HERZLIA SENIOR HIGH SCHOOL

"If you will it, it is no legend"

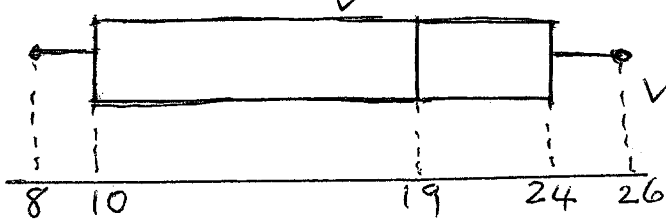
MATHEMATICS PAPER 2
GRADE 12
MONDAY 5TH SEPTEMBER 2016
ANSWER BOOK

QUESTION	MARK	TOTAL
1		12
2		9
3		21
4		18
5		25
6		6
7		9
8		18
9		13
10		19
TOTAL		150

NAME: MATHS (P2) Memo

TEACHER: J. FORTUIN / A. SONNENBERGER

QUESTION 1

	Solution	Marks
1.1	$\bar{x} = \frac{189}{11}$ $= 17,18$	(2)
1.2	Min : 8 Q_1 : 10 Q_2 : 19 Q_3 : 24 Max : 26	(3)
1.3		(2)
1.4	Skewed to the left	(1)
1.5	$\sigma = 6,46$	(2)
1.6	$x > 17,18 + 6,4$ $> 23,65$ Passengers : 24; 25; 26 (3 destinations)	(2)
		[12]

QUESTION 2

	Solution	Marks
2.1	$(30{,}53)$ ✓	(1)
2.2	$a = -38{,}51$ ✓ $b = 2{,}68$ ✓ $\hat{y} = -38{,}51 + 2{,}68x$ ✓	(3)
2.3	37 (on scatterplot) From $\hat{y} = -38{,}51 + 2{,}68(28)$ ✓ $= 36{,}53$ ≈ 37 ✓	(2)
2.4	Very strong ✓ (positive) since $r = 0{,}98$ ✓	(2)
2.5	Above a certain temperature humans will not survive. ✓	(1)
		[9]

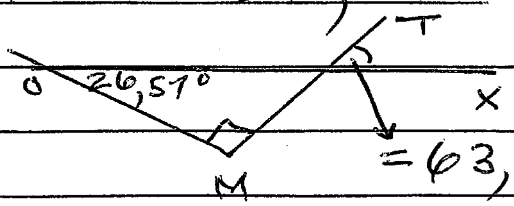
QUESTION 3

	Solution	Marks
3.1	Angle between radius (diameter) & Tangent ✓	(1) (1)
3.2	From Pyth: $RC^2 = 20^2 + 10^2$ ✓ $= 500$ $\therefore RC = \sqrt{500}$ ✓ or $10\sqrt{5}$	(2) (2)
3.3	$(K-3)^2 + (21+1)^2 = 500$ ✓ $(K-3)^2 = 16$ $\quad$ $K^2 - 6K - 7 = 0$ ✓ $\therefore K - 3 = \pm 4$ $\quad$ $(K-7)(K+1) = 0$ ✓ $\therefore K = 7$ or -1 or $K^2 - 6K - 7 = 0$ $\therefore K = 7$ ✓ (first quad)	(4) (4)
3.4	$(x-3)^2 + (y+1)^2 = 100$ ✓ ✓	(2) (2)

3.5	Since CS(radius) ⊥ PS ✓ $S(3;y)$ $\therefore y=-11$ ✓	(2) (2)
3.6.1	subst $P(x;-11)$ into $3y-4x=35$ ✓ $\therefore -33-4x=35$ $-68=4x$ $\therefore x=-17$ ✓	(2) (2)
3.6.2	$P(-17;-11)$ and $C(3;-1)$ $\therefore PC^2=(-17-3)^2+(-11+1)^2$ $=500$ ✓ $CP=10\sqrt{5}$ Since PC = RC and TC is common ✓ in $\triangle PTC$ and $\triangle RTC$ (They are ≡ ; 90°; H; S) $\therefore PT = 20$ units ✓ (or Pyth)	(3) (3)
3.7.1	$M(3;-16)$ ✓	(1) (1)
3.7.2	$r=4$ units ✓	(1) (1)
3.7.3	Radius(R_1) of circle centre $C=10$ Radius(R_2) of circle centre $M=4$ $\therefore R_1+R_2=14$ ✓ whilst $CM^2=(3-3)^2+(-16+1)^2$ $=225$ ✓ $\therefore CM=15$ Since $CM > R_1+R_2$ ✓, result	(3) (3)
		[21]

