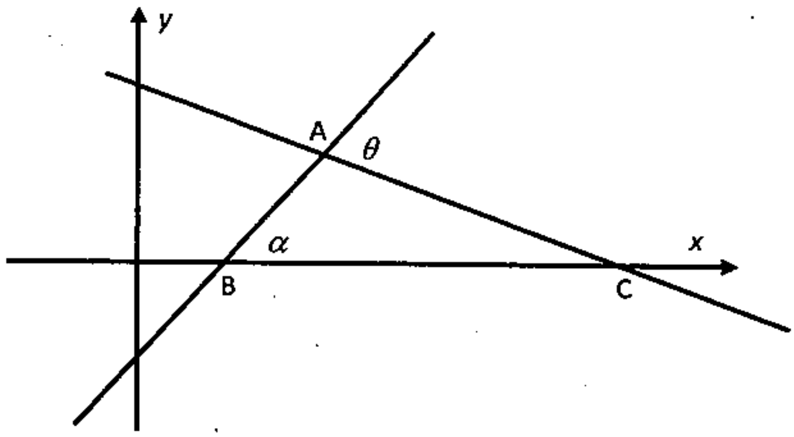
Out of	14	11	25	12	15	15	10	12	14	14	8	150
Mark												

SECTION A (77 marks)

Question 1

Two straight lines are drawn with equations: $y = x - 2$ and $5y + 3x = 30$.



(a) Determine the coordinates of A, B and C (6)

B(2;0) ✓ C(10;0) ✓

C: $5(x-2) + 3x = 30$ ✓

$8x = 40$ ✓

$x = 5$ ✓

$y = 3$ ✓

(b) Calculate the area of ΔABC (2)

Area = $\frac{1}{2} \times 8 \times 3$ ✓

= 12 units² ✓

(c) Calculate the size of α and θ (4)

$\alpha = \tan^{-1}(1) = 45°$ ✓

$\tan^{-1}\left(-3/5\right) = -30,96°$ ✓✓

$\theta = 45° + 30,96°$

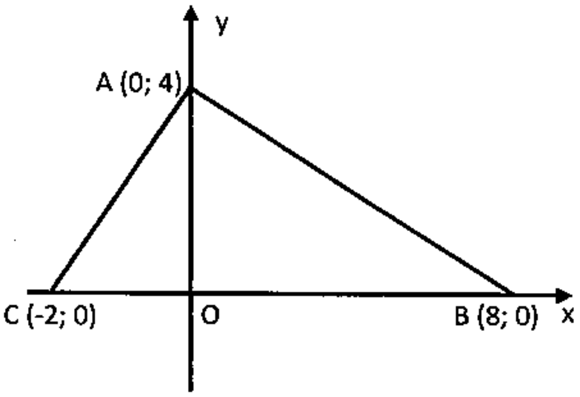
$= 76,0°$ ✓

(d) Write down the coordinates of D, such that ACDB is a paralleogram. (2)

$(7; -3)$ ✓✓

[14]

Question 2



Refer to the diagram

(a) Verify that $C\hat{A}B$ is a right-angle (3)

$m_{AC} = \frac{4}{2} = 2$ ✓ $m_{AB} = -\frac{4}{8} = -1/2$ ✓

$2 \times -1/2 = -1.$ ✓

(b) Find the equation of the circle passing through C, A and B (4)

m.p. BC = (3;0) ✓ radius = 5 ✓

$\therefore (x-3)^2 + y^2 = 25$ ✓ (∠ in a semi-circle) ✓

(c) Establish whether the circle you have found in (b) is tangential to the circle with equation: $(x+3)^2+(y-8)^2=225$ (4)

Distance between centres:

$d = \sqrt{6^2+8^2} = 10$ ✓✓

$r_1 = 5 \quad r_2 = 15$ ✓

$r_2 - r_1 = 10 \quad \therefore$ tangential ✓

[11]

Question 3

(a) Simplify, without the use of a calculator:

$\sin 124°.\sin 64° + \sin 214°.\sin 26°$ (6)

