ST MARY'S DSG, KLOOF

GRADE: 12 SEPTEMBER 2015

MATHEMATICS: PAPER II

Examiner: J van Rooyen Moderator: P Norton

TIME: 3 HOURS TOTAL: 150 MARKS

INSTRUCTIONS:

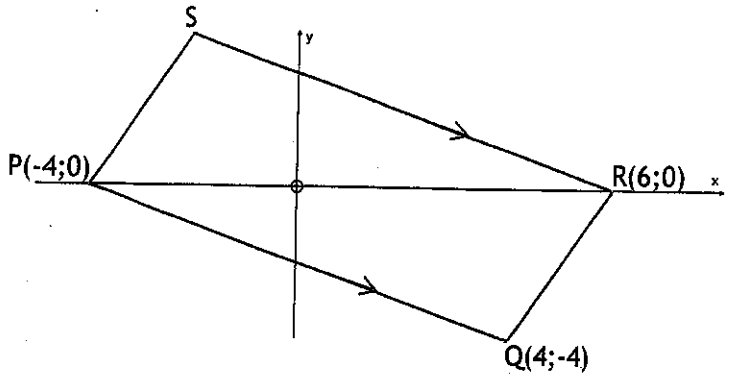
1. This question paper consists of 21 pages.
2. All answers will be written on the question paper.
3. There are 3 sections.
4. **Write your number and Maths teacher's name** on the top of each section.
5. A formula sheet has been provided.
6. Diagrams are not drawn to scale.
7. Please give all answers correct to 2 decimal places unless otherwise indicated.
8. Read all the questions carefully.
9. An approved non-programmable and non-graphical calculator may be used, unless otherwise specified.
10. Make sure that your calculator is in degree mode.

NUMBER: MEMO

TEACHER'S NAME: _____

QUESTION 1

In the diagram below PQRS is a rectangle with vertices P(-4;0), Q(4;-4), R (6;0) and S.



a) Give the co-ordinates of S (2)

$(-2; 4)$

b) Find t, if T $(t; t-3)$ is collinear with points Q and R. (3)

$\frac{0+4}{6-4} = \frac{t-3}{t-6}$ OR $\frac{0+4}{6-4} = \frac{t-3+4}{t-4}$

$\frac{4}{2} = \frac{t-3}{t-6}$ $\quad 2 = \frac{t+1}{t-4}$

$4t-24 = 2t-6$ $\quad 2t-8 = t+1$

$2t = 18$ $\quad t = 9.$

$t = 9$

c) Determine the equation of the circle with diameter PR. Give your answer in the form $(x-a)^2 + (y-b)^2 = r^2$ (3)

centre $(1;0)$ $r = 5$

$(x-1)^2 + y^2 = 25$

d) Give the equation of the tangent to the above mentioned circle at Q (4)

centre $OQ_m = \frac{0+4}{1-4} = \frac{4}{-3}$

tangent $m = 3/4$

$y = 3/4x + c$

$(4; -4)$ $\quad -4 = 4(3/4) + c$

$-7 = c$

$y = 3/4x - 7$

[12]

QUESTION 2

A runner has recorded her times, in seconds, for six different laps of the running track.

53 57 58 60 55 56

a) Calculate:

i) Her mean lap time (1)

56,5 seconds

ii) The standard deviation of these lap times (2)

2,22

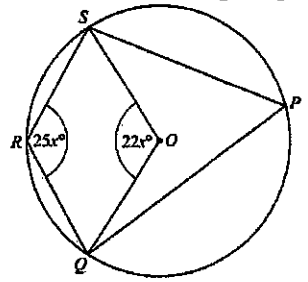
b) She changes her training routine hoping to improve her consistency. She then records her times for another six laps. Her mean for the second six laps is 55 seconds and the standard deviation is 3,2 seconds. Has the new training routine improved her consistency? Give a reason for your answer. (2)

no, higher std deviation ∴ less consistent

[5]

QUESTION 3

PQRS is a cyclic quadrilateral in the circle, centre O. $Q\hat{O}S = 22x^\circ$ and $Q\hat{R}S = 25x^\circ$. Find the value of x, giving reasons. [3]



$\hat{P} = 11x$ (∠ at centre)

