GRADE 12 EXAMINATION
AUGUST 2016

Durban Girls' College
A Unique Educational Journey

# MATHEMATICS Paper 1

Time: 3 hours 150 marks
Examiners: DGC Mathematics Department

---

**PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY**

1. Read the questions carefully. Answer all the questions.

2. Number your answers exactly as the questions are numbered.

3. You may use an approved, non-programmable and non-graphical calculator, unless otherwise stated.

4. Round off your answers to **ONE DECIMAL PLACE**, where necessary, unless otherwise indicated.

5. All the necessary working details must be clearly shown.

6. It is in your own interest to write legibly and to present your work neatly.

7. Diagrams are not drawn to scale.

Name:_____ Teacher:_____

**SECTION A** **[78 marks)**

## QUESTION 1

Solve for $x$:

a) $6x^2 - 12x = -1$ (3)

b) $2^{x-1} + 2^{x+1} + 2^x = 28$ (3)

c) $\sqrt{12+x} + x = 0$ (5)

d) $x^2 - 3x - 28 \geq 0$ (4)

**[15]**

## QUESTION 2

Given the arithmetic sequence: 3; 7; 11; ….

a) 1) Calculate the sum of the first twenty terms. (3)

2) If the $n^{th}$ term is 191, determine the value of n. (3)

b) Calculate the following: $\sum_{n=0}^{15} 81\left(\frac{2}{3}\right)^n$ (4)

Show all your working, using an appropriate formula.

**[10]**

## QUESTION 3

Anna buys a flat for R640 000. She pays a 20% deposit and secures a loan for the balance, to be repaid in equal monthly instalments over a period of 20 years.

a) If interest is calculated at 8,5% p.a. compounded monthly, what will the monthly instalment be, if payments start 1 month after the loan is granted? (5)

b) After 8 years, Anna has some financial problems and is unable to pay her original instalments.

1) Calculate the balance outstanding on Anna's loan at the end of 8 years. (4)

2) If Anna is now (after 8 years) able to pay equal monthly instalments of only R7800 per month. Calculate how many payments Anna will be required to make in order to pay off the balance of the loan . (5)

**[14]**

**QUESTION 4**

a) Determine $f'(x)$ from first principles if $f(x) = 2 - 2x^2$ (5)

b) Determine the derivative of:
(Leave answers with positive exponents)

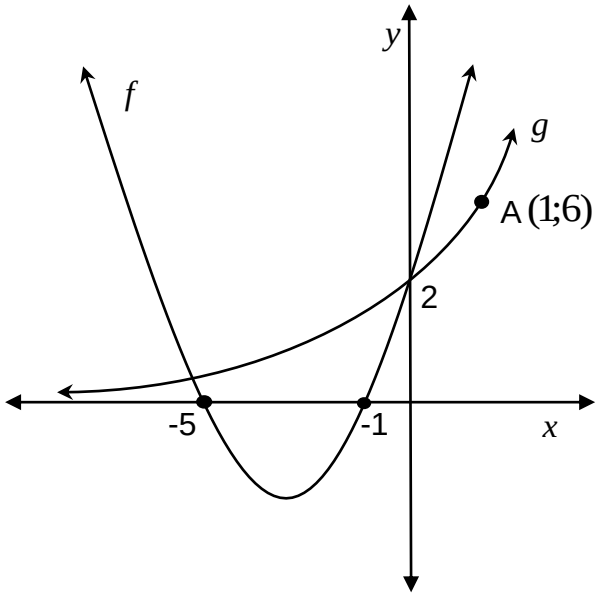
1) $y = \dfrac{2x^2 + 3x + 1}{x + 1}$ (3)

2) $g(x) = \dfrac{-2x + \sqrt{x}}{x^2}$ (4)

**[12]**

**QUESTION 5**

Given: $f(x) = ax^2 + bx + c$ and $g(x) = 2.3^x$
Both graphs have a y-intercept of 2.
Point A $(1;6)$ lies on $g(x)$




a) Write down:

1) The equation of the axis of symmetry of the parabola. (1)

2) The equation of the asymptote of the exponential curve. (1)

3) The values of $x$ for which $f(x)$ is decreasing. (1)

4) The values of $x$ for which $f(x) > 0$. (2)

5) The range of $g(x) + 1$. (2)