✓ $\sin 56° \cdot \cos 26°$ ✓ $- \cos 56° \sin 26°$ ✓

$= \sin(56° - 26°)$ ✓

$= \sin 30°$ ✓

$= \frac{1}{2}$ ✓

(b) If $p = \sin 25°$, write $\frac{\cos 50°}{\sin 205°}$ in terms of p: (4)

$$\frac{1 - 2\sin^2 25°}{-\sin 25°} = \frac{1-2p^2}{-p}$$

or $\frac{2p^2 - 1}{p}$

(c) Given that $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

(i) Hence or otherwise, prove that $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ (3)

$$\cos^2\theta = 1 - \sin^2\theta$$

$$= 1 - \frac{1-\cos 2\theta}{2}$$

$$= \frac{2 - 1 + \cos 2\theta}{2}$$

$$= \frac{1+\cos 2\theta}{2}$$

(ii) Hence, or otherwise, prove: $\sin^2\theta\cos^2\theta = \frac{1-\cos 4\theta}{8}$ (5)

$$\frac{(1-\cos 2\theta)}{2}\frac{(1+\cos 2\theta)}{2}$$

$$= \frac{1-\cos^2 2\theta}{4}$$

$$= \frac{2 - 2\cos^2 2\theta}{8}$$

$$= \frac{1 + 1 - 2\cos^2 2\theta}{8}$$

$$= \frac{1 - \cos 4\theta}{8}$$

(d) Solve for x if $2\cos 2x + 5\cos x = 4$ and $x \in [-90°; 360°]$ (7)

$2(2\cos^2 x - 1) + 5\cos x - 4 = 0$ ✓

$4\cos^2 x + 5\cos x - 6 = 0$

$(4\cos x - 3)(\cos x + 2) = 0$ ✓✓

$\cos x = 3/4$ ✓

$x = \pm 41{,}4° + 360°k \quad k \in \mathbb{Z}$ ✓

$\therefore x = \{-41{,}4°; 41{,}4°; 318°{,}6°\}.$ ✓✓

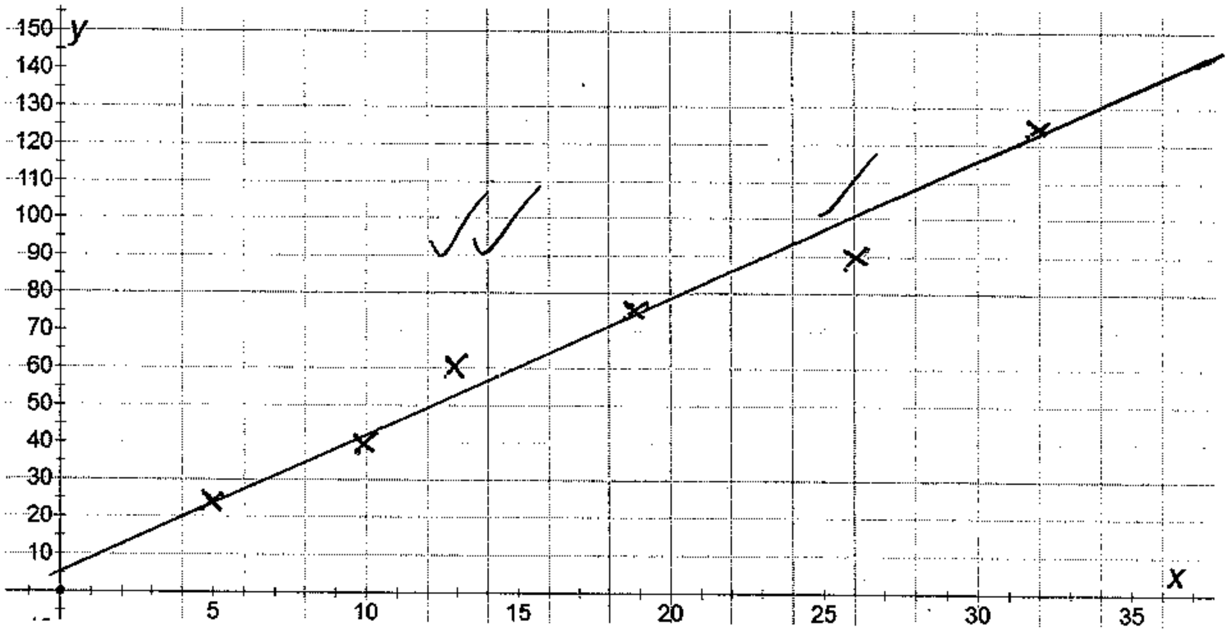
[25]

