ME Q4 – 6

(39)

Section A

1a) $6x^2 - 12x = -1$ ✓m

$6x^2 - 12x + 1 = 0$

$x = \frac{12 \pm \sqrt{120}}{12}$

$x = 0{,}1$ ✓ or $x = 1{,}9$ ✓ (3)

b) $2^x \cdot 2^{-1} + 2^x \cdot 2^1 + 2^x = 28$ ✓m

$2^x(\frac{1}{2} + 2 + 1) = 28$

$2^x = 8$ ✓

$x = 3$ ✓ca (3)

c) $\sqrt{12+x} + x = 0$

$\sqrt{12+x} = -x$ ✓m

$12 + x = x^2$ ✓sq both sides

$x^2 - x - 12 = 0$ } ✓=0

$(x-4)(x+3) = 0$ }

$x = 4$ or $x = -3$ ✓ ✓ (5)

d) $x^2 - 3x - 28 \geq 0$

$(x-7)(x+4) \geq 0$

c.v. 7 and -4 ✓ ✓m

$\therefore x \leq -4$ or $x \geq 7$ (4)

[15]

Question 2

3; 7; 11; ...

a.1) $S_{20} = \frac{20}{2}[2(3) + 19(4)]$ ✓a ✓M

$= 820$ ✓CA (3)

2) $T_n = a + (n-1)d$

$191 = 3 + (n-1)(4)$ ✓M

$191 = 3 + 4n - 4$

$192 = 4n$

$48 = n$

$\therefore T_{48} = 191$ ✓a (3)

b) $\sum_{n=0}^{15} 81\left(\frac{2}{3}\right)^n = 81 + 54 + 36 \ldots$ ✓

$a = 81 \quad r = \frac{2}{3} \quad n = 16$

$S_{16} = \frac{81\left[\left(\frac{2}{3}\right)^{16} - 1\right]}{\frac{2}{3} - 1}$ ✓M

$= 242,6$ ✓CA (4)

[10]

Question 3

a) Deposit = R128 000 ✓

$512\,000 = \frac{x\left[1 - \left(1 + \frac{8,5\%}{12}\right)^{-240}\right]}{\frac{8,5\%}{12}}$ ✓CA ✓a

$R4443,25 = x$ ✓ (5)

b1) $F = \frac{4443,25\left[1 - \left(1 + \frac{8,5\%}{12}\right)^{-144}\right]}{\frac{8,5\%}{12}}$ ✓a ✓

$F = R400\,272,76$ ✓ (4)

2) $400\,272,76 = \frac{3800\left[1 - \left(1 + \frac{8,5\%}{12}\right)^{-n}\right]}{\frac{8,5\%}{12}}$ ✓a

$\left(1 + \frac{8,5\%}{12}\right)^{-n} = 0,2538 \ldots$ ✓

$\log_{\left(1 + \frac{8,5\%}{12}\right)} 0,2538 = -n$ ✓M

$-194,2 \ldots = -n$

$\therefore n = 195$ payments ✓CA (5)

[14]

Question 4

a) $f(x) = 2 - 2x^2$ ✓a

$f'(x) = \lim_{h \to 0} \frac{2 - 2x^2 - 4xh - 2h^2 - (2 - 2x^2)}{h}$ ✓m

$= \lim_{h \to 0} \frac{2 - 2x^2 - 4xh - 2h^2 - 2 + 2x^2}{h}$ ✓ca

$= \lim_{h \to 0} \frac{h(-4x - 2h)}{h}$ ✓m

$= \lim_{h \to 0} -4x - 2h$

$= -4x$ ✓ca (5)

$f(x) = 2 - 2x^2$

$f(x+h) = 2 - 2(x+h)^2$

$= 2 - 2(x^2 + 2xh + h^2)$

$= 2 - 2x^2 - 4xh - 2h^2$

(-1) FIN

b i) $y = \frac{(2x+1)(x+1)}{x+1}$ ✓

$y = 2x + 1$ ✓

$\frac{dy}{dx} = 2$ ✓ca (3)

(2) $g(x) = \frac{-2x}{x^2} + \frac{x^{\frac{1}{2}}}{x^2}$ ✓m

$= -2x^{-1} + x^{-\frac{3}{2}}$ ✓a

$g'(x) = 2x^{-2} - \frac{3}{2} x^{-\frac{5}{2}}$

$= \frac{2}{x^2} - \frac{3}{2x^{\frac{5}{2}}}$ ✓ca ✓ca (4)

[12]

Multiply only one / Badly correct / for that one / only (2)

Question 5

a) 1) $x = -3$ ✓ (1) (1)

