Q1 – 4 39 Margie
Q5 – 7 40 Carmen
Q8 – 12 33 Tiffs
Q13 – Q15 38 Malin

Durban Girls' College
A Unique Educational Journey

GRADE 12 EXAMINATION

AUGUST 2016

MATHEMATICS Paper 2

Time: 3 hours 150 marks

Examiners: DGC Mathematics Department

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. Read the questions carefully. Answer all the questions.
2. Number your answers exactly as the questions are numbered.
3. You may use an approved, non-programmable and non-graphical calculator, unless otherwise stated.
4. Round off your answers to **ONE DECIMAL PLACE**, where necessary, unless otherwise indicated.
5. All the necessary working details must be clearly shown.
6. It is in your own interest to write legibly and to present your work neatly.
7. Diagrams are not drawn to scale.

Name:______________________ Teacher:______________________

SECTION A

QUESTION 1

Given the points R(1 ; 3), S(3 ; 7) and T(– 1; – 1)

Determine :

a) the gradient of RS

$m_{RS} = \frac{7-3}{3-1}$ ✓m $= 2$ ✓a

(2)

b) the equation of the line through T parallel to RS

$y = 2x + c$

$-1 = 2(-1) + c$ ✓m

$y = 2x + 1$ ✓ca

(2)

c) the equation of a straight line which is perpendicular to RS and bisects RS.

$\left(\frac{1+3}{2} ; \frac{3+7}{2}\right)$ $m = -\frac{1}{2}$ ✓a

$(2 ; 5)$ ✓a

$y = -\frac{1}{2}x + c$

$5 = -\frac{1}{2}(2) + c$ ✓m

$c = 6$

$y = -\frac{1}{2}x + 6$ ✓ca

(4)

d) the equation of a circle whose diameter is the line joining points R and S

$(3-2)^2 + (7-5)^2 = r^2$ ✓m

$\sqrt{5} = r$ ✓a

$(x-2)^2 + (y-5)^2 = 5$ ✓ca

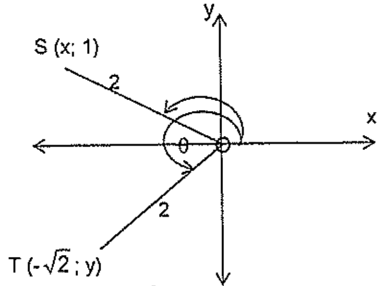
(3)

[11]

QUESTION 2

a) In the given diagram, S(x ; 1) and T($-\sqrt{2}$; y) are two points in the Cartesian plane such that OS = OT = 2 units.

Without a calculator and using trigonometric ratios determine the value of θ, i.e. $S\hat{O}T$



$\sin\alpha = \frac{1}{2}$ ✓m $\qquad \sin\alpha = \frac{1}{2}$ ✓m

$\alpha = 30^\circ$ ✓a $\qquad \alpha = 150^\circ$ ✓a

or

$\cos\beta = \frac{\sqrt{2}}{2}$ ✓m $\qquad \cos\beta = -\frac{\sqrt{2}}{2}$ ✓m

$\beta = 45^\circ$ ✓a $\qquad \beta = 225^\circ$ ✓a

$\theta = 30^\circ + 45^\circ$ $\qquad 225 - 150^\circ$

$= 75^\circ$ ✓ca $\qquad = 75^\circ$ ✓ca (5)

b) Without the use of a calculator, determine the value(s) of x if :

$\tan^2 x = \dfrac{\cos 140^\circ . \tan(-320^\circ)}{\sin 220^\circ}$ and $x \in [-90°;90°]$

$\tan^2 x = \dfrac{-\cos 40^\circ \text{ ✓a} . \tan 40^\circ \text{ ✓a}}{-\sin 40^\circ \text{ ✓a}}$

$\tan^2 x = 1$ ✓ca

$\tan x = \pm 1$ ✓a

✓ca

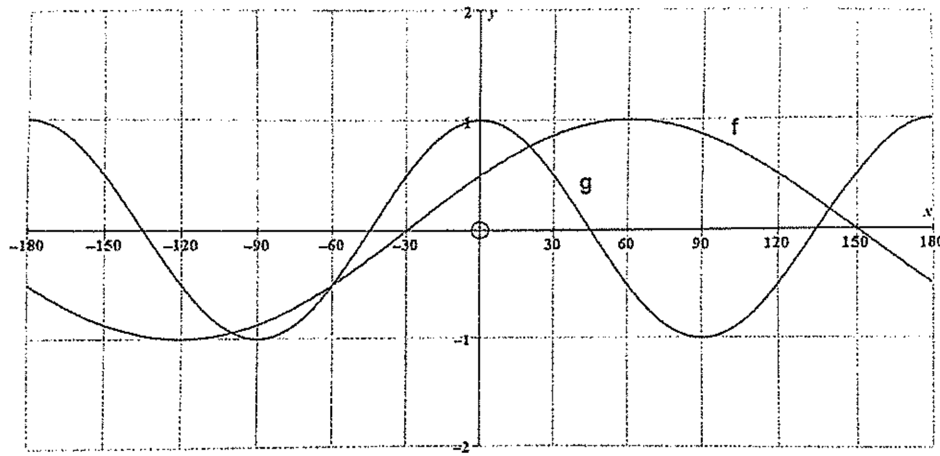
$x = 45^\circ + k.180^\circ$ $\qquad x = 135^\circ + k.180^\circ$

$x \in \{-45^\circ ; 45\}$ ✓a (7)

[12]

QUESTION 3

The graphs of $f(x) = \sin(x + a)$ and $g(x) = \cos bx$ for $x \in [-180°;180°]$ are drawn below.



a) Determine the values of a and b respectively.

