



# HERZLIA SENIOR HIGH SCHOOL

*"If you will it, it is no legend"*

**GRADE 12**

**MATHEMATICS PAPER 1  
WEDNESDAY 31<sup>ST</sup> AUGUST 2016**

**MARKS: 150**

**TIME: 3 HOURS**

**This question paper consists of 12 pages and 1 information sheet.**

**INSTRUCTIONS AND INFORMATION**

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams and graphs that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
9. An INFORMATION SHEET, with formulae, is included.
10. Write neatly and legibly.

**QUESTION 1**1.1 Solve for  $x$ :

1.1.1  $4x^2 - 25 = 0$  (2)

1.1.2  $(2x - 5)(x + 3) = 1$  (correct to TWO decimal places) (4)

1.1.3  $2x^{\frac{-5}{3}} = 64$  (3)

1.1.4  $-3(x - 2)(x + 4) \geq 0$  (3)

1.2 Solve simultaneously for  $x$  and  $y$ :

$2x - y + 1 = 0$

$x^2 - 3x - 4 - y = y^2$  (6)

1.3 Given:  $f(x) = x + 1$  and  $g(x) = \frac{-4}{x - 3}$ 

1.3.1 Solve for  $x$  if  $f(x) = g(x)$ . (3)

1.3.2 State whether the graph of  $f$  is a tangent to the graph of  $g$  when  $f(x) = g(x)$ . Motivate your answer. (1)

1.4 Simplify:

$$\frac{3^{2015} + 3^{2013}}{9^{1006}}$$
 (3)

**[25]**

**QUESTION 2**

The first four terms of a quadratic number pattern are  $-1 ; 2 ; 9 ; 20$ .

2.1 Determine the general term of the quadratic number pattern. (4)

2.2 Calculate the value of the 48<sup>th</sup> term of the quadratic number pattern. (1)

2.3 Show that the sum of the first differences of this quadratic number pattern can be given by  $S_n = 2n^2 + n$ . (3)

2.4 Determine  $n$  if the sum of the first differences in QUESTION 2.3 is 9 591. (3)

**[11]**

**QUESTION 3**

3.1 Given the arithmetic series:  $6 + w + 20 + \dots$

3.1.1 Determine the value of  $w$ . (1)

3.1.2 Calculate the sum of the first 20 terms of the series. (2)

3.1.3 Write the series in QUESTION 3.1.2 in sigma notation. (2)

3.2  $6 - x$ ,  $5$  and  $\sqrt{4x + 12}$  are the first three terms of an arithmetic sequence.

3.2.1 Determine the value of  $x$ . (5)

3.2.2 Calculate the value of the 10<sup>th</sup> term of this arithmetic sequence. (2)

3.3 Calculate  $\sum_{p=4}^{21} (-3)^p$ . (3)

3.4 Given the geometric series:  $(x - 2) + (x^2 - 4) + \dots$

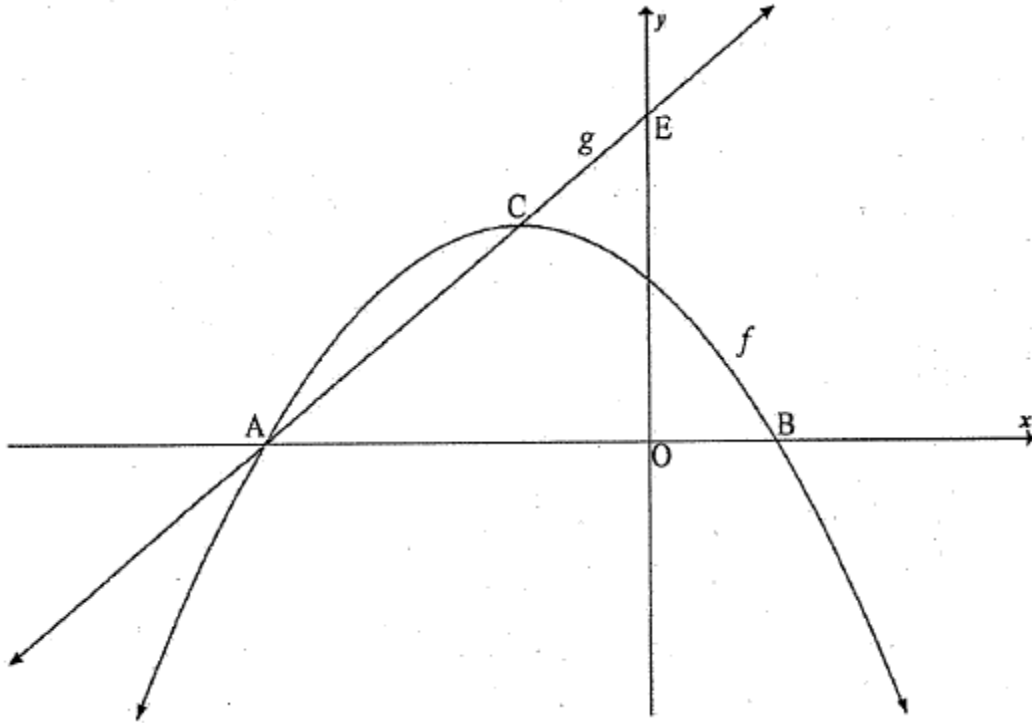
3.4.1 Determine the values of  $x$  for which the series converges. (3)