2) $y = 0$ ✓ (1) (1)

3) $x < -3$ ✓ca (1) (1)

4) $x < -5$ ✓ or $x > -1$ ✓ (2) (2)

5) $y \in (1; \infty)$ ✓✓ (2) (2)

b) $y = a(x+5)(x+1)$ ✓m

a ✓ $2 = a(0+5)(0+1)$

$2 = 5a$ ✓ca

$\frac{2}{5} = a$ ✓ca

$\therefore y = \frac{2}{5}(x^2 + 6x + 5)$

$f(x) = \frac{2}{5}x^2 + \frac{12}{5}x + 2$ ✓ca (5)

c) $\sum_{k=0}^{2} g(k) = 2 \cdot 3^0 + 2 \cdot 3^1 + 2 \cdot 3^2$ ✓a

$= 2 + 6 + 18$

$= 26$

$\therefore \underset{g(k)}{26} - \underset{g(3)}{2 \cdot 3^3}$ ✓a $+ \underset{f(0)}{2}$ ✓a

$= -26$ ✓ca (4)

[16]

$0 = 25a - 5b + 2$ ✓a

$0 = a - b + 2$ ✓a

$b = a + 2$

$b = 2 \ldots$ ✓a

$0 = 25a - 5(a+2) + 2$ ✓m

$0 = 25a - 5a - 10 + 2$

$0 = 20a - 8$

$8 = 20a \ldots$

Question 6

a) $h(x) = 2^{-x}$

b) $y = 2^{-x}$

$x = 2^{-y}$

$\log_2 x = -y$

$\therefore y = -\log_2 x$

or

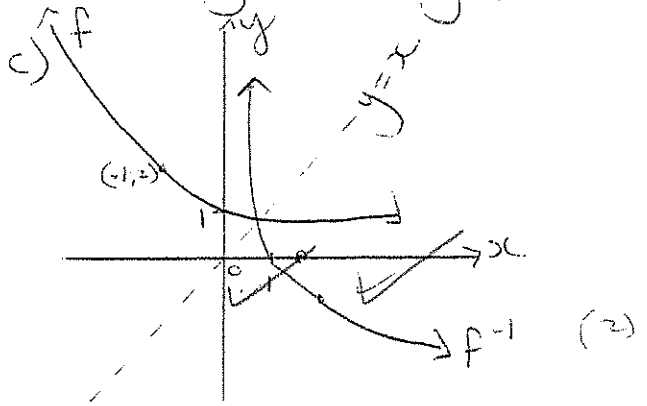
$y = \left(\frac{1}{2}\right)^x$

$x = \left(\frac{1}{2}\right)^y$

$\log_{\frac{1}{2}} x = y$

(3)

c)



(2)

d) d: $x > 0$ r: $y \in \mathbb{R}$ (2)

e) $f(x) \geq 8$

$2^{-x} \geq 8$

$2^{-x} \geq 2^3$

$-x \geq 3$

$x \leq -3$

(3)

[11]

SECTION B

Question 7

a) $\frac{x^4 + 1}{x^4} = \frac{1}{2}$

$2x^4 + 2 = x^4$

$x^4 = -2$

$x = \sqrt[4]{-2}$

$\sqrt[4]{\ } \geq 0$

$\therefore$ no real roots. (4)

b) $(x+1)(x-3)(x+c) = x^3 - ax^2 + ax + b$

$= (x^2 - 2x - 3)(x + c)$

$= x^3 + cx^2 - 2x^2 - 2cx - 3x - 3c$

$= x^3 + (c-2)x^2 + (-2c-3)x - 3c$

$\therefore c - 2 = -a$ $\quad -2c - 3 = a + b = -3c$

$c - 2 = -(-2c - 3)$

$c - 2 = 2c + 3$

$-5 = c$

$\therefore a = -2(-5) - 3$

$a = 7$

$b = -3(-5)$

$b = 15$

(6)

[10]

Question 8

a) 2; 12; 20; 26; ....