$a = 30^\circ$ ✓a

$b = 2$ ✓ (2)

b) For which values of x will:

(1) $f(x) \geq 0$ for $x \in [-180°;180°]$

$x \in [-30^\circ ; 150^\circ]$ ✓a ✓a (2)

(2) $f(x).g(x) > 0$ for $x \in [-60°;60°]$

$x \in (-30^\circ ; 45^\circ)$ ✓a ; $x \in (-60^\circ ; -45^\circ)$ ✓a (2)

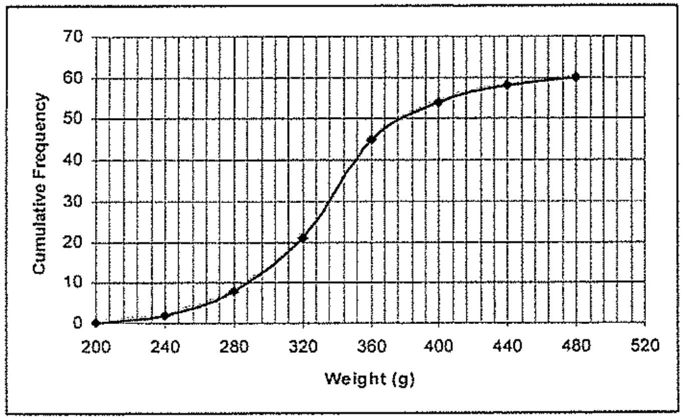
(c) Give the new equation if f is shifted 20° to the left.

$f(x) = \sin(x + 50^\circ)$ ✓a (1)

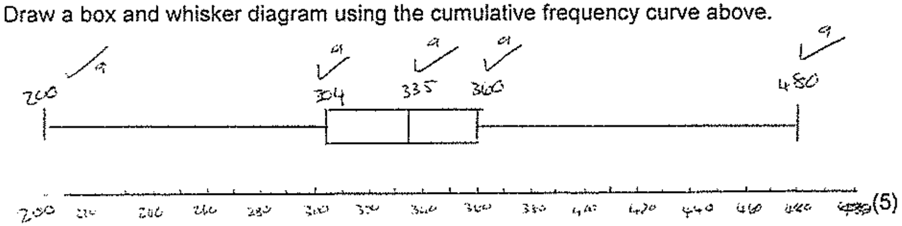
[7]

QUESTION 4

A sample of 60 baking potatoes was harvested on the farm in February 2016.
Each one was weighed, and the results are shown on this cumulative frequency curve.
Potatoes weighing more than 440 grams are usually good for exporting.

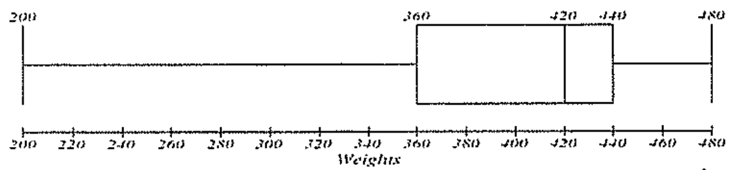


a) Draw a box and whisker diagram using the cumulative frequency curve above.



(5)

b) The following box and whisker diagram represents a sample of 60 baking potatoes that was harvested in February 2015. Each one was weighed, and the results are shown on the following box and whisker diagram.



1) What percentage of potatoes was approved for exporting? 25% (1)

2) Which year yielded higher quality potatoes and give 2 reasons for your answer? (3)

2015, ~~data skewed to the left~~ : more data grouped on RHS above 420 (median)

50% of potatoes are heavier than 420g

75% of potatoes weigh more than 360g

[9]

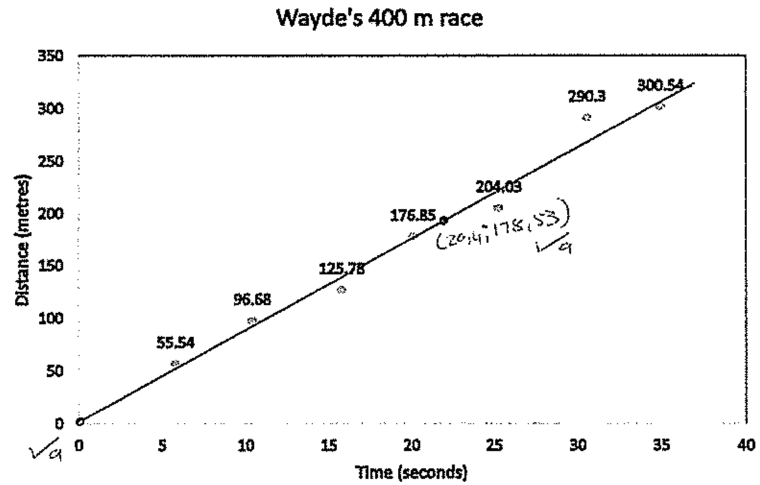
QUESTION 5

Wayde van Niekerk ran the 400m race in the 2016 Olympics in Rio.