Question 4

(a) The energy saving lightbulb, also known as a compact fluorescent light bulb (CFL), is a type of fluorescent lamp that fits into a standard light bulb socket. An energy saving lightbulb uses far less wattage to produce the same amount of light as a regular incandescent light bulb (IL). The table below compares the wattage of an IL bulb to a CFL bulb.

CFL (watts) (x)	IL bulb (watts) (y)
5	25
10	40
13	60
19	75
26	90
32	125

(i) Plot the information given in the table on the axes given below and draw in a line of best fit. (3)



(ii) Use your calculator to find the equation of the line of best fit through this data and the correlation coefficient. (3)

$y = 8{,}04 + 3{,}49x$

$r = 0{,}98$

(iii) Determine the equivalent CFL wattage for a 55W IL bulb. (1)

$55 = 8{,}04 + 3{,}49x$

$x = 13{,}5$

(iv) Comment on the validity of your answer in part (iii). (1)

Accurate due to high correlation.

(b) State the skewness of the data in each case: (4)

A: 20, 35, 40, 55, 68, 75, 92

Symmetrical

B: $Q_3 + Q_1 < 2Q_2$

$Q_3 - Q_2 < Q_2 - Q_1$

$\therefore$ Skew left

C:

Skew right

D:

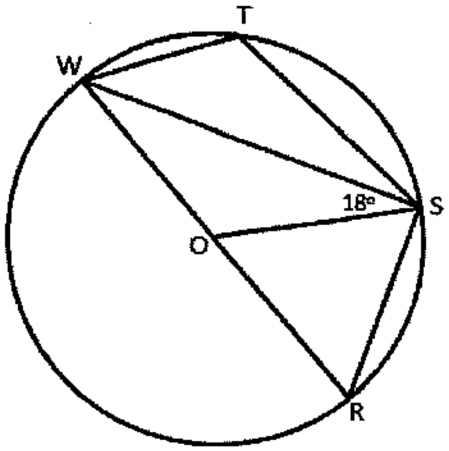
Skew right

[12]

Question 5

(a) O is the centre of the circle and WR is a diameter. R, S, T and W are points on the circle and $O\hat{S}W = 18°$

Calculate, giving clear reasons, the size of $S\hat{T}W$.



(4)

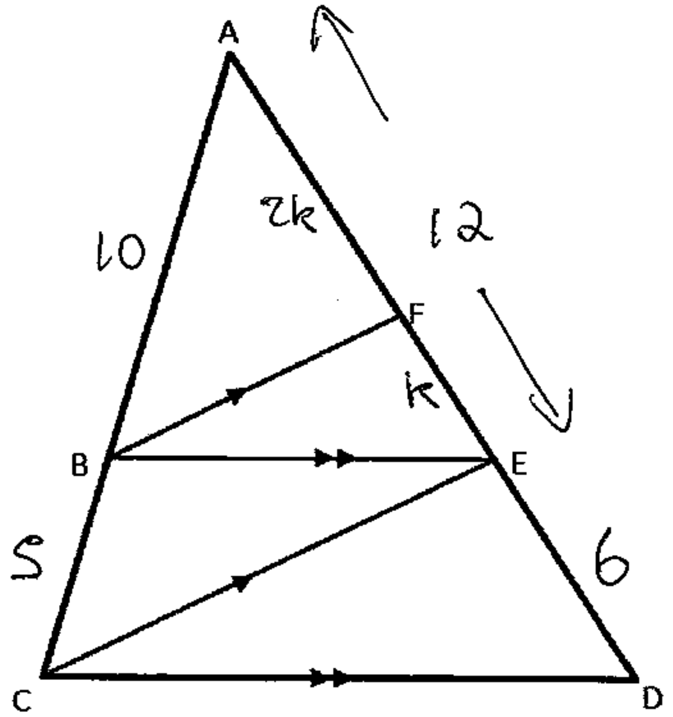
$R\hat{W}S = 18°$ ($\angle$ opp equal sides, radii) ✓