3.4.2 If  $x = -\frac{3}{2}$ , calculate the sum to infinity of the given series. (2)

**[20]**

## QUESTION 4

The sketch below shows the graphs of  $f(x) = -x^2 - 2x + 3$  and  $g(x) = mx + q$ . Graph  $f$  has  $x$ -intercepts at A and B(1;0) and a turning point at C. The straight line  $g$ , passing through A and C, cuts the  $y$ -axis at E.



- 4.1 Write down the coordinates of the  $y$ -intercept of  $f$ . (1)
- 4.2 Determine the coordinates of C. (3)
- 4.3 Write down the coordinates of A. (1)
- 4.4 Calculate the length of CE. (4)
- 4.5 Determine the value of  $k$  if  $h(x) = 2x + k$  is a tangent to the graph of  $f$ . (5)
- 4.6 Determine the equation of  $g^{-1}$ , the inverse of  $g$ , in the form  $y = \dots\dots\dots$  (2)
- 4.7 For which value(s) of  $x$  is  $g(x) \geq g^{-1}(x)$ ? (3)

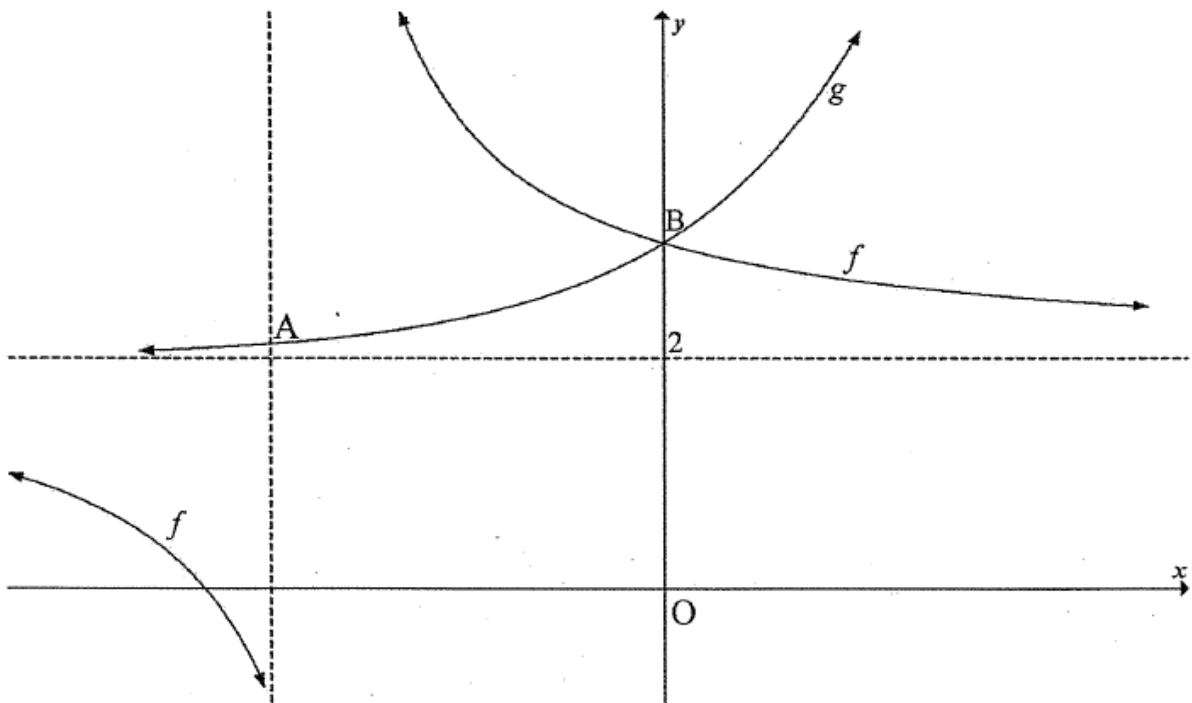
[19]

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## QUESTION 5

The sketch below shows the graphs of  $f(x) = \frac{3}{x-p} + q$  and  $g(x) = 2^x + r$ .