$PR=\sqrt{1600}$
$PT=40-20$

QUESTION 4

	Solution	Marks
4.1	$M(8;-4)$ ✓	(1) (1)
4.2	$OM^2 = 8^2 + (-4)^2 = 80$ ✓ $\therefore OM = \sqrt{80}$ ✓ $(4\sqrt{5})$	(2) (2)
4.3	$ON = OM - NM$ ✓ (Radius) $= \sqrt{80} - \sqrt{45}$ ✓ $= \sqrt{5}$ ✓	(3) (3)
4.4	$P\hat{T}M = 90°$ ✓ (radius ⊥ tangent) $=$ alt $O\hat{M}T$ $(OM \parallel TP)$ ✓ both reasons required	(2) (2)
4.5	$m_{OM} = -\frac{4}{8} = -\frac{1}{2}$ ✓ $\therefore M\hat{O}X = 26{,}57°$ (accept $26{,}56°$)  $= 63{,}43°$ (or . . dec.) Thus $m_{MT} = \text{Tan } 63{,}43° = 2$ ✓ $\therefore y = 2x + c$	(4) (4)

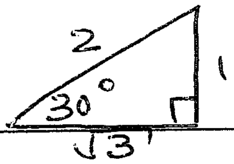
Subst $(8;-4)$ ✓ $\therefore -4 = 16 + c$ $\therefore c = -20$ ✓

$y = 2x - 20$

4.6	$y = 2x - 20$ into $(x-8)^2 + (y+4)^2 = 45$ $\therefore (x-8)^2 + (2x - 20 + 4)^2 = 45$ ✓ $x^2 - 16x + 64 + 4x^2 - 64x + 256 = 45$ ✓ $5x^2 - 80x + 275 = 0$ ✓ $5(x^2 - 16x + 55) = 0$ $(x-11)(x-5) = 0$ ✓ $\therefore x = 11$ ✓ $\therefore y = 2$ ✓ $T(11; 2)$	(6) (6)
		[18]

QUESTION 5

	Solution	Marks
5.1.1	$OP = 13$ (Pythagoras) $\therefore \cos\alpha = \frac{-5}{13}$ ✓ ✓	(2) (2)



5.1.2	$\sin(30° - \alpha)$ $= \sin 30° \cos\alpha - \cos 30° \sin\alpha$ ✓ $= \left(\frac{1}{2}\right)\left(\frac{-5}{13}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{-12}{13}\right)$ ✓ $= \frac{-5 + 12\sqrt{3}}{26}$ ✓	(3) (3)
5.2	$= \frac{1}{\cos\theta \cdot \cos\theta} - \tan^2\theta$ ✓ ✓ ✓ $= \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$ ✓ $= \frac{1 - \sin^2\theta}{\cos^2\theta}$ $= \frac{\cos^2\theta}{\cos^2\theta} = 1$ ✓	(5) (5)

5.3	$4\sin x + 2(1 - 2\sin^2 x) = 2$ $4\sin x + 2 - 4\sin^2 x = 2$ $4\sin^2 x - 4\sin x = 0$ $4\sin x(\sin x - 1) = 0$ $\sin x = 0$ or $\sin x = 1$ $x = 0° + K180°$ $x = 90° + K.360°$ or (3 answ $+ K360°$) $K \in Z$	(6) (6)
5.4	$LHS = \dfrac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)}$ $= \dfrac{\cos^2 x + 2\sin x\cos x + \sin^2 x - (\cos^2 x - 2\sin x\cos x + \sin^2 x)}{\cos^2 x - \sin^2 x}$ $= \dfrac{2(2\sin x\cos x)}{\cos^2 x - \sin^2 x}$ $= 2\dfrac{\sin 2x}{\cos 2x}$ $= RHS$	(5)

5.5	Square both sides: $\sin^2 x - 2\sin x\cos x + \cos^2 x = \frac{9}{16}$ $1 - 2\sin x\cos x = \frac{9}{16}$ $1 - \sin 2x = \frac{9}{16}$ $\therefore \sin 2x = 1 - \frac{9}{16}$ $= \frac{7}{16}$	4 (4)
		[25]