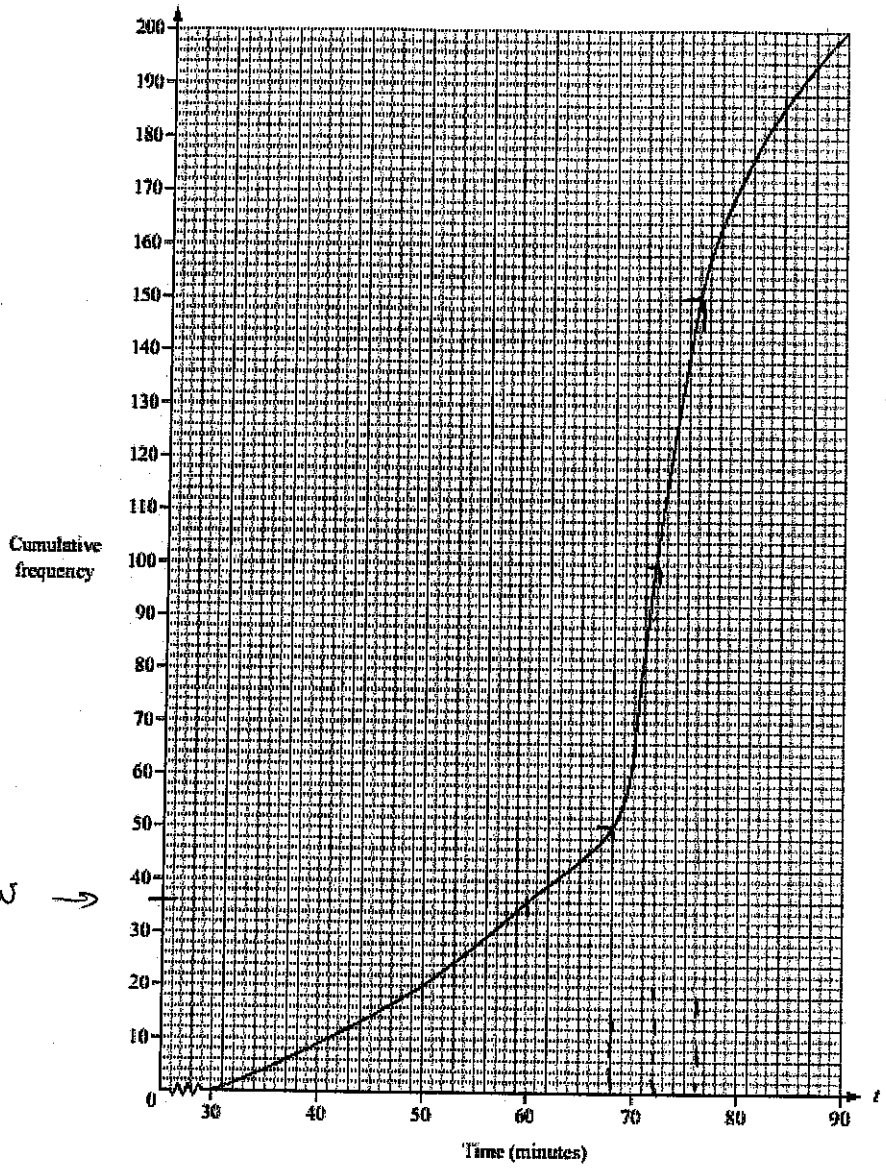
$25x + 11x = 180^\circ$ (opp ∠'s cyclic quad)

$36x = 180^\circ$

$x = 5^\circ$

QUESTION 4

200 students write a Mathematics examination. The cumulative frequency diagram below shows information about the time taken, t minutes, to complete the examination.



min ✓
Q1 ✓
Q2 ✓
Q3 ✓
max ✓
Values ✓

a) Draw the box and whisker plot for this data set in the space above, below the cumulative frequency diagram. (3)

b) How many students took more than one hour to complete the examination? Indicate on the graph how you get your result. (1)

$200 - 36 = 164$

c)i) Use the cumulative frequency diagram to complete the grouped frequency table (1)

Time, t minutes	$30 < t \leq 40$	$40 < t \leq 50$	$50 < t \leq 60$	$60 < t \leq 70$	$70 < t \leq 80$	$80 < t \leq 90$
Frequency	9	11	16	28	108	28

ii) Calculate an estimate of the mean time taken by the 200 students to complete the examination (2)

69,95 minutes.

d) There is a power outage during the writing of the mathematics paper and all the students are given an extra x minutes in which to finish the paper.

i) What will the new mean time to complete the paper become? (1)

69,95 + x

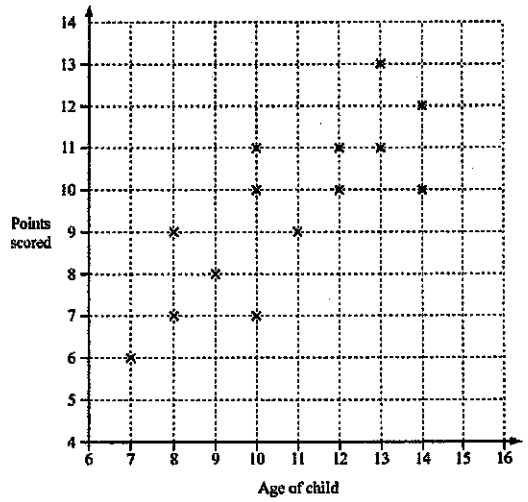
ii) How will this change the standard deviation ? (1)