$R\hat{O}S = 36°$ ($\angle$ at centre twice $\angle$ at circ) ✓

$O\hat{R}S = 72°$ ($\angle$'s in isos triangle, radii) ✓

$\therefore S\hat{T}W = 108°$ (opp $\angle$'s cyclic quad) ✓

(b) In the digram, $BE \parallel CD$ and $BF \parallel CE$. AB = 10 mm, BC = 5 mm and DE = 6 mm.



(i) Calculate, with reasons, the length EF. (5)

$\frac{AE}{ED} = \frac{AB}{BC}$ (BE ∥ CD in △ACD) ✓

$\frac{AE}{6} = \frac{10}{5}$ ∴ AE = 12 ✓

$\frac{AF}{FE} = \frac{AB}{BC}$ (BF ∥ CE in △ACE) ✓

∴ $2k + k = 12$ ✓

∴ $k = 4$ ∴ FE = 4 ✓

(ii) Write down the value of the ratio:

1. $\frac{CD}{BE}$. (1)

$\frac{10}{15} = \frac{2}{3}$ ✓

2. $\frac{Area\ \Delta ABF}{Area\ \Delta EBF}$ (2)

$\frac{2}{1}$ ✓

3. $\frac{Area\ \Delta ABE}{Area\ BEDC}$ (3)

$\frac{ABE}{ACD} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

∴ $\frac{ABE}{BEDC} = \frac{4}{5}$ ✓✓✓

[15]

SECTION B (73 marks)

Question 6

(a) Prove the identity: $\cos x = \dfrac{1-\tan^2 \frac{1}{2}x}{1+\tan^2 \frac{1}{2}x}$ (Hint: let $y = \frac{1}{2}x$) (5)

$$\text{R.H.S.} = \frac{1-\tan^2 y}{1+\tan^2 y}$$

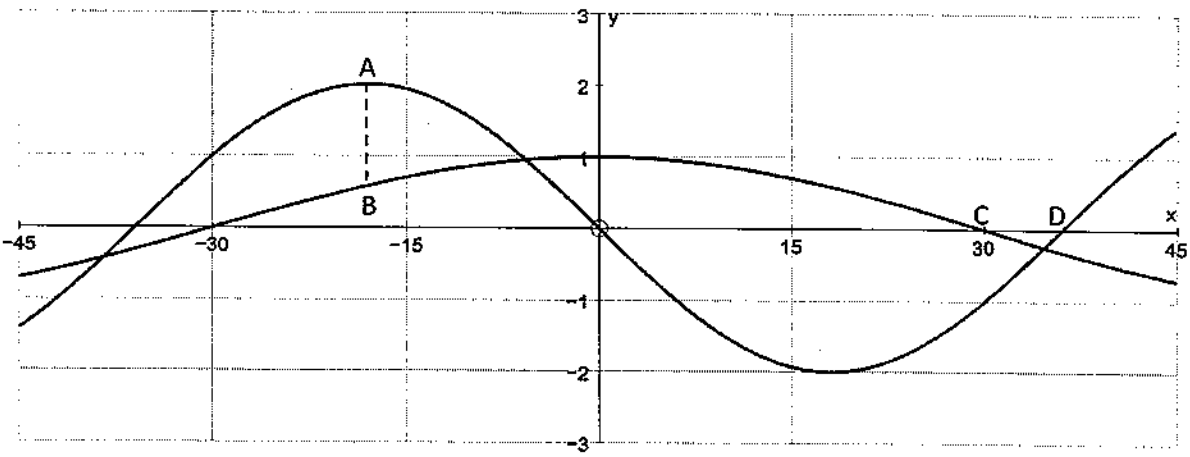
$$= \frac{\cos^2 y - \sin^2 y}{\cos^2 y + \sin^2 y}$$

$$= \frac{\cos 2y}{1}$$

$$= \cos x$$

(b) The diagram below shows two graphs:

$f(x) = \cos 3x$ and $g(x) = -2\sin 5x$ on the interval $-45° \le x \le 45°$.