At different times (in seconds) during the race, the distance (in metres), that he covered from the starting line is noted in the table below:

T (seconds)	5,8 s	10,4 s	15,8 s	20,1 s	25,3 s	30,6 s	35 s
D (metres)	55,54 m	96,68 m	125,78 m	176,85 m	204,03 m	290,30 m	300,54 m

A scatter plot is drawn from the information above:



a) Find the equation of the line of regression. Round your answers to 1 decimal place (2)

$y = 8{,}7x$

b) Draw the exact line of regression on the scatter plot above giving at least 2 points. (2)

(20,4 ; 178,53

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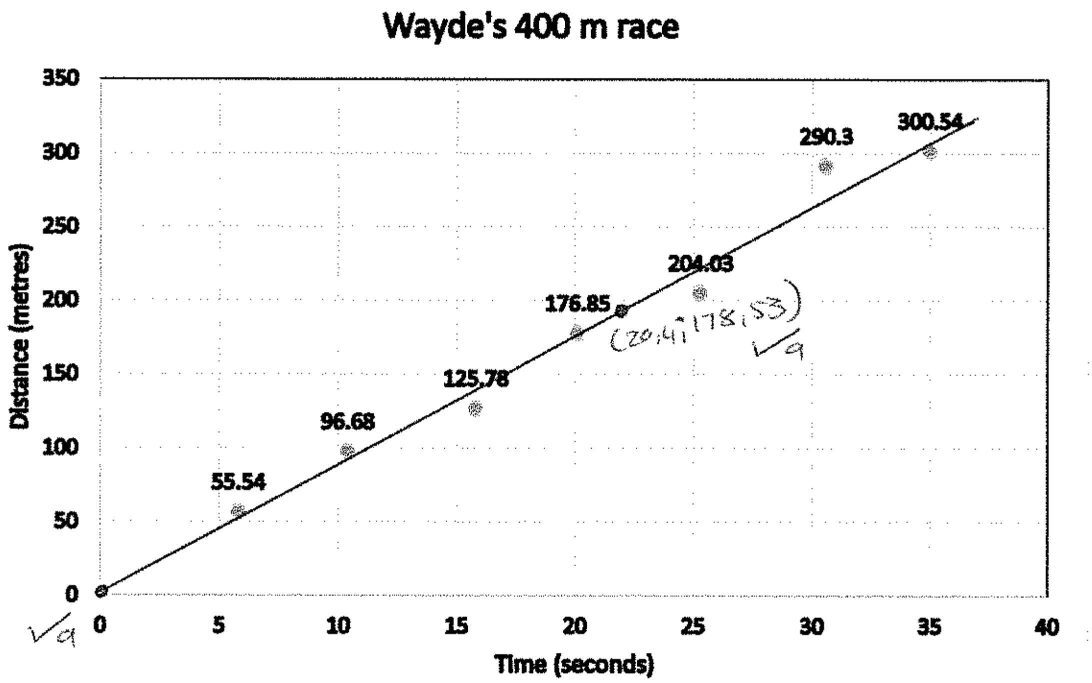


a) Find the equation of the line of regression. Round your answers to 1 decimal place (2)

$y = 8,7x$ √a √a

b) Draw the exact line of regression on the scatter plot above giving at least 2 points. (2)

(20,4 ; 178,53

c) From the information above, what is the approximate average speed in m/s that Wayde ran? (1)

8,7 m/s ✓ca ~~8,4 m/s~~

d) Between which times did Wayde increase his speed the most? (1)

25 and 30 seconds ✓a

e) Would Wayde have finished the race at 40 seconds? Explain your answer using the data above. (3)

$y = 8,7(40)$ ✓m

$y = 348$ ✓a ∴ no ✓a

OR $400 = 8,7x$

$45,7 = x$

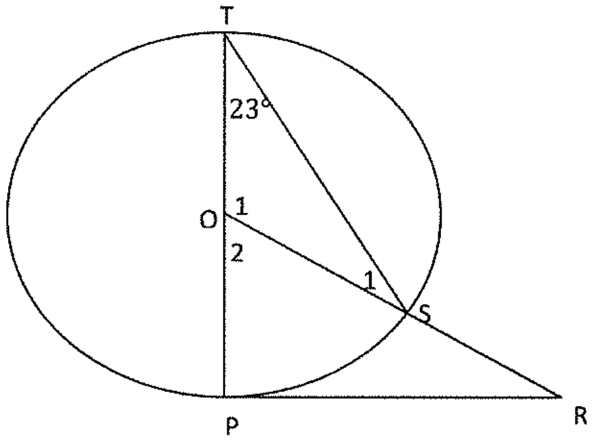
f) Is your answer reliable or not and give a reason for your answer. (2)

not reliable ✓a, extrapolation ✓a

[11]

QUESTION 6

a) O is the centre of the circle. (5)