No change

[9]

QUESTION 5

14 children played a game. The age of each child and the number of points they scored are plotted on the scatter diagram.



a) Give the equation of the linear regression line for the above data (3)

$y = 2,04 + 0,698x = 2,04 + 0,7x$.

b) Give the correlation co-efficient for the above data. (1)

$r = 0,79$

c) Use the above equation to predict what score a $13\frac{1}{2}$ year old would achieve in the game. (2)

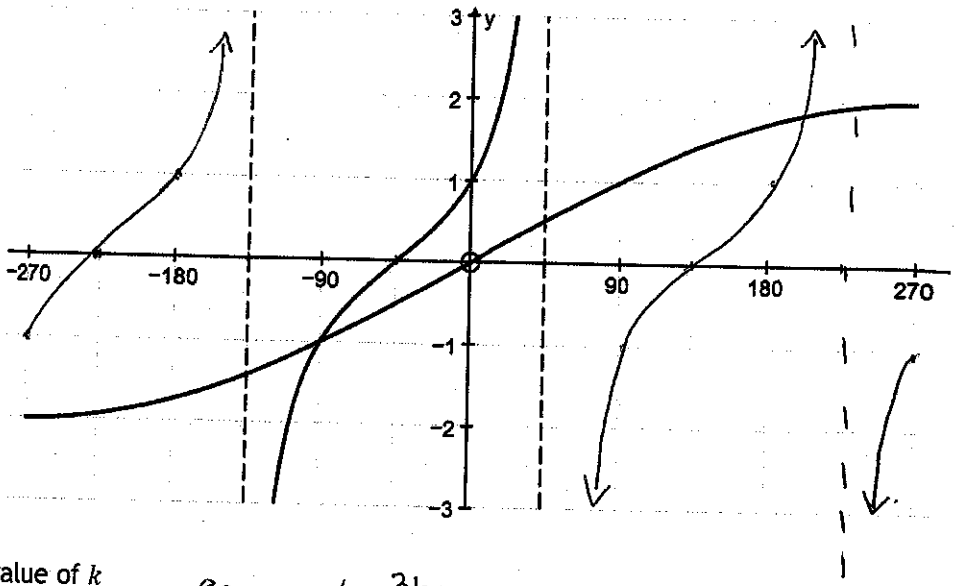
$y = 2,04 + 0,7(13,5) = 11,49$

$= 11,5$ points.

[6]

QUESTION 6

Given are the graphs of $f(x) = 2\sin kx$ and $g(x) = \tan(x + n)$ for $x \epsilon (-270^\circ; 270^\circ)$



a) Give the value of k (2)

$\frac{90}{270} \;/\; \frac{360}{1080} = \frac{1}{3}$

b) Give the value of n (1)

45°

c) Complete the graph of $f(x)$ 1 mark per 3RD, last 3RD includes asymptote (3)

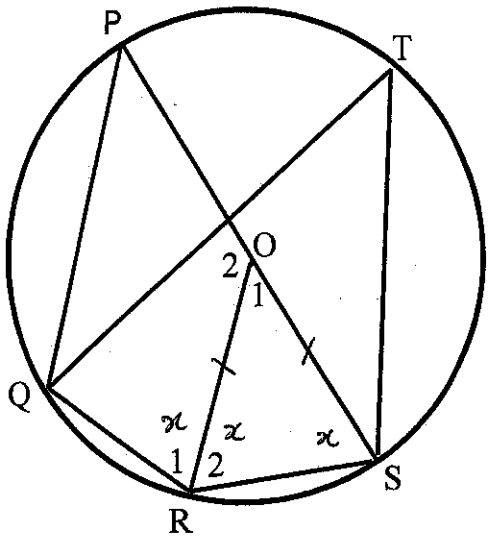
d) For which values of x is $g(x) \leq f(x)$ when $x \in [-270^\circ; 0^\circ]$ (2)

$x \in (-135^\circ; -90^\circ]$

[8]

QUESTION 7

In circle centre O, $\hat{R}_1 = \hat{R}_2 = x$, PQRS and TQRS are cyclic quads.