- $g$  intersects the vertical asymptote of  $f$  at A.
- B is the common  $y$ -intercept of  $f$  and  $g$ .
- $y = 2$  is the common horizontal asymptote of  $f$  and  $g$ .



- 5.1 Write down the value of  $r$ . (1)
- 5.2 Determine the coordinates of B. (1)
- 5.3 Determine the value of  $p$ . (2)
- 5.4 Determine the coordinates of A. (2)
- 5.5 For which value(s) of  $x$  is  $f(x) \geq g(x)$ ? (2)
- 5.6 If  $h(x) = g(x) - 2$ , sketch the graphs of  $h(x)$  and  $h^{-1}(x)$  on the same set of axes in your ANSWER BOOK. Clearly indicate ALL intercepts with the axes. (4)

**QUESTION 6**

- 6.1 How many years will it take for an article to depreciate to half its value according to the reducing balance method at 7% per annum? (4)
- 6.2 A farmer bought a tractor for R $x$  on 1 April 2016.
- He will trade in this tractor when he replaces it with a similar one in 5 years' time on 1 April 2021.
  - The tractor depreciates by 20% p.a. according to the reducing-balance method.
  - The price of a similar tractor increases by 18% annually.
  - The farmer calculated that if he deposited R8 000 per month into a sinking fund, which paid interest at 10% p.a. compounded monthly, he would have enough money to cover the replacement cost of the tractor. He made the first deposit in this fund on 30 April 2016 and will continue to do so at the end of every month until 31 March 2021.
- 6.2.1 Determine, in terms of  $x$ , what the book value of the current tractor will be on 1 April 2021 (that is, 5 years after it was bought). Give your answer correct to THREE decimal places. (1)
- 6.2.2 Determine, in terms of  $x$ , what the price of a similar new tractor will be on 1 April 2021. Give your answer correct to THREE decimal places. (1)
- 6.2.3 Calculate the amount accumulated in the sinking fund on 1 April 2021. (4)
- 6.2.4 Calculate the value of  $x$ , the price of the current tractor. Round off your answer to the nearest thousand. (3)

**[13]**

**QUESTION 7**

7.1 Given:  $f(x) = 3x^2 - 5$

Determine  $f'(x)$  from first principles. (5)

7.2 Determine  $\frac{dy}{dx}$  if:

7.2.1  $y = 4x^6 + \frac{5}{x^3}$  (3)

7.2.2  $y = (\sqrt{x} - x^2)^2$  (4)