TOP is the diameter and

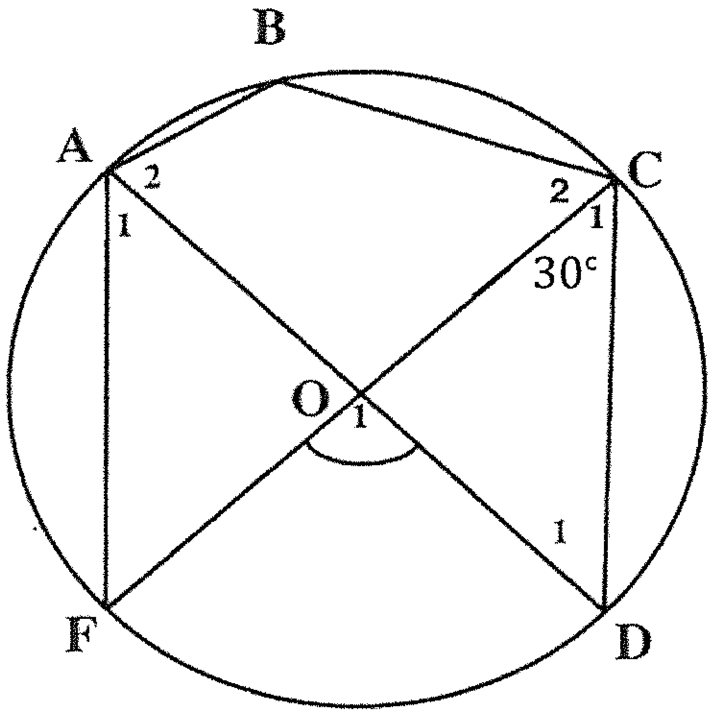
PR is a tangent.

If $\hat{T} = 23°$, determine the size of $\hat{R}$ by completing the following table:

Statements	**Reasons**
$TO = OS$	radii ✓a
$\hat{S}_1 = 23°$	isosceles △ ✓a
$\hat{O}_2 = 46°$	Ext ∠ of △ ✓a
$O\hat{P}R = 90°$	tan ⊥ radius ✓a
$\hat{R} = 44°$	int. ∠'s of △ ✓a

b) In the given diagram, O is the centre of the circle.

$\hat{C}_1 = 30^0$



1) Name, with reasons, **three** other angles each equal to 30^0.

$\hat{A} = 30°$ ($\angle$'s in same segment) ✓a

$OC = OD = OF = OA$ (radii) ✓a

$\hat{F} = 30°$ (Isosceles $\triangle$) ✓a

$\hat{D}_1 = 30°$ (isosceles $\triangle$) ✓a

(4)

2) Calculate, with reasons, the following angles:

(i) $\hat{B}$ = 150° (opp $\angle$'s of cyclic quad) ✓a (1)

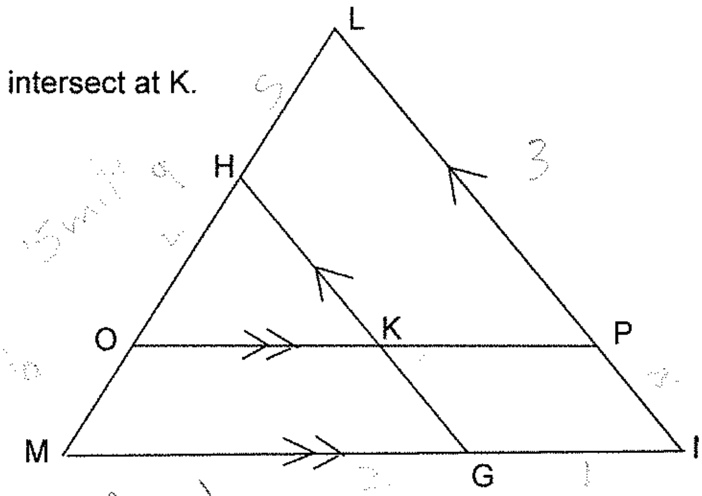
(ii) $\hat{O}_1$ = 60° ($\angle$ at centre) ✓a (1)

(ext $\angle\triangle$)

c) In ΔLMN, HG // LN and OP // MN. HG and OP intersect at K.

$\frac{LP}{PN}=\frac{3}{2}$ and $\frac{MG}{GN}=\frac{2}{1}$

LM = 15 units.



Determine:

1) The length of LO.

$\frac{LO}{LM}=\frac{LP}{LN}$ (Line ∥ to one side of △) ✓a

$\frac{LO}{15}=\frac{3}{5}$ ✓m

$\therefore LO=9$ ✓ca

(3)

2) The length of LH

$\frac{LH}{LM}=\frac{NG}{NM}$ (line ∥ to one side of △) ✓a

$\frac{LH}{15}=\frac{1}{3}$ ✓m

$LH=5$ ✓ca

(3)