Show that $\hat{O}_1 = \hat{T} = \hat{P}$. Give reasons for working. (4)

In $\triangle ORS$ $\hat{R}_2 = \hat{S} = x$ (∠'s opp = radii)

$\hat{O}_1 = 180° - 2x$ (∠ sum of $\triangle$)

$\hat{P} = 180° - 2x$ (opp ∠'s cyclic quad PQRS)

$\hat{T} = 180° - 2x$ (" " " QRST)

OR (∠'s in same seg)

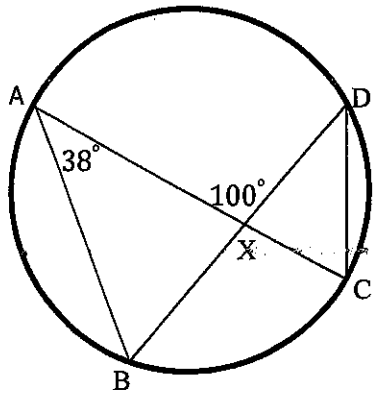
$\therefore \hat{O}_1 = \hat{P} = \hat{T}$

SECTION B [54] NUMBER: MEMO

MATHS TEACHER:

QUESTION 8

a) A, B, C and D lie on a circle. AC is a diameter of the circle.



i) Calculate the size of $X\hat{C}D$, giving reasons. (2)

$\hat{B} = 100° - 38°$ (ext ∠ of Δ)

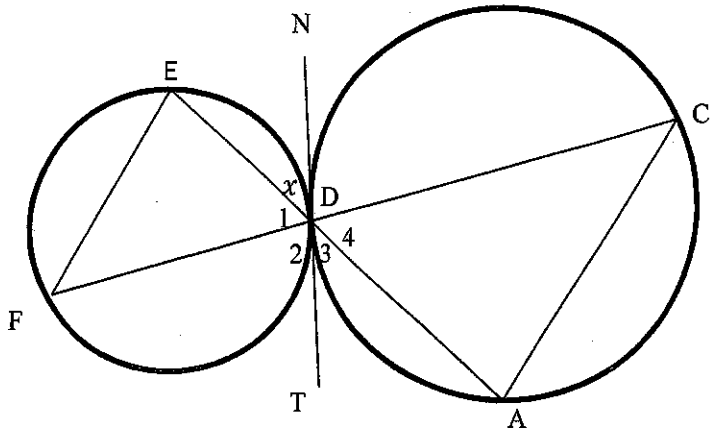
$= 62°$

$\hat{C} = \hat{B} = 62°$ (∠'s in same seg)

ii) If lines AB and AC are fixed and the position of D moves along the circle, what will $A\hat{X}D$ measure when BD is also a diameter of the circle? (2)

$2(38°) = 76°$ (angles opp = radii, ext ∠ of Δ).

as above ....



i) Complete the following: (3)

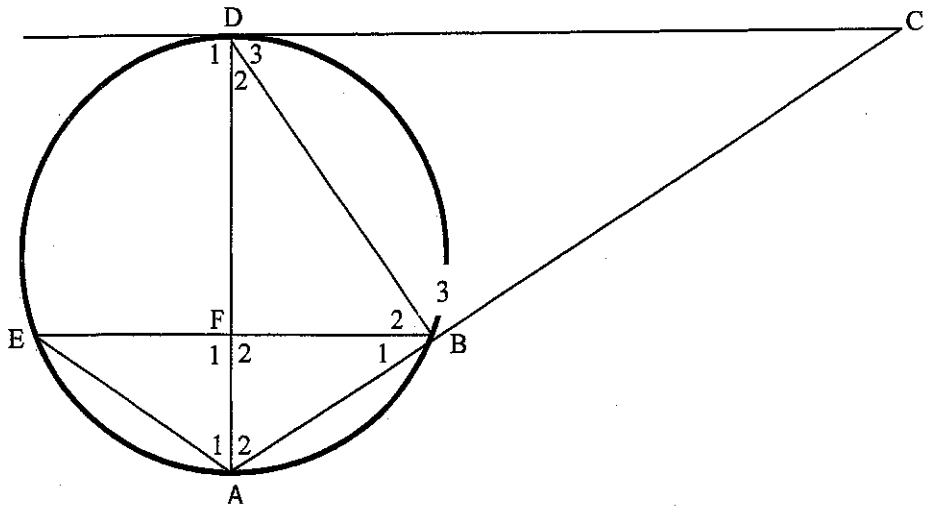
STATEMENT	REASON
$\hat{F} = x$	tan- chord
$\hat{D}_3 = x$	vert opp ∠'s
$\hat{D}_3 = \hat{C} = x$	tan- chord

ii) If the above statements are true, what can be concluded about EF and CA? Explain (2)

EF // CA corr. alt ∠'s =

alt ∠'s are equal

c) AD is a diameter of a circle. Tangent DC meets line AB at C. BE ⊥ AD.



i) The following are possible reasons why $\Delta AFE \equiv \Delta AFB$. Circle the correct one. (2)

A) RHS B) SSS C) AAA D) SAS E) SAA

ii) Prove ΔBDC III ΔFAE (4)