(i) Write down the period of $g(x)$. (1)

$72°$

(ii) Write down the amplitude of $f(x)$ (1)

$\cancel{2}$ 1

(iii) Find the vertical distance AB where A is a turning point. (4)

x-coord of A = $-18°$ ✓

$\therefore AB = -2\sin(5x-18) - \cos(3x-18)$ ✓✓

$= 1{,}41$ ✓

(iv) Find the horizontal distance CD, where C and D are x-intercepts. (4)

D is half the period $\therefore$ $36°$ ✓✓

$36 - 30° = 6°$ ✓

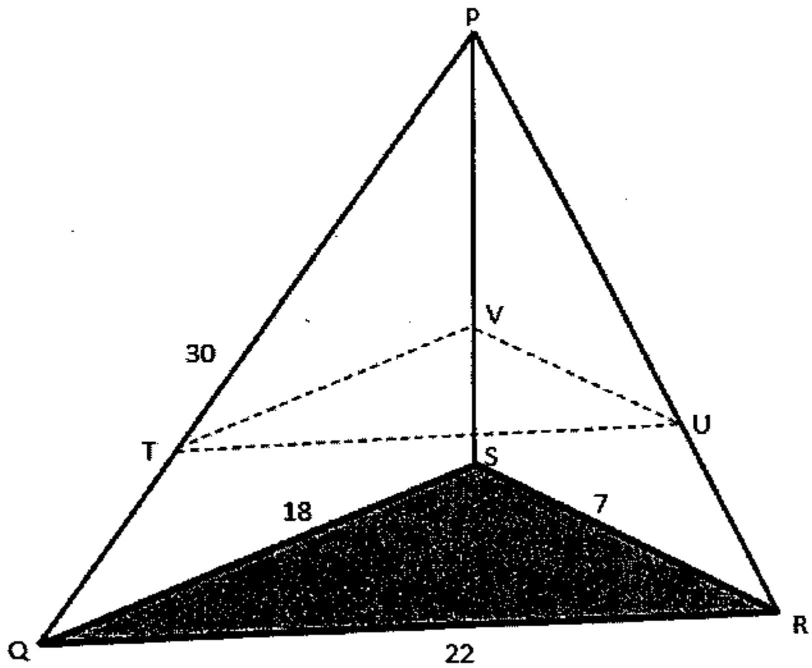
✓

[15]

Question 7

Formula: Volume of a pyramid = $\frac{1}{3} \times base\ area \times \perp height$

A triangular pyrmid, lying on a flat horizontal surface has base QSR with QR = 22 cm, QS = 18 cm and SR = 7 cm. The slant height PQ = 30 cm. P is vertically above S and thus PS is the height of the pyramid.



(a) Calculate the area of the base QSR. (5)

$22^2 = 18^2 + 7^2 - 2 \times 18 \times 7 \cos \hat{S}$ ✓

$\cos \hat{S} = -\frac{37}{84}$ ✓

$\therefore \hat{S} = 116,13°$ ✓

$\therefore \text{Area} = \frac{1}{2} \times 18 \times 7 \times \sin 116,13°$ ✓

$= 56,6 \text{ units}^2$ ✓

(b) Calculate the volume of the pyramid. (3)

$PS = \sqrt{30^2 - 18^2} = 24$ ✓

$\therefore V = \frac{1}{3} \times 56,6 \times 24$ ✓

$= 452,8 \text{ units}^3$ ✓

(c) The pyramid is now cut along TUV such that the plane TUV is parallel to the original base QRS and PT : TQ = 2 : 1.