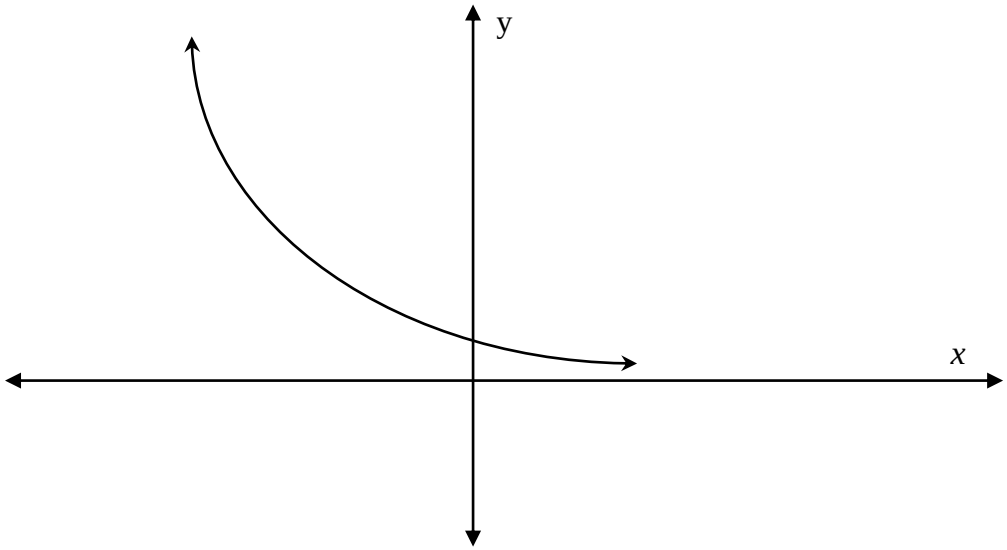
b) Determine the equation of $f(x)$ (5)

c) Show ALL working details to evaluate the following. (4) **[16]**

$$\sum_{k=0}^{2} g(k) - g(3) + f(0)$$

**QUESTION 6**

The graph of $f(x) = 2^{-x}$ is sketched below.




a) The graph of *f* is reflected about the $x$ – axis to form the graph of h; determine the equation of the graph of $h$ in the form $h(x) =$ ..... (1)

b) Determine the equation of the inverse of *f*, in the form $f^{-1}(x) =$ ..... (3)

c) Redraw $f(x)$ on your answer sheet and then, on the same set of axes, draw $f^{-1}(x)$. Indicate all intercepts with the axes. (2)

d) State the domain and range of graph $f^{-1}(x)$. (2)

e) Calculate the values of $x$ if $f(x) \geq 8$ (3)

**[11]**

**SECTION B** **[72 marks]**

**QUESTION 7**

a) Prove that: $\frac{x^4+1}{x^4}=\frac{1}{2}$ has no real roots (4)

b) $(x+1)(x-3)(x+c) = x^3-ax^2+ax+b$. Determine a, b and c (6)

**[10]**

**QUESTION 8**

The following sequence is given: 2 ; 12 ; 20 ; 26 ; …

a) Determine a formula for the $n^{th}$ term of the sequence, showing all calculations. (4)

b) If it is given that $T_n=-n^2+13n-10$, show by calculation that no term in the sequence will have an integral value greater than 32. (5)

**[9]**

**QUESTION 9**

a) Given: $f(x)=3x^2-x$. Determine the following in simplest form: $f\left(\frac{1}{x}\right)\times\frac{1}{x-3}$ (5)

b) If $\log_m 70 = x+1$ and $x=\log_m 10$. Calculate the numerical value of m. (4)

c) The world population P, T years after the year 2000, can be predicted by the equation:

$$P=6\times10^9\times(1{,}015)^T$$

Calculate in which year the population will be 12 billion. (1 billion = $10^9$) (4)

**[13]**

**QUESTION 10**

A small boy standing on the side of a still lake, picks up a flat stone and throws it horizontally across the water. The stone flies 30 metres through the air, then bounces several times across the surface of the lake, before finally coming to rest and sinking.
The distance $(d)$ in metres that the stone travels **after each bounce** $(n)$ on the water is given by the equation: $d_n=18\left(\frac{1}{2}\right)^{n-1}$

a) How far has the stone travelled at the 3rd bounce? (2)

b) After which bounce does the stone travel less than 56,25 cm for the first time? (4)

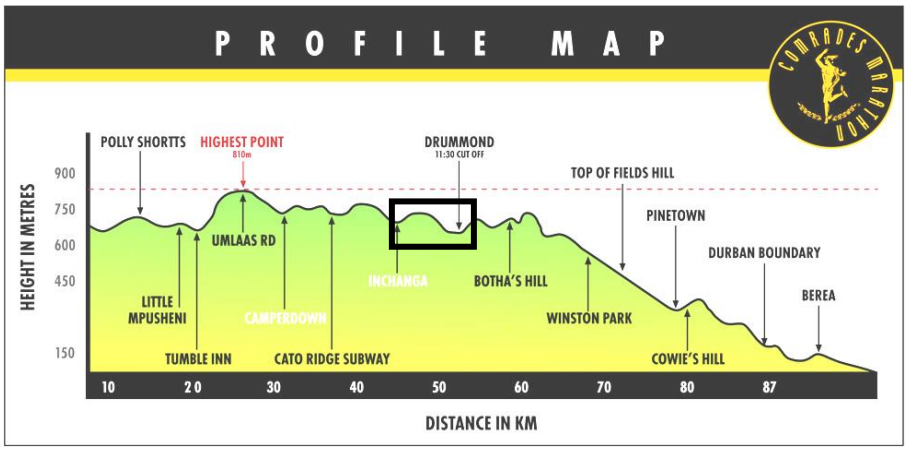
c) What is the total distance travelled by the stone before it sinks? (5)

**[11]**

**QUESTION 11**

a) This year in the Comrades Marathon the 2016 athletes ran from Pietermaritzburg to Durban. Below is a profile map of the their run.