1) $\hat{B}_{1+2} = 90°$ (∠ in semi-⊙)

$\hat{B}_3 = 90°$ (adj ∠'s St line)

$\hat{F}_1 = 90°$ (given ⊥)

2) $\hat{D}_2 = \hat{A}_2$ (tan-chord)

$\hat{A}_2 = \hat{A}_1$ (≡ Δ's)

3) $\hat{C} = \hat{E}$ (∠ sum Δ)

∴ Δ BDC III Δ FAE (AAA)

iii) Prove that $BC.AB = CD.FE$ (2)

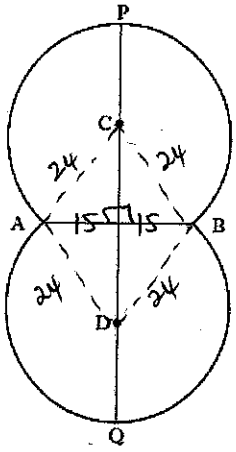
$\Rightarrow \frac{BD}{FA} = \frac{DC}{AE} = \frac{BC}{FE}$

$DC.FE = BC.AE$

$AE = AB$ (≡ Δ's)

∴ $DC.FE = BC.AB$.

the centres of the circles. Each circle has a radius of 24cm. AB is a common chord of length 30cm.



i) Prove that ACBD is a rhombus (2)

AC = CB = BD = DA = 24cm (radii)

∴ ACBD is a rhombus (= adj sides)

ii) Calculate the height of the shape, PQ. Give reasons for working. (5)

CD ⊥ AB (diags rhombus)

∴ 15cm + 15cm (rad ⊥ chord)

$\sqrt{24^2 - 15^2} = 3\sqrt{39}$

$PQ = 2(3\sqrt{39}) + 2(24)$

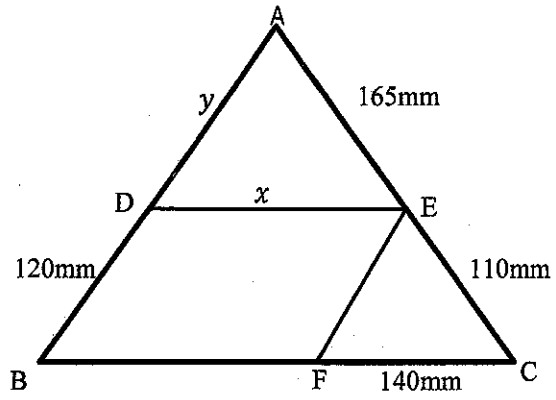
$= 6\sqrt{39} + 48$

$= 85,47\text{cm}$

[24]

QUESTION 9

a) BDEF is a parallelogram. Calculate, with reasons, the ratio $x : y$ (5)



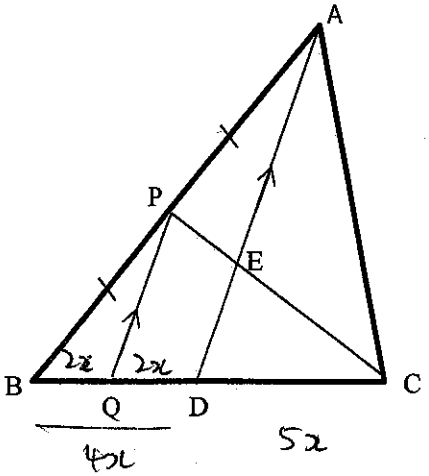
$\frac{BF}{FC} = \frac{AE}{EC}$ (prop int th, AB // EF)

$BF = \frac{165}{110} \times 140 = 210 = x$ (opp sides parm =)

$y = \frac{165}{110} \times 120 = 180$ (prop int th, DE // BC)

$\therefore \quad x : y = 210 : 180 = 7 : 6$

b) In ΔABC, P is the midpoint of AB. $PQ \parallel AD$. $\frac{CD}{DB} = \frac{5}{4}$



Find the ratio $\frac{CP}{CE}$, giving reasons

$\frac{AP}{PB} = \frac{DQ}{DB}$ (prop int th, AD // PQ)

$= \frac{1}{1} = \frac{2x}{2x}$

$\frac{CP}{CE} = \frac{CQ}{CD}$ (prop int th, PQ // ED)

$= \frac{7x}{5x}$

$= \frac{7}{5}$ (3)

[8]

QUESTION 10

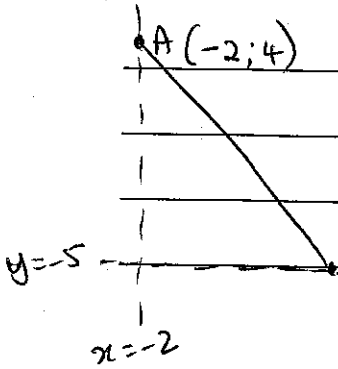
a) C $(\frac{1}{2};-\frac{1}{2})$ is the midpoint of the line joining points A $(a;4)$ and B $(3;b)$.