Write down the volume of the pyramid which is cut off. (2)

Ratio of volumes $= (2:3)^3$

$= 8 : 27$

$\frac{8}{27} \times 452,8$

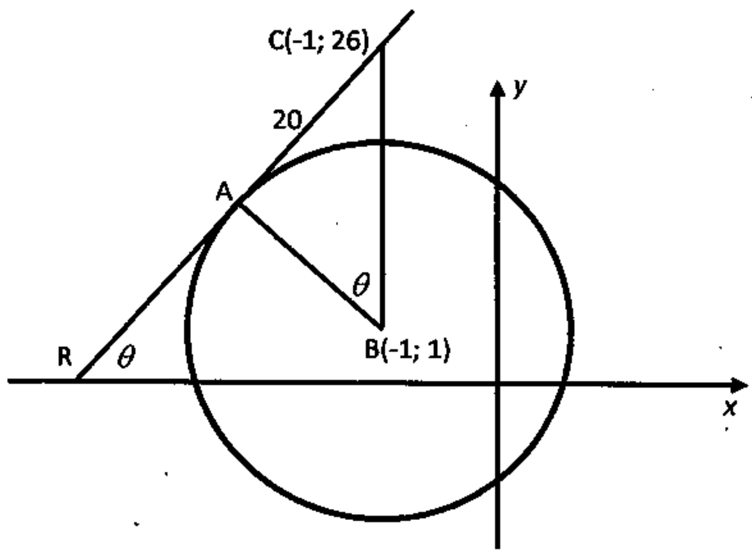
$= 134,16$ units^3

[10]

Question 8

In the figure below (not to scale), B(-1; 1) is the centre of the circle.

A tangent is drawn at A which intercepts the x-axis at R. C(-1; 26) is a point on the tangent and AC = 20 mm. $\hat{R} = \hat{B} = \theta$.



(a) Prove that the radius of the circle is 15 mm. (3)

BC = 25

$\therefore AB = \sqrt{25^2 - 20^2}$

$= 15.$

(b) Show that the gradient of the tangent is $\frac{4}{3}$. (2)

In $\triangle ABC$: $\tan\theta = \frac{20}{15} = \frac{4}{3}$ ✓

$m_{tangent} = \tan\theta = 4/3$. ✓

(c) Determine the equation of the tangent to the circle at A. (4)

$y - 26 = \frac{4}{3}(x+1)$ ✓✓✓

$\therefore y = \frac{4}{3}x + \frac{82}{3}$ ✓

(d) Establish whether the point P(13; 7) lies inside or outside the circle. (3)

