QUESTION 1

a.1.

$$x^2 - 7x - 18 = 0$$
$$\therefore x = 9 \text{ or } x = -2$$

a.2.

$$x^2 \leq x$$
$$x^2 - x \leq 0$$
$$x(x-1) \leq 0$$
$$0 \leq x \leq 1$$

a.3.

$$x^2 - 4 = -x + 1$$
$$x^2 + x - 5 = 0$$
$$\therefore x = \frac{-1 \pm \sqrt{21}}{2}$$
$$\therefore x = 1{,}79 \ \ or \ \ x = -2{,}79$$

a.4.

$$(2^x - 5)^3 = 1$$
$$2^x - 5 = 1$$
$$2^x = 6$$
$$x = \log_2 6$$
$$x = 2{,}58$$

b)

$$y = x^2 - x^{-2}$$
$$\frac{dy}{dx} = 2x + 2x^{-3}$$
$$= 2x + \frac{2}{x^3}$$

c)

$$\frac{dy}{dx} = 2x$$
$$m = 2.2 = 4$$
$$y = 4 + 2 = 6$$
$$y = 4x + c$$
$$6 = 4.2 + c$$
$$c = -2$$
$$\therefore y = 4x - 2$$

d)

$$S_{30} = \frac{30}{2}\big(2.31 + (30-1)(-3)\big)$$
$$= -375$$

e)

$$(x^{-1} + y^{-1})^{-1} = \left(\frac{1}{x} + \frac{1}{y}\right)^{-1}$$
$$= \left(\frac{y+x}{xy}\right)^{-1}$$
$$= \frac{xy}{x+y}$$

QUESTION 2

a) $x = 2;\ y = 3$

b) 1 right; 4 up

c)

$$j(x) = g(-x)$$
$$= (-x)^2 - 2(-x) + 1$$
$$= x^2 + 2x + 1$$

QUESTION 3

a)

$$2 = b^1$$
$$b = 2$$

b.1) $(2;\ 1)$

b.2) $(1;\ -2)$

c) $h^{-1} = \log_2 x$

d) $y > -5$

e)

$$-2^x.x + 3.2.2^{x-1} < 0$$
$$-2^x.x + 3.2^x < 0$$
$$2^x(-x+3) < 0$$
$$\therefore x > 3, \text{ from graph}$$

QUESTION 4

a)

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$
$$0{,}6 = 0{,}3 + 0{,}4 - x$$
$$x = 0{,}1$$

b.1) $\frac{145}{188} = 0{,}77$

b.2) $\frac{20}{43} = 0{,}47$

SECTION B

QUESTION 5

a)

$$y = ax^3 + bx^2 + 4$$

$$0 = -a + b + 4$$
$$a = b + 4 \dots \mathbb{A}$$

$$2 = a + b + 4 \dots \mathbb{B}$$
$$2 = (b + 4) + b + 4$$
$$-2b = 6$$
$$b = -3$$
$$a = 1$$

$$\therefore y = x^3 - 3x^2 + 4$$

b)

$$\frac{dy}{dx} = 3x^2 - 6x$$
$$\frac{d^2y}{dx^2} = 6x - 6$$
$$0 = 6x - 6$$
$$x = 1$$

$$\therefore y = 1 - 3 + 4 = 2$$

Therefore, inflection is at $A(1; 2)$

c)

$$-4 < k < 0$$

QUESTION 6

a)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{\frac{2}{x+h} + 3 - \left(\frac{2}{x} + 3\right)}{h}$$
$$= \lim_{h\to 0} \left(\frac{2}{x+h} - \frac{2}{x}\right) \times \frac{1}{h}$$
$$= \lim_{h\to 0} \frac{2x - 2(x+h)}{hx(x+h)}$$
$$= \lim_{h\to 0} \frac{-2h}{xh(x+h)}$$
$$= -\frac{2}{x^2}$$

b)

$$f^{-1}(x): x = \frac{2}{y} + 2$$
$$x - 2 = \frac{2}{y}$$
$$y = \frac{2}{x-2}$$

$$-\frac{2}{x^2} = \frac{2}{x-2}$$
$$-2(x-2) = 2x^2$$
$$0 = 2x^2 + 2x - 4$$
$$0 = x^2 + x - 2$$
$$\therefore x = 1 \text{ or } x = -2$$