i) Show that $a = -2$ and $b = -5$. (3)

$\frac{a+3}{2} = \frac{1}{2}$ $\qquad$ $\frac{b+4}{2} = -\frac{1}{2}$

$\therefore a = -2$ $\qquad$ $b = -5$

ii) Determine the area of the triangle formed by lines AB, $y = b$ and $x = a$. (4)



$A = \frac{1}{2}(5)(9)$

$= \frac{45}{2}$ units^2

[7]

QUESTION 11

Formulae you may need. $V = \frac{1}{3}\pi r^2 H$ $\qquad$ $V = \frac{4}{3}\pi r^3$ $\qquad$ $V = \pi r^2 H$

A soild sphere of radius 10,5 cm is melted and recast into smaller solid cones, each of radius 3,5cm and height 3cm. Find the number of cones so formed. [4]

Sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}(\pi)(10{,}5)^3$

$= \frac{3087\pi}{2}$

Cone $= \frac{1}{3}\pi r^2 H$

$= \frac{1}{3}\pi(3{,}5)^2(3)$

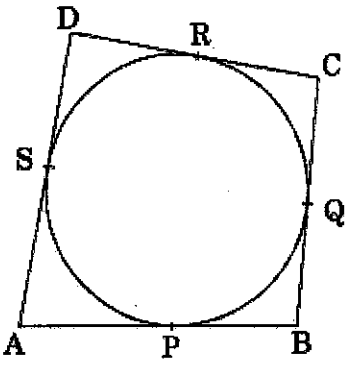
$= \frac{49}{4}\pi$

$\frac{\text{Sphere}}{\text{Cone}} = \frac{\frac{3087\pi}{2}}{\frac{49}{4}\pi} = 126$ cones.

QUESTION 12

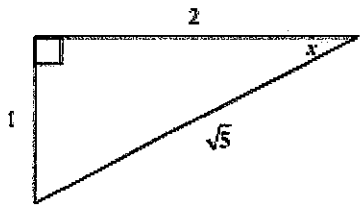
In each of the following, select the one correct answer and ring the letter corresponding to the correct answer. You may work in the space alongside any diagram.

a) In the figure, Quadrilateral ABCD is drawn around a circle so that P,Q,R and S are each the one point of contact for each side. If $AB = x\ cm,\ BC = 7\ cm, CR = 3\ cm\ and\ AS = 5\ cm,$ then $x =$ (3)



(A) 10 cm (B) 8 cm (C) 9 cm (D) 7 cm

b) The diagram shows a right angled triangle. Use the given information to find $\cos 2x$. (3)

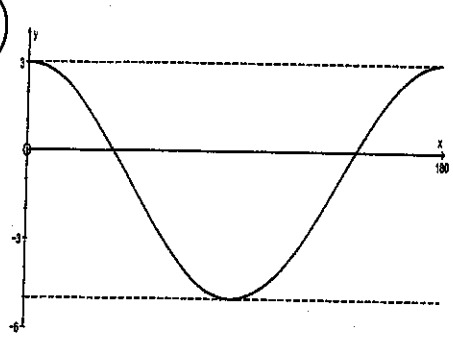


(A) $\frac{3}{5}$ (B) $\frac{4}{5}$ (C) $\frac{3}{\sqrt{5}}$ (D) $\frac{2}{\sqrt{5}}$

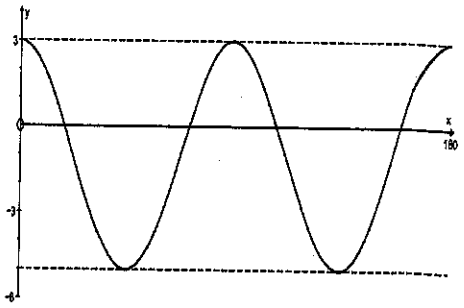
c) Which of the following is equivalent to $\cos(a - 90^\circ)$ if $0^\circ < a < 90^\circ$ (2)

(A) $\cos a$ (B) $\sin a$ (C) $-\cos a$ (D) $-\sin a$

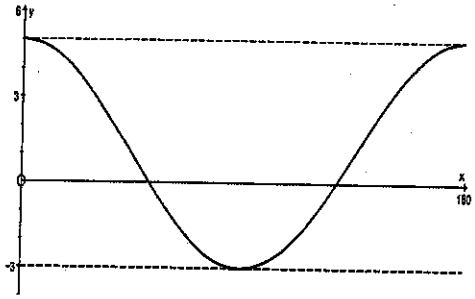
(A)



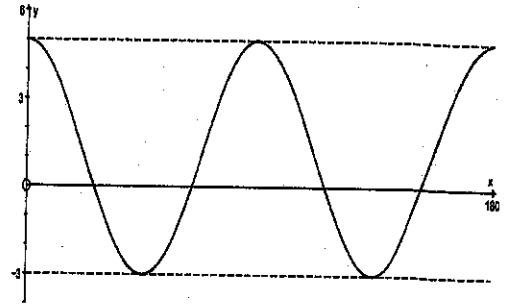
(B)



(C)



(D)



[11]

SECTION C [49]

NUMBER: MEMO.