3) The value of $\frac{GK}{KH}$

$OH=9-5=4$ ✓ca

$OM=15-9=6$ ✓ca

$\frac{GK}{KH}=\frac{MO}{OH}$ (line ∥ to one side of △) ✓a

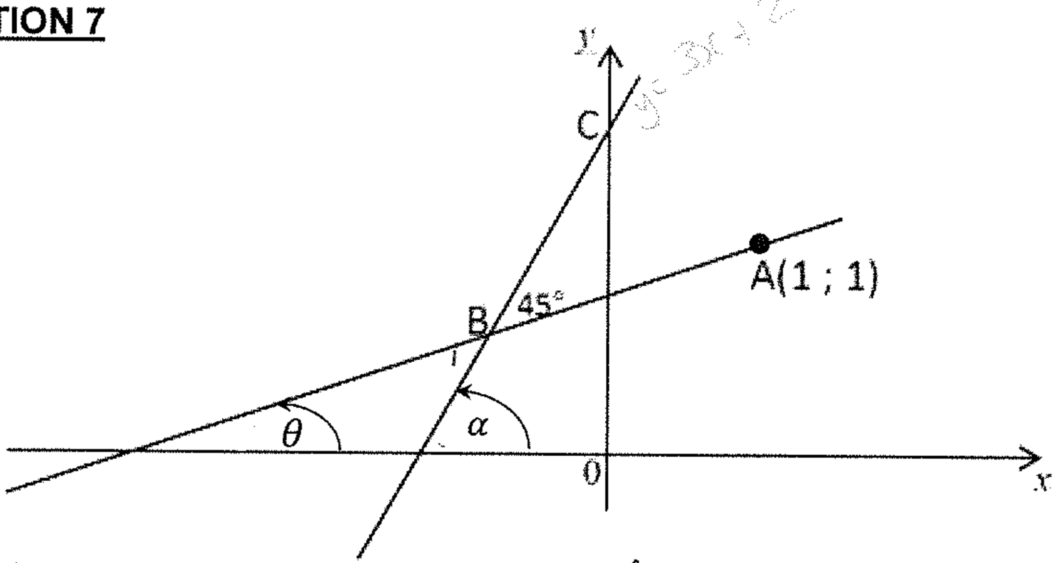
$\frac{6}{4}=\frac{3}{2}$ ✓ca

(4)

SECTION A = 71 MARKS **[21]**

SECTION B

QUESTION 7



In the given sketch A(1 ; 1) is a point on AB and $A\hat{B}C = 45°$.

Points B and C lie on the line $y = 3x + 2$

Determine the the equation of AB. (Giving reasons where necessary)

$\tan\alpha = 3$ ✓m $\quad \hat{B}_1 = 45°$ (vert. opp ∠'s) ✓a

$\alpha = 71,57°$ ✓a

$\theta = 71,57° - 45°$ (Ext ∠ of △)

$\theta = 26,57°$ ✓ca

$\tan\theta = m$

$\tan 26,57° = m$

$\frac{1}{2} = m$ ✓ca

$y = \frac{1}{2}x + c$

$1 = \frac{1}{2}(1) + c$ ✓m

$c = \frac{1}{2}$ ✓ca

[8]

$y = \frac{1}{2}x + \frac{1}{2}$ ✓ca

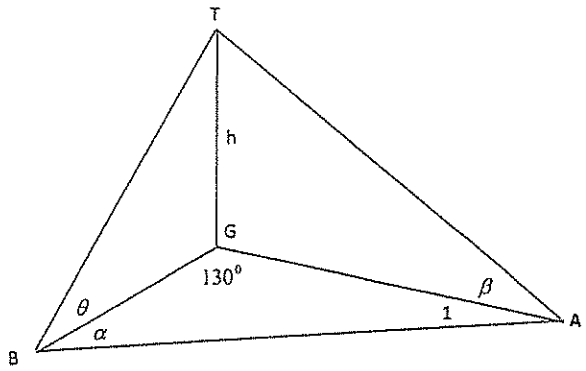
QUESTION 12

A, B and G are points on horizontal ground.

TG is a vertical tower.

The angles of elevation of T from B and A are θ and β respectively.

$G\hat{B}A = \alpha$ and TG = h and $B\hat{G}A = 130^0$



$180 - 130 - \alpha$

a) Find $\hat{A}_1$ in terms of ∝ (giving reasons where necessary) (1)

$50^\circ - \alpha$ ✓ (int ∠s of △)

b) Find BG in terms of h and θ. (2)

sine rule

$\tan\theta = \frac{h}{BG}$ ✓

$BG = \frac{h}{\tan\theta}$ ✓

$\frac{h\cos\theta}{\sin\theta}$ simplify

c) Prove that $\sin\alpha = \dfrac{\tan\theta.\sin(50^0-\alpha)}{\tan\beta}$ (5)

$\frac{\sin\alpha}{GA} = \frac{\sin(50-\alpha)}{BG}$ ✓

$\tan\beta = \frac{TG}{GA}$

$\frac{\sin\alpha}{\frac{h}{\tan\beta}} = \frac{\sin(50^\circ-\alpha)}{\frac{h}{\tan\theta}}$ ✓

$\tan\beta = \frac{h}{GA}$ ✓

$GA = \frac{h}{\tan\beta}$ ✓

$\sin\alpha = \frac{h}{\tan\beta} \cdot \sin(50^\circ - \alpha) \cdot \frac{\tan\theta}{h}$ ✓

$\sin\alpha = \frac{\tan\theta(\sin(50^\circ-\alpha))}{\tan\beta}$

[8]