**Profile map for the Comrades Marathon Down-Run**.

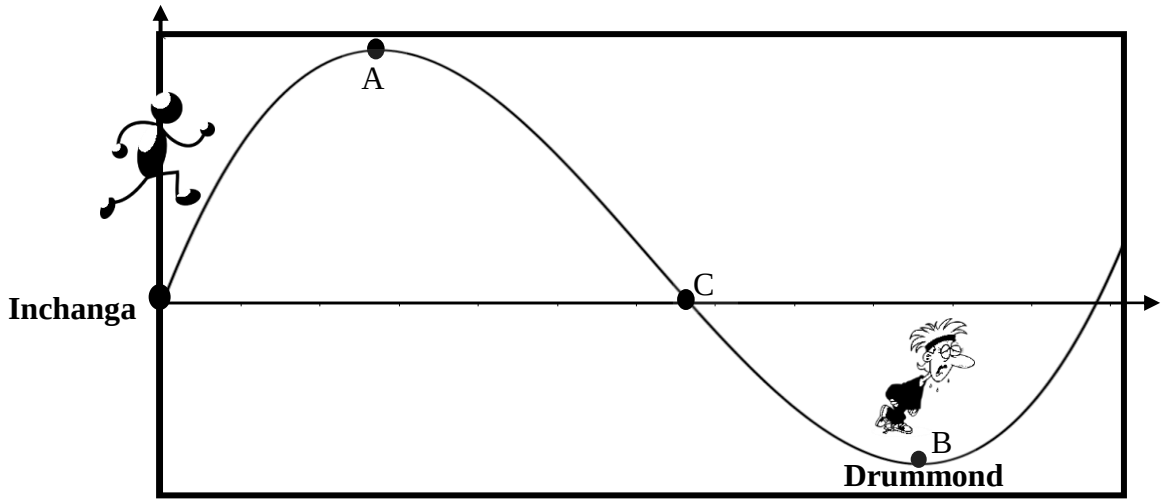



**Pietermaritzburg** → **Durban**

The name "down–run" is a deceptive title, as the race profile above shows that much of the race is uphill. The section from Inchanga to Drummond can be modelled by the equation:

$$y = x^3 - 18{,}5x^2 + 79x$$

where x is the horizontal distance in km from Inchanga, and y is the altitude above Inchanga in metres. This is depicted in the graph below.




Calculate the following, giving answers correct to the **nearest metre**:

1) How much higher, after leaving Inchanga, must the runners climb to before running downhill again? (i.e. how much higher is point A than Inchanga?) (4)

2) The downhill continues until Drummond.
What is the difference in altitude between point A and Drummond? (2)

3) At C, the runners are again at the same altitude as Inchanga.

i) If the horizontal distance from Inchanga to point C is 7km, what will the gradient of the road be at C? (2)

ii) Show by calculation whether the road at this point will be concave up or concave down. (3)

b) At a later point in the race, a cyclist decides to follow the runners.
The height above sea level is now defined by the equation $h(t) = 5t^3 - 65t^2 + 200t + 100$, where ***h*** is the height above sea level in metres and ***t*** is the time in minutes that has elapsed.

1) What is his rate of change of height after 4 minutes of cycling? (3)

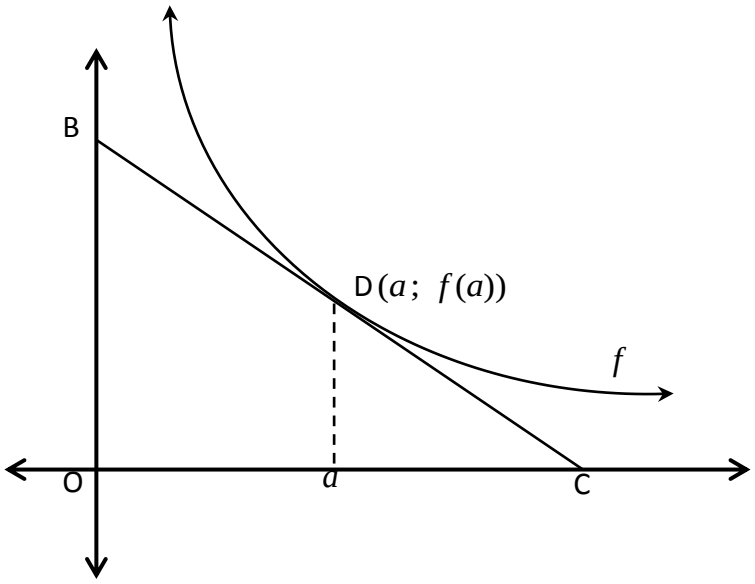
2) How many minutes after his journey has started, will the rate of change of height be a maximum? (3)

**[17]**

**QUESTION 12**

a) At what point (where $x > 0$) on the curve of $f(x) = 3x^3$, does the tangent form an angle of 45° with the positive $x$-axis? (6)

b) Given: $f(x) = \frac{1}{x}$.
The tangent at D, where $x = a$, is drawn.




Show that the equation of the tangent at D is: $x + a^2 y = 2a$. (6)

**[12]**