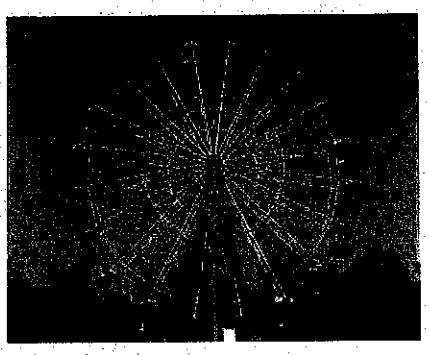
MATHS TEACHER:

QUESTION 13

A Ferris wheel is turning at a steady rate. The height, h metres, of one of the cars above the ground at a time t seconds, is given by the formula [4]

$$h = 7 + 5\sin t$$

Find at which two times during the first turn, the car is at a height of 10,8 metres above the ground.



$10{,}8 = 7 + 5\sin t$

$3{,}8 = 5\sin t$

$\frac{3{,}8}{5} = \sin t$

$t = 49{,}46$ secs OR $130{,}54$ secs

QUESTION 14

Determine the general solution to the following :

$$\sin^2 x - \sin x \cdot \cos x = \sin x - \cos x$$ [7]

$\sin x(\sin x - \cos x) = \sin x - \cos x$

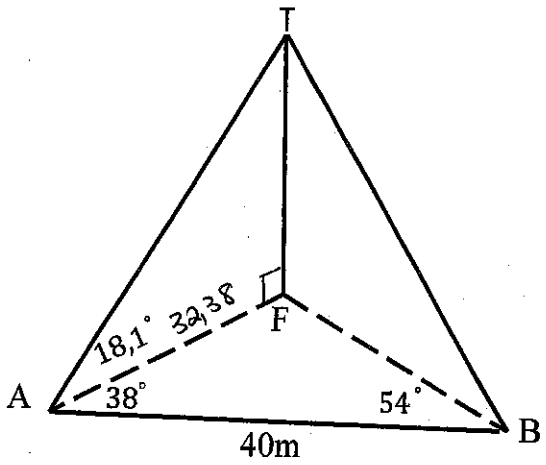
$\sin x(\sin x - \cos x) - (\sin x - \cos x) = 0$

$(\sin x - \cos x)(\sin x - 1) = 0$

$\sin x - \cos x = 0$ OR $\sin x = 1$

$\tan x = 1$ $x = 90° + 360k,\ k \in Z$

$x = 45° + 180k$



Abby and Brendan are battling to get cell phone coverage so that they can talk to each other. They are near a cell phone tower TF and have to stand 40m apart to achieve full signal coverage. The angle of elevation from Abby to the top of the tower is 18,1°. The angle between Abby and the foot of the tower is 38°. The angle between Brendan and the foot of the tower is 54°. How tall is the tower? (5)

$\hat{F} = 180° - (38° + 54°)$

$= 88°$

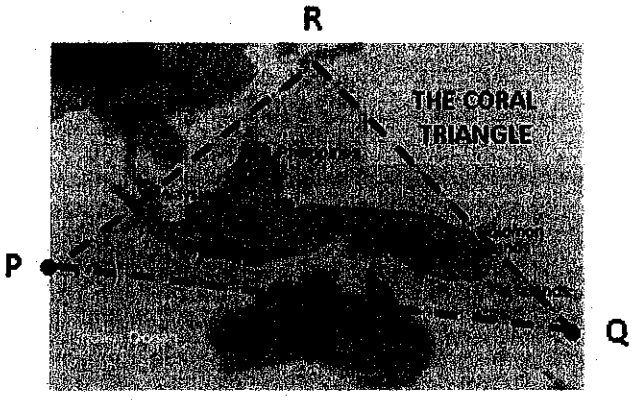
$\frac{FA}{\sin 54°} = \frac{40}{\sin 88°}$

$FA = 32,38$

$TF = 32,38 \tan 18,1°$

$= 10,58m$

b) The map below shows the extent of what is known as the CORAL TRIANGLE.