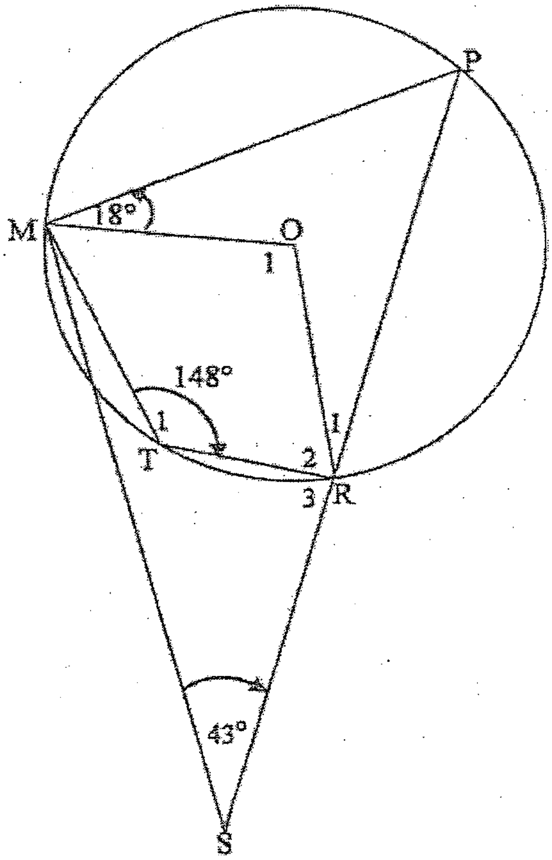
QUESTION 6

	Solution	Marks
6.1	$p = 30°$ $q = -\frac{1}{2}$	(2)
6.2	$-120° < x < 0°$ or $x \in (-120°; 0)$	(2)
6.3	Shift the graph of g 60° to the left, and reflect in the x-axis 120° to the right 240° to the left	(2)
		[6]

QUESTION 7

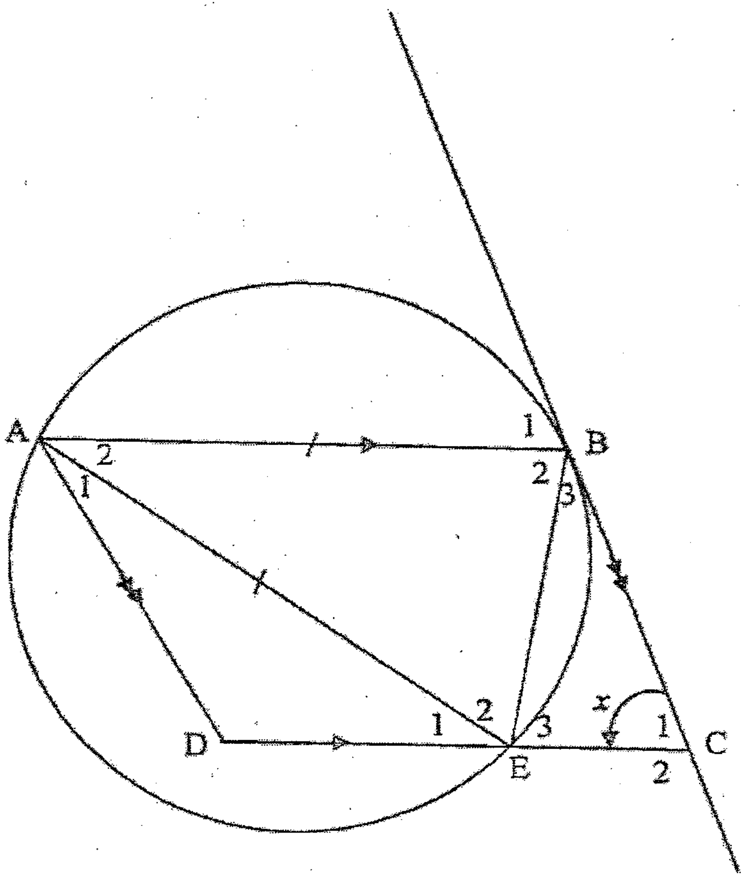
	Solution	Marks
7.1	$\cos\theta = \frac{64}{81}$ ✓ $\therefore \theta = 37,80°$ $= 38°$ ✓ $\frac{\sin\theta}{49,65} = \frac{\sin 90°}{81}$ $\sin\theta = \frac{49,65}{81}$ $(AD = 49,65\ m)$	(3) (3)
7.2	$BC^2 = 81^2 + 87^2 - 2(81)(87)\cos 82,6°$ ✓ $= 14130 - 1815,245$ $= 12314,75$ ✓ $BC = 110,97\ m$ ✓	(3) (3)
7.3	$\frac{\sin D\hat{C}B}{64} = \frac{\sin 110°}{110,97}$ ✓ $\therefore \sin D\hat{C}B = \frac{64 \sin 110°}{110,97}$ ✓ $\therefore D\hat{C}B = 32,82°$ ✓	(3) (3)
		[9]

QUESTION 8