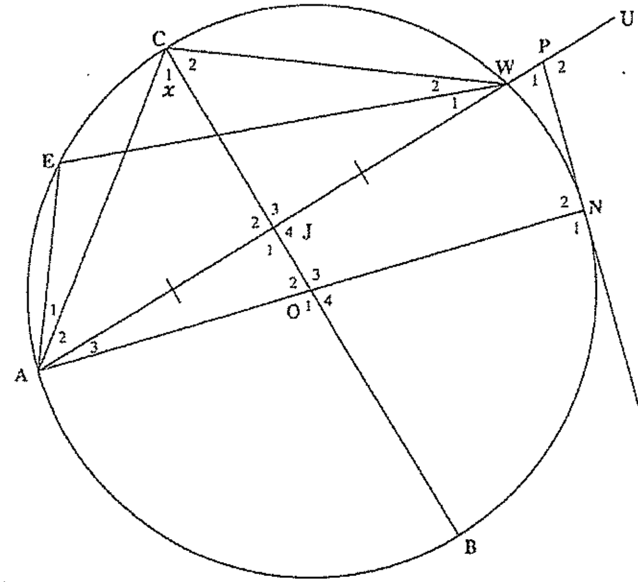
QUESTION 13

In the diagram below, O is the centre of a circle. A, E, C, W and N are points on the circle.

Diameter BC is drawn to bisect chord AW at J. AN is a diameter of the circle.

A tangent is drawn to the circle at N. Chord AW is extended to U and meets the tangent at P.

Let $\hat{C}_1 = x$



a) Prove that ONPJ is a cyclic quadrilateral. (4)

$\hat{J}_4 = 90^\circ$ (midp. chord) ✓

$\hat{N}_2 = 90^\circ$ (tangent ⊥ radius) ✓

$\therefore \hat{J}_4 + \hat{N}_2 = 180^\circ$ ✓

$\therefore$ ONPJ is cyclic quad (opp ∠s are suppl.) ✓

QUESTION 13

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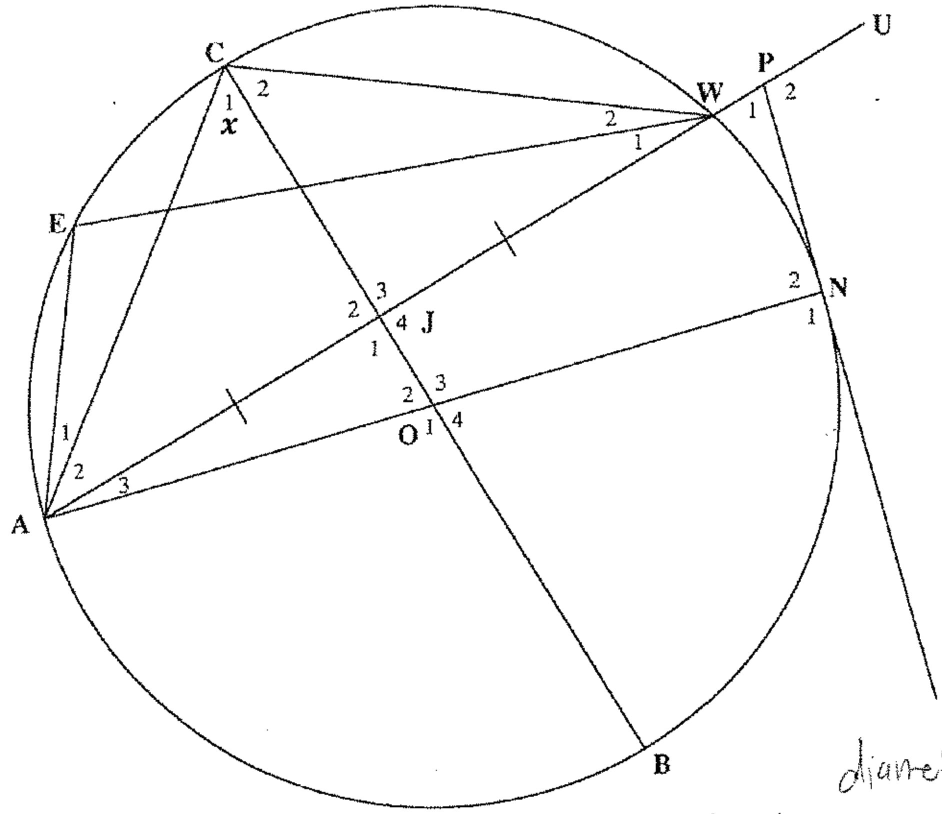


diameter bisects

had to say or ~~midpt~~ (Line from centre bisects chord)

a) Prove that ONPJ is a cyclic quadrilateral.

$\hat{J}_4 = 90°$ (midp. chord) ✓

$\hat{N}_2 = 90°$ (tangent ⊥ radius) ✓

$\therefore \hat{J}_4 + \hat{N}_2 = 180°$ ✓

$\therefore$ ONPJ is cyclic quad (opp ∠'s are suppl.) ✓

(4)

• they can't say line bisects AW, what line? What theorem? any line can bisect another line what will make it ⊥??

b) Prove that OC bisects $A\hat{C}W$.

$CO = OA$ (radii) ✓a

$\hat{C}_1 = \hat{A}_2 + \hat{A}_3$ (isos $\triangle$) ✓a

$\hat{O}_2 = 180° - 2x$ (int ∠'s of $\triangle$) ✓a

$\hat{W}_1 + \hat{W}_2 = 90° - x$ (∠ at centre) ✓a

$\hat{C}_2 = 180° - 90° - (90° - x)$ (int ∠'s of $\triangle$) ✓a

$\hat{C}_2 = x$

$\therefore \hat{C}_1 = \hat{C}_2$

(5)

c) Express $\hat{P}_1$ in terms of x, giving reasons.