If it is given that $R\hat{P}Q = 48°, PR:PQ = 13:19$. Also, that the area of the CORAL TRIANGLE is approximately 6 million square kilometres. Determine the lengths of PR and PQ correct to the nearest kilometre. (5)

$PR \rightarrow 13x \qquad PQ \rightarrow 19x$

$6\,000\,000 = \frac{1}{2}(13x)(19x)\sin 48°$

$653\,74,87\ldots = x^2$

$255,69 = x$

$PR = 13(255,69) \approx 3324 \text{ km}$

$PQ = 19(255,69) \approx 4858 \text{ km}$

[10]

QUESTION 16

Prove that,

$$\frac{\sin A + \sin 2A}{1 + \cos A + \cos 2A} = \tan A$$ [4]

LHS $\quad \dfrac{\sin A + 2\sin A\cos A}{1 + \cos A + 2\cos^2 A - 1}$

$= \dfrac{\sin A(1 + 2\cos A)}{\cos A(1 + 2\cos A)}$

$= \tan A$

$=$ RHS

Given the circles $(x+1)^2+(y-2)^2=36$ and $(x-3)^2+(y+4)^2=16$. Will these circles intersect once, twice or never? Show working to substantiate your answer. [5]

A

centre $(-1;2)$

$r=6$

B

centre $(3;-4)$

$r=4$

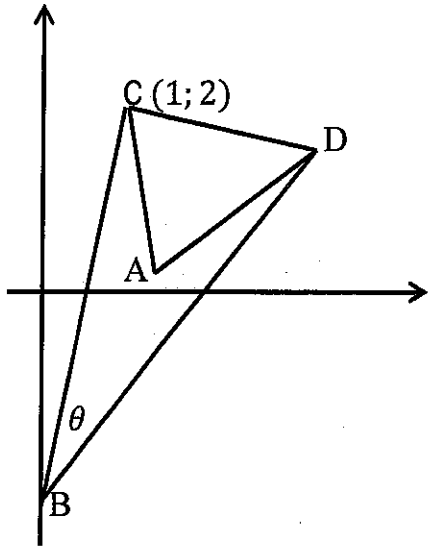
$|AB| = \sqrt{4^2+6^2} = \sqrt{52} = 7,2.$

$r_1 + r_2 = 10.$

$\therefore$ Intersect twice.

QUESTION 18

In the diagram BD is part of the line $y = \frac{6}{5}x - 3\frac{1}{2}$. Point C (1; 2) is given.



a) Give the co-ordinates of B (1)

$(0; -3\frac{1}{2})$

a) Find the size of θ to the nearest degree. (4)

$BD_m = \frac{6}{5}$ $\tan\hat{B} = \frac{6}{5}$ $\hat{B} = 50°$

$BC_m = \frac{-5\frac{1}{2}}{-1} = 5\frac{1}{2}$ $\tan \text{angle} = \frac{11}{2}$

$\text{angle} = 80°$

$\therefore \theta = 80° - 50°$

$= 30°$

b) If $\cos\hat{A} = \frac{1}{2}$, could a circle centre A, pass through B,C and D ? Give reasons where necessary. (3)

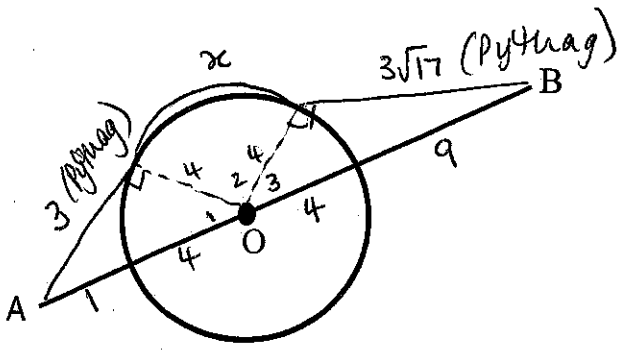
$\hat{A} = 60°$

$\therefore$ yes; angle at centre = 2x $\angle$ at circumf.

[8]

QUESTION 19

Circle centre O is a circular lake of radius 4km. One of the journey groups wants to walk from point A to point B which is 18km away, as the crow flies. If AO is 5km, find the shortest distance the hikers must travel to avoid the lake. [11]
Working on the diagram will be marked. Reasons should be given where needed.



distance = $3 + x + 3\sqrt{17}$.

$\hat{O}_1 = 36,86°$

$\hat{O}_3 = 72,08°$

$\therefore \hat{O}_2 = 180° - (36,86° + 72,08°)$

$= 71,06°$

$\frac{71,06}{360} = \frac{x}{2\pi(4)}$

$4,96 = x$

distance $= 3 + 4,96 + 3\sqrt{17}$

$= 20,33$ km.