Distance from centre $= \sqrt{14^2 + 6^2}$

$\approx 15,23$ ✓

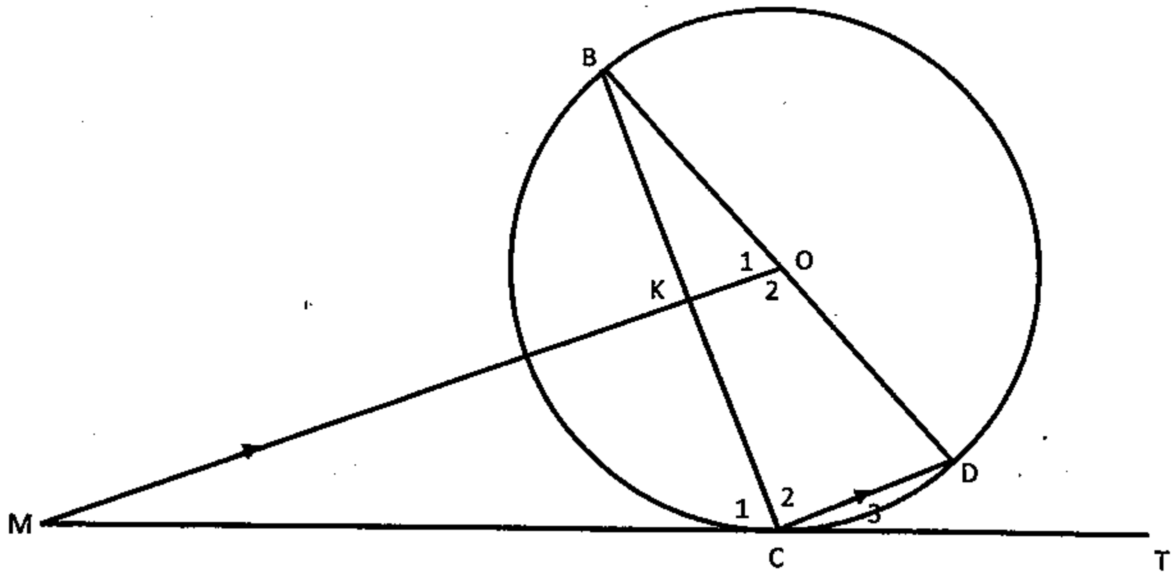
$15,23 > 15$ ✓

$\therefore$ outside the circle. ✓

[12]

Question 9

In the figure, O is the centre of the circle. MCT is a tangent. OM || DC.



(a) Prove:

(i) BK = KC (4)

$\hat{C}_2 = 90°$ (∠ in a semi-circle) ✓

$B\hat{K}O = C\hat{K}O = 90°$ (corres ∠'s OM || CD) ✓

∴ BK = KC (perp. from centre to m.p. chord)
✓ ✓

(ii) BOCM is a cyclic quad. (3)

Let $\hat{C}_3 = x$

∴ $\hat{B} = x$ (tan - chord) ✓

and $\hat{M} = x$ (corres ∠'s OM || CD) ✓

∴ BOCM cyclic (equal ∠s subt by same line) ✓

(iii) $\Delta BOK \,|||\, \Delta MCK$. (3)

In ΔBOK and ΔMCK

$\hat{B} = \hat{M}$ (already shown)

$B\hat{K}O = M\hat{K}C = 90°$ (already shown)

$\therefore \Delta BOK \,|||\, \Delta MCK$ (equiangular)

(b) Calculate BC (in surd form) if OK = 1 and MK = 7. (4)

$\frac{BO}{MC} = \frac{BK}{MK} = \frac{OK}{CK}$ (similar Δ's)

$BK = \frac{1}{2}BC$. and $CK = \frac{1}{2}BC$.

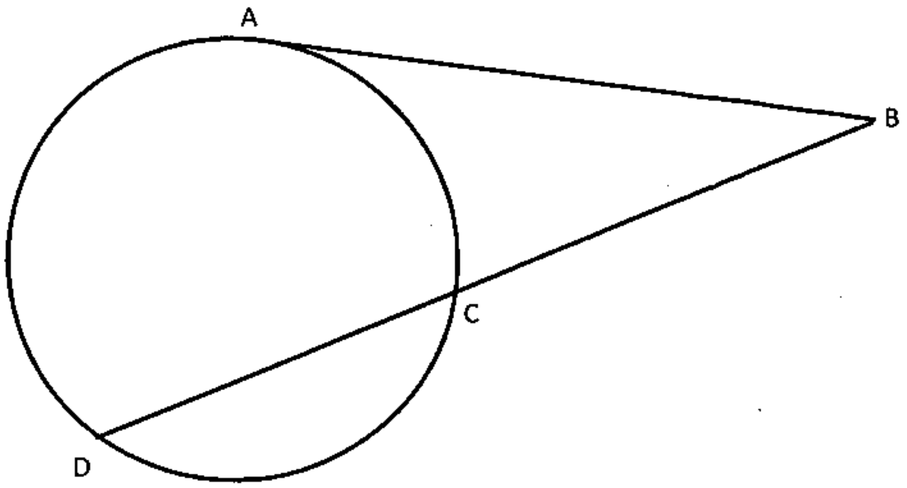
$\therefore \frac{\frac{1}{2}BC}{7} = \frac{1}{\frac{1}{2}BC}$ $\therefore BC^2 = 28$

$BC = 2\sqrt{7}$

[14]

Question 10

The diagram below illustrates the ***tangent-secant*** theorem (N.B. this is not one of the standard theorems you learn in class):



AB is a tangent to the circle at A. The line BCD cuts the circle at C and D.