10 8 6 ✓M

-2 -2

$2a = -2$ $3a + b = 10$ $a + b + c = 2$

$a = -1$ ✓ $-3 + b = 10$ $-1 + 13 + c = 2$

$b = 13$ ✓ $c = -10$ ✓

(4)

$\therefore T_n = -n^2 + 13n - 10$

b) $-n^2 + 13n - 10 > 32$ ✓M

$-n^2 + 13n - 42 > 0$

$n^2 - 13n + 42 < 0$

$(n - 6)(n - 7) < 0$

CV 6 ; 7 ✓

+ c − c + ✓M

6 7

$\therefore 6 < n < 7$ ✓

impossible for $6 < n < 7$ ✓

$n \in \mathbb{N}$

$\therefore T_n < 32$

(5)

[9]

Question 9

a) $f(x) = 3x^2 - x$

$f\left(\frac{1}{x}\right) \times \frac{1}{x-3}$ ✓M

$= \left[3\left(\frac{1}{x}\right)^2 - \left(\frac{1}{x}\right)\right] \times \frac{1}{(x-3)}$

$= \left(\frac{3}{x^2} - \frac{1}{x}\right) \times \frac{1}{(x-3)}$

$= \frac{\left(3 - x\right)}{x^2} \times \frac{1}{(x-3)}$ ✓ ✓M −1 ✓

$= \frac{-1}{x^2}$ ✓

(5)

b) $\log_m 70 = x + 1$ ✓M and $x = \log_m 10$

$\log_m 70 = \log_m 10 + 1$

$\log_m 70 - \log_m 10 = 1$ ✓

$\log_m 7 = 1$ ✓

$m^1 = 7$

$\therefore m = 7$ ✓

(4)

c) ✓M $P = 6 \times 10^9 \times (1,015)^T$

$12 \times 10^9 = 6 \times 10^9 \times (1,015)^T$

$2 = (1,015)^T$

✓ $\log_{1,015} 2 = T$

$46,5\ldots = T$

46 years + 7 months

$\therefore$ 2047 ✓A

(4)

[13]

Question 10

a) $d_3 = 18\left(\frac{1}{2}\right)^{3-1}$ ✓m
$= 4,5\text{ m}$ ✓ (2)

b) $18\left(\frac{1}{2}\right)^{n-1} < 0,5625$ ✓m
$\left(\frac{1}{2}\right)^{n-1} < \frac{1}{32}$ ✓
$n-1 < 5$ ✓
$n < 6$. ✓ (4)

∴ after the 6th ✓ bounce.

c) $S_\infty = \frac{a}{1-r}$ ✓m
$= \frac{18}{1-\frac{1}{2}}$ ✓
$= 36$ ✓
∴ $30 + 36 = 66\text{ m}$. ✓ca (5)

[11]

Question 11

9) i) $y = x^3 - 18,5x^2 + 79x$ ✓m
$\frac{dy}{dx} = 3x^2 - 37x + 79 = 0$ ✓m
$x = 10\text{m}$ or $x = 3\text{m}$ ✓ca

∴ At A, $x = 3$

$y = (3)^3 - 18,5(3)^2 + 79(3)$ ✓m
$y = 98\text{ m}$ ✓ (4)

2) $y = (10)^3 - 18,5(10)^2 + 79(10)$
$= -60$ ✓

∴ diff in altitude is 158m. ✓ca (2)

3) i) $\frac{dy}{dx} = 3(7)^2 - 37(7)$ ✓m $+ 79$
$= -33$ ✓ (2)

ii) $y'' = 6x - 37$ ✓m
$= 6(7) - 37$
$= 5$ ✓
$y'' > 0$, ∴ at $x = 7$ the road will be concave up. ✓ (3)

b) 1) $h'(t) = 15t^2 - 130t + 200$ ✓
$h'(4) = 15(4)^2 - 130(4) + 200$ ✓m
$= -80$. ✓ (3)

2) $15t^2 - 130t + 200 = 0$ ✓m
$t = 6,7$ or $t = 2$ ✓
2 minutes ✓ (3)

Question 12

a) $m = \tan 45° = 1$ ✓

$\therefore f'(x) = 9x^2 = 1$ ✓M

$x^2 = \frac{1}{9}$

$x = \pm \frac{1}{3}$

but $x > 0$; $\therefore x = \frac{1}{3}$ ✓

$y = 3(\frac{1}{3})^3$ ✓M

$y = \frac{1}{9}$ $\therefore pt(\frac{1}{3}, \frac{1}{9})$ ✓ (6)

b) $f(x) = \frac{1}{x}$

$f(x) = x^{-1}$

$f'(x) = -x^{-2}$

$= -\frac{1}{x^2}$ ✓

$f'(a) = -\frac{1}{a^2}$ ✓M $\quad f(a) = \frac{1}{a}$ ✓M

$\therefore pt(a, \frac{1}{a})$

$\therefore y = mx + c.$

$y = -\frac{1}{a^2} \cdot x + c$ ✓

$\frac{1}{a} = -\frac{1}{a^2} \times a + c$

$\frac{2}{a} = c$ ✓

$\therefore y = -\frac{1}{a^2} x + \frac{2}{a}$ ✓

$a^2 y = -x + 2a$ (6)