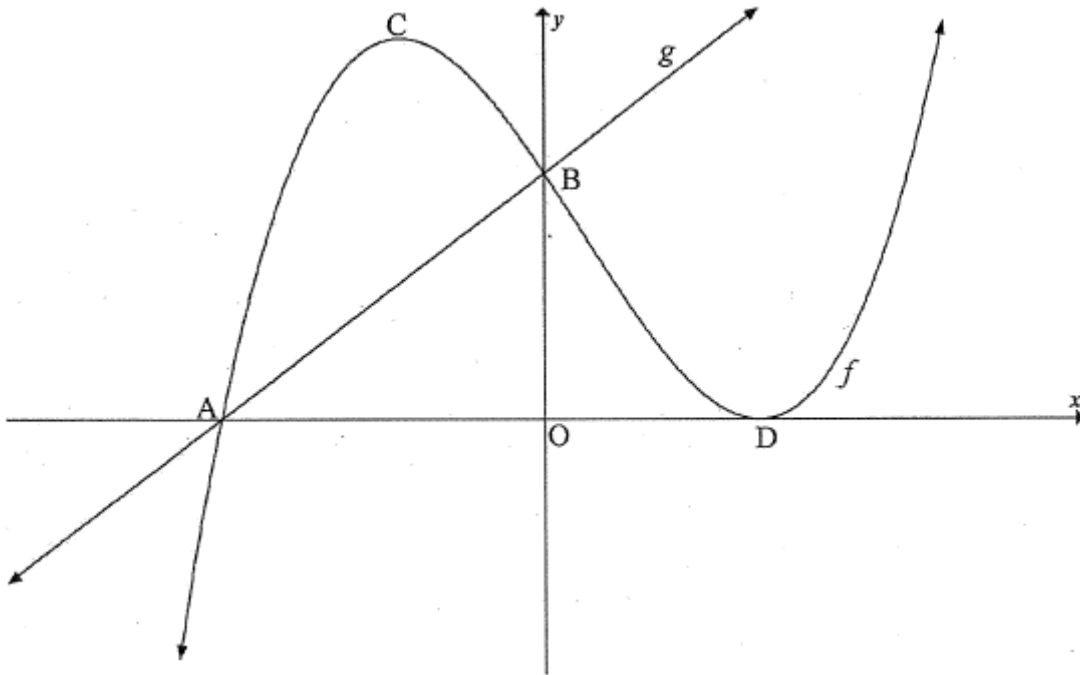
**[12]**



## QUESTION 8

Sketched below are the graphs of  $f(x) = (x-2)^2(x-n)$  and  $g(x) = mx+12$ .

- A and D are the  $x$ -intercepts of  $f$ .
- B is the common  $y$ -intercept of  $f$  and  $g$ .
- C and D are the turning points of  $f$ .
- The straight line  $g$  passes through A.



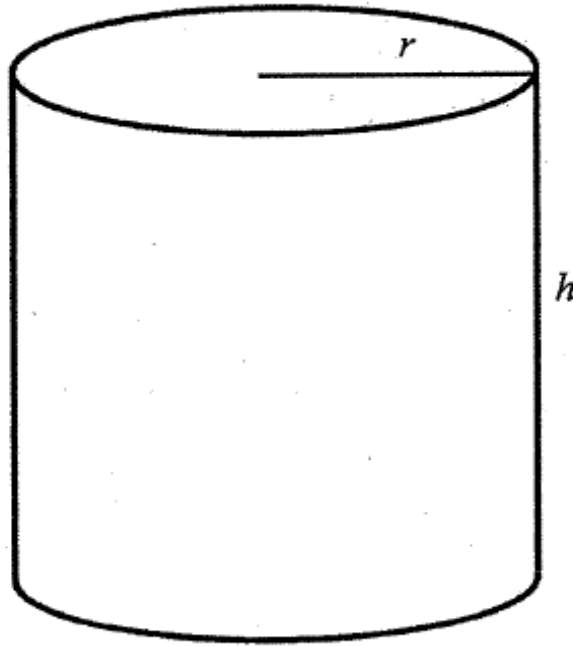
- 8.1 Write down the  $y$ -coordinate of B. (1)
- 8.2 Calculate the coordinates of A. (3)
- 8.3 If  $n = -3$ , calculate the coordinates of C. (6)
- 8.4 For which value(s) of  $x$  will:
- 8.4.1  $f$  be concave down (2)
- 8.4.2  $f'(x) > 0$  (2)
- 8.5 Write down the value(s) of  $p$  for which  $f(x) = p$  will have THREE distinct roots. (2)

[16]

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## QUESTION 9

A 340 m<sup>2</sup> closed can with height  $h$  cm and radius  $r$  cm is shown below.



9.1 Determine the height of the closed can in terms of the radius  $r$ . (2)

9.2 Calculate the length of the radius of the can, in cm, if the surface area is to be a minimum. (6)

**[8]**

**QUESTION 10**

- 10.1 A tournament organiser conducted a survey among 150 members at a local sports club to find out whether they play tennis or not. The results are shown in the table below.

	<b>PLAY TENNIS</b>	<b>DO NOT PLAY TENNIS</b>
<b>Male</b>	50	30
<b>Female</b>	20	50

- 10.1.1 What is the probability that a member selected at random is:
- (a) Male (1)
- (b) Female and plays tennis (1)
- 10.1.2 The events 'female' and 'plays tennis' are NOT independent. Show ALL calculations to support this statement. (3)

- 10.2 The events A, B and C are such:

A and B are independent, B and C are independent and A and C are mutually exclusive. Their probabilities are  $P(A) = 0,3$ ,  $P(B) = 0,4$  and  $P(C) = 0,2$ .

Calculate the probability of the following events occurring:

- 10.2.1 Both A and C occur (1)
- 10.2.2 Both B and C occur (1)
- 10.2.3 At least one of A or B occur (2)

[9]

**QUESTION 11**

Five boys and four girls go to the movies. They are all seated next to each other in the same row.

11.1 One boy and girl are a couple and want to sit next to each other at any end of the row of friends. In how many different ways can the entire group be seated? (2)

11.2 If all the friends are seated randomly, calculate the probability that all the girls are seated next to each other. (3)

[5]

**TOTAL : 150**