By the tangent-secant theorem:

$$AB^2 = BC \times BD$$

*(Note that a **secant** is a line which cuts a curve in two places)*

(a) By making suitable construction lines and using similar triangles, prove the tangent-secant theorem. (7)

Draw AC and AD ✓

Let $B\hat{A}C = x$ ✓

$\therefore \hat{D} = x$ (tan-chord) ✓

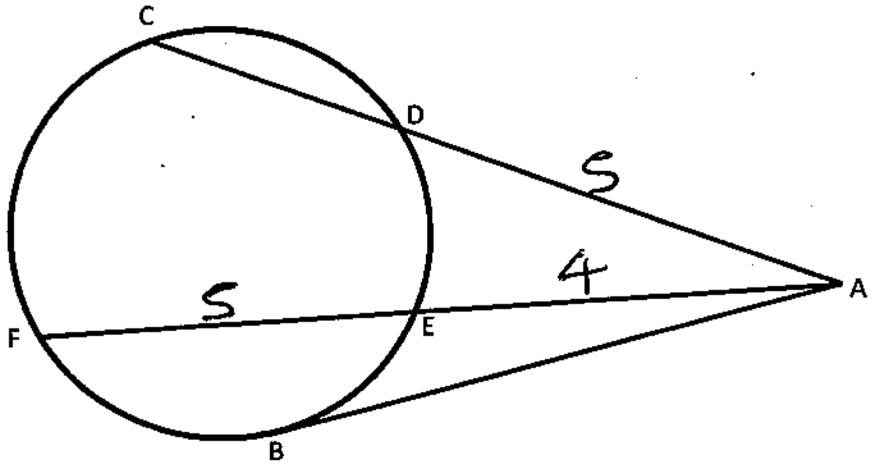
$\hat{B}$ is common ✓

$\therefore \triangle B\hat{A}C \mathrel{|||} \triangle BDA$ (equiangular) ✓

$\therefore \frac{BA}{BD} = \frac{BC}{BA} = \frac{AC}{DA}$ (similarity) ✓

$\therefore AB^2 = BC \times BD$ ✓

(b) Refer to the diagram below. AB is a tangent to the circle at B. AEF is a secant which cuts the circle at E and F. ADC is a secant which cuts the circle at C and D.



(i) Using the tangent-secant theorem, or otherwise, prove:

$$\frac{AE}{AD} = \frac{AC}{AF}$$ (3)

$AB^2 = AE \times AF$ (tan-sec) ✓

$AB^2 = AD \times AC$ (tan-sec) ✓

$\therefore AE \times AF = AD \times AC$ ✓

$\therefore \frac{AC}{AF} = \frac{AE}{AD}$

(ii) You are now given that AE = 4 and AD = FE = 5.

Find:

(1) the length of the tangent AB (2)

$AB^2 = 4 \times 9 = 36$ ✓

$\therefore AB = 6$ ✓

(2) the length of CD. (2)

$5(x+5) = 36$ ✓

$\therefore x = \frac{11}{5}$ ✓

[14]

Question 11

(a) Daniel wants to find the mean of the following set of numbers:

504, 512, 523, 529, 532

He doesn't have a calculator but uses the following clever strategy:

$4+12+23+29+32=100$

$\frac{100}{5}=20$

$Mean = 20+500$

$\quad = 520$

(i) Explain why this method works. (2)

Subtracting 500 from each number decreases the mean by 500.

(ii) How does the standard deviation of the original set of numbers compare to the numbers 4, 12, 23, 29, 32? Explain your answer. (2)

Does not change.
The difference between each number and the mean is the same.

(b) Let x_r represent a set of 30 numbers. It is given that $\sum_{r=1}^{30}(x_r - 20) = 420$.

Find the mean of the set of numbers. (4)

$\frac{420}{30} = 14$

$14+20 = 34$

[8]

TOTAL: 150 marks