$\hat{O}_3 = 180° - (180° - 2x)$ ✓a (∠'s on str. line)

$\hat{O}_3 = 2x$ ✓a

✓a

$\therefore \hat{P}_1 = 180° - 2x$ (opp ∠'s of cyclic quad)

(3)

d) Prove that $\hat{P}_2 = \hat{E}$.

✓a

$\hat{C}_1 + \hat{C}_2 = \hat{E}$ (∠'s in same segment)

$\hat{E} = 2x$ ✓a

$\hat{P}_2 = 180° - (180° - 2x)$ (∠'s on str line) ✓a

$\hat{P}_2 = 2x$ ✓a

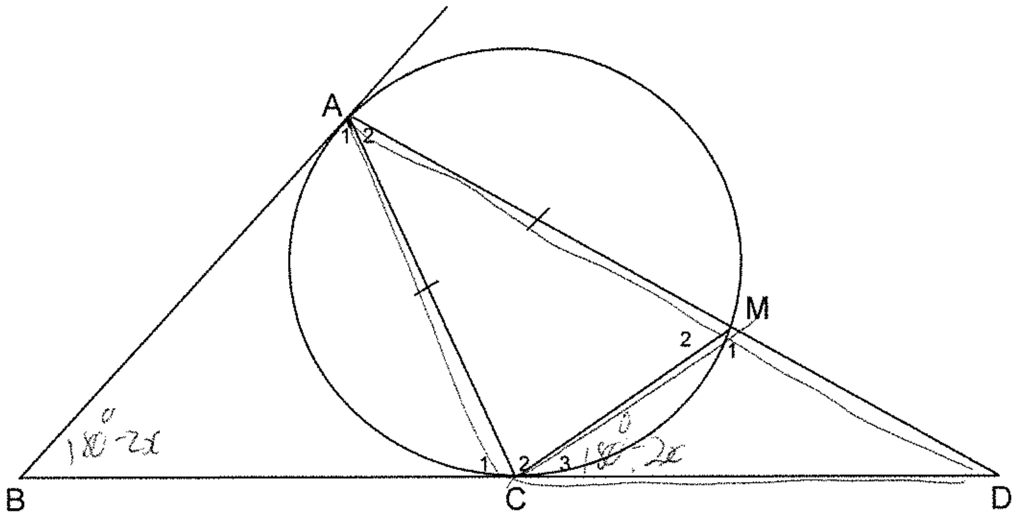
$\therefore \hat{P}_2 = \hat{E}$

(4)

[16]

$\triangle ABC$ and $\triangle DMC$:

QUESTION 14



BA and BD are tangents. AC = AM.

Prove:

a) $CD^2 = DM.AD$

In $\triangle$ CDM and $\triangle$ CDA ✓a

$\hat{D}$ is common ✓a

$\hat{C}_3 = \hat{A}_2$ (tan chord) ✓a

$\therefore \hat{M}_1 = A\hat{C}D$ (int ∠s of $\triangle$)

$\therefore \triangle CDM \parallel\parallel \triangle ADC$ (AAA) ✓a

$\therefore \frac{CD}{AD} = \frac{DM}{DC}$ ($\parallel\parallel$ $\triangle$'s) ✓a

$\therefore DC^2 = DM.AD$ (5)

b) $\frac{DM}{MA} = \frac{DC}{CB}$

$\hat{A}_1 = \hat{M}_2$ (tan chord) ✓a

$\hat{C}_2 = \hat{M}_2$ (∠s op = sides) ✓a

$\therefore \hat{A}_1 = \hat{C}_2$

$\therefore AB \parallel MC$ (alt ∠s are =) ✓a

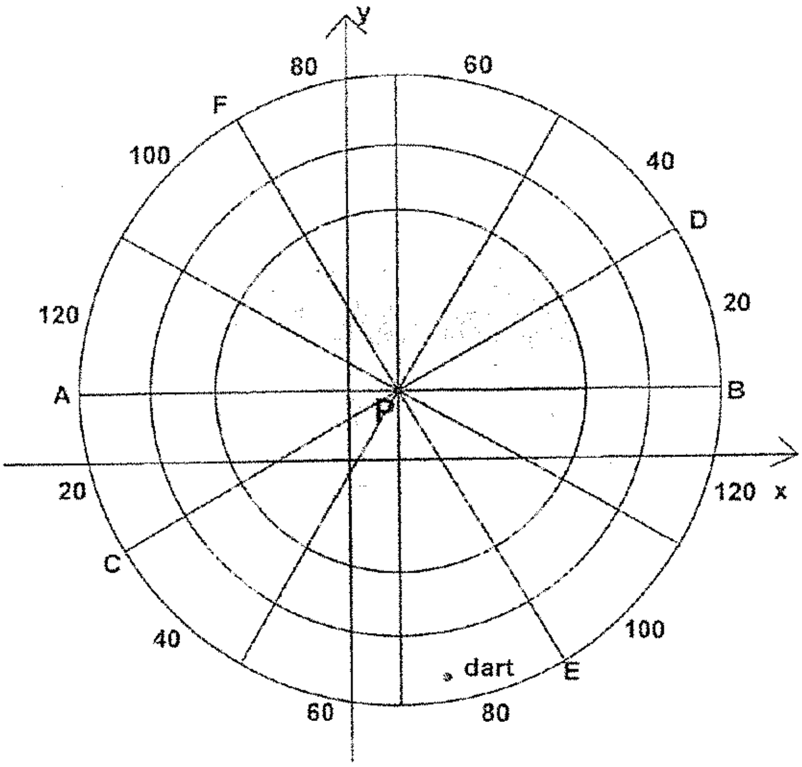
$\therefore \frac{DM}{MA} = \frac{DC}{CB}$ (line $\parallel$ to one side of $\triangle$) ✓a