QUESTION 7

a) A
b) D
c) A
d) B
e) C

QUESTION 8

$$(x - a)(x^2 + 3x + 2) = 0$$
$$(x - a)(x + 1)(x + 2) = 0$$

$$\therefore x = a, x = -1 \text{ or } x = -2$$

QUESTION 9

a)

$$a = 1$$
$$r = x$$
$$S_\infty = \frac{3}{2}$$
$$\frac{3}{2} = \frac{1}{1-x}$$
$$3(1-x) = 2$$
$$-3x = -1$$
$$x = \frac{1}{3}$$

b)

$$a^x = a\left(a.a^{\frac{1}{2}}\right)^{\frac{1}{2}}$$
$$= a\left(a^{\frac{3}{2}}\right)^{\frac{1}{2}}$$
$$= a.a^{\frac{3}{4}}$$
$$= a^{\frac{7}{4}}$$
$$\therefore x = \frac{7}{4}$$

SECTION C

QUESTION 10

a) $7! = 5040$

b.1) $5 \times 5 \times 4 \times 3 \times 2 \times 1 \times 1 = 600$

b.2) $\frac{600}{5040} = \frac{5}{42} = 0{,}12$

QUESTION 11

a.1)

$$4b^2 - c - (b^2 + 2c) = 3d$$
$$3b^2 - 3c = 3d$$
$$d = b^2 - c$$

a.2)

$$11 = b^2 + 2c \ (\mathbb{A})$$
$$b^2 - c = -4$$
$$c = b^2 + 4 \ (\mathbb{B})$$

Sub $\mathbb{B}$ into $\mathbb{A}$:

$$11 = b^2 + 2(b^2 + 4)$$
$$11 = 3b^2 + 8$$
$$3b^2 = 3$$
$$b^2 = 1$$
$$\therefore b = \pm 1$$
$$\therefore c = 5$$

b.1)

$$\frac{x^2 - 3x + 2}{2(x-1)} = \frac{x-2}{x^2 - 3x + 2} = r$$
$$\frac{(x-1)(x-2)}{2(x-1)} = \frac{x-2}{(x-1)(x-2)}$$
$$\frac{x-2}{2} = \frac{1}{x-1}$$
$$x^2 - 3x + 2 = 2$$
$$x^2 - 3x = 0$$
$$x(x-3) = 0$$
$$\therefore x = 0 \text{ or } x = 3$$

b.2)

If $x = 0, r = -1$. Therefore $x \neq 0$.

If $x = 3, r = \frac{1}{2}$. Therefore $x = 3$.

$$S_\infty = \frac{2.3 - 2}{1 - \frac{1}{2}} = 8$$

QUESTION 12

a)

$$A = 150000\left(1 + \frac{0{,}085}{12}\right)^{36}$$
$$= 193395{,}33$$

b)

$$193395{,}33 = x \times \frac{1 - \left(1 + \frac{0{,}085}{12}\right)^{-60}}{\frac{0{,}085}{12}}$$
$$x = \frac{193395{,}33}{\frac{1 - \left(1 + \frac{0{,}085}{12}\right)^{-60}}{\frac{0{,}085}{12}}}$$

$$x = 3967{,}80$$

c)

$$b = 3967{,}80 \times \frac{1 - \left(1 + \frac{0{,}085}{12}\right)^{-48}}{\frac{0{,}085}{12}}$$

$$b = 160976{,}59\ldots$$

$$x = \frac{160976{,}59\ldots}{\frac{1 - \left(1 + \frac{0{,}09}{12}\right)^{-48}}{\frac{0{,}09}{12}}}$$

$$x = 4005{,}91$$

QUESTION 13

a)

$$A = \frac{1+x}{2} \times \left(-\frac{x^2}{4} + x + 3\right)$$
$$= \left(\frac{1}{2} + \frac{x}{2}\right)\left(-\frac{x^2}{4} + x + 3\right)$$
$$= -\frac{x^2}{8} + \frac{x}{2} + \frac{3}{2} - \frac{x^3}{8} + \frac{x^2}{2} + \frac{3x}{2}$$
$$= -\frac{x^3}{8} + \frac{3x^2}{8} + 2x + \frac{3}{2}$$

b)

$$\frac{dA}{dx} = -\frac{3x^2}{8} + \frac{3x}{4} + 2$$
$$0 = -3x^2 + 6x + 16$$
$$\therefore x = 3{,}52 \text{ or } x \neq -1{,}52$$

Therefore Area = 7,73 units^2.