(4)

[9]

R forgetting reasons $\parallel\parallel$ $\triangle$s AAA

QUESTION 15



A dartboard consists of three concentric circles, with diameters dividing it into 12 equal sectors, as shown in the first diagram.
The radii of the circles are 10 cm, 8 cm and 6 cm respectively.
The diameter AB is parallel to the x-axis.

Diagonals CD and EF have equations $y = \frac{1}{\sqrt{3}}x + 1$ and $y = -\sqrt{3}x + 5$ respectively.

a) Show that the coordinates of P is $(\sqrt{3}; 2)$.

$\frac{x}{\sqrt{3}} + 1 = -\sqrt{3}x + 5$ ✓m $\quad y = \frac{\sqrt{3}}{\sqrt{3}} + 1$ ✓a

$x + \sqrt{3} = -3x + 5\sqrt{3}$ ✓a $\quad y = 2$

$x = \sqrt{3}$ (3)

b) Hence determine the equation of the second circle.

$(x - \sqrt{3})^2 + (y - 2)^2 = 8^2$

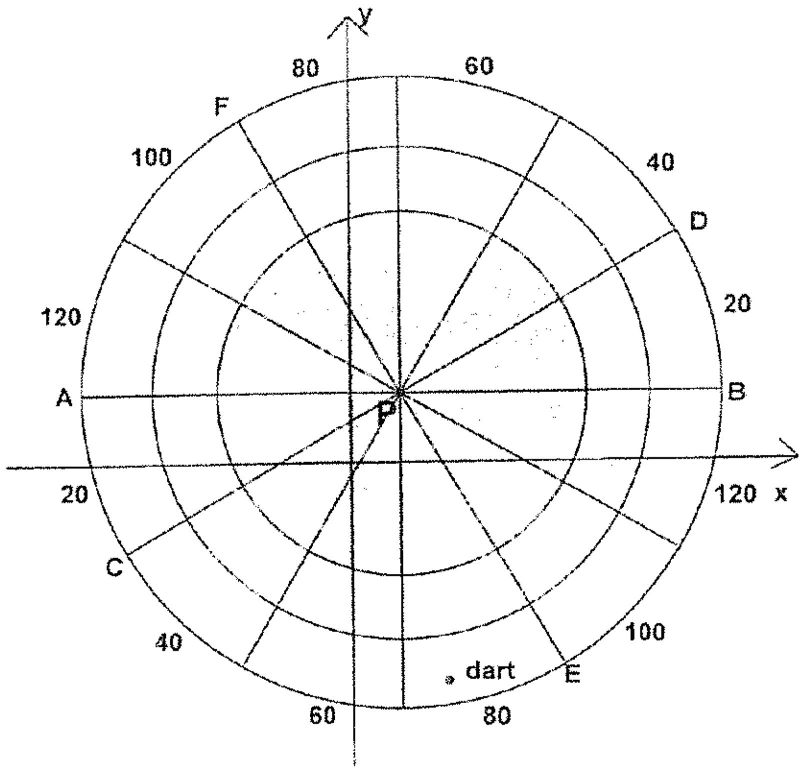
✓m ✓ca

(2)

c) Write down the equation of the diameter AB.

$y = 2$ ✓a (1)

* Show means cannot use prove paint!!



d) Scores for a particular sector are shown on the outer ring of the dartboard.
If a dart lands in the inner ring of that sector, the score remains unchanged.
If a dart lands between the inner and outer rings of that sector, i.e. in the middle ring, the score is doubled.
If a dart lands in the outer ring of that sector, the score is halved.
For example, the dart shown in the diagram scores 40 points.

1) In which ring is the point R(8 ; 6) located ? Explain.

$\sqrt{(8-\sqrt{3})^2+(6-2)^2} = 7,44$ ✓m ✓a

ring 2 ✓a

(3)

2) In which sector is the point R(8 ; 6) located ? Explain.

$m_{PR} = \frac{6-2}{8-\sqrt{3}} = 0,64$ ✓
$\tan\theta = 0,64$
$\theta = 32,54$
$32,54° > 30°$ ✓

$y = \frac{1}{\sqrt{3}}(8)+1$ $(8-\sqrt{3})^2+(y-2)^2 = 64$

$y = 5,62$ ✓a $y = 6,97$ ✓a

sector 40 ✓ca

(3)

3) Determine the score obtained by a dart landing at R.

80 ✓ca

(1)

SECTION B = 79 MARKS **[13]**

writing on scrap paper
ask for another answer booklet
never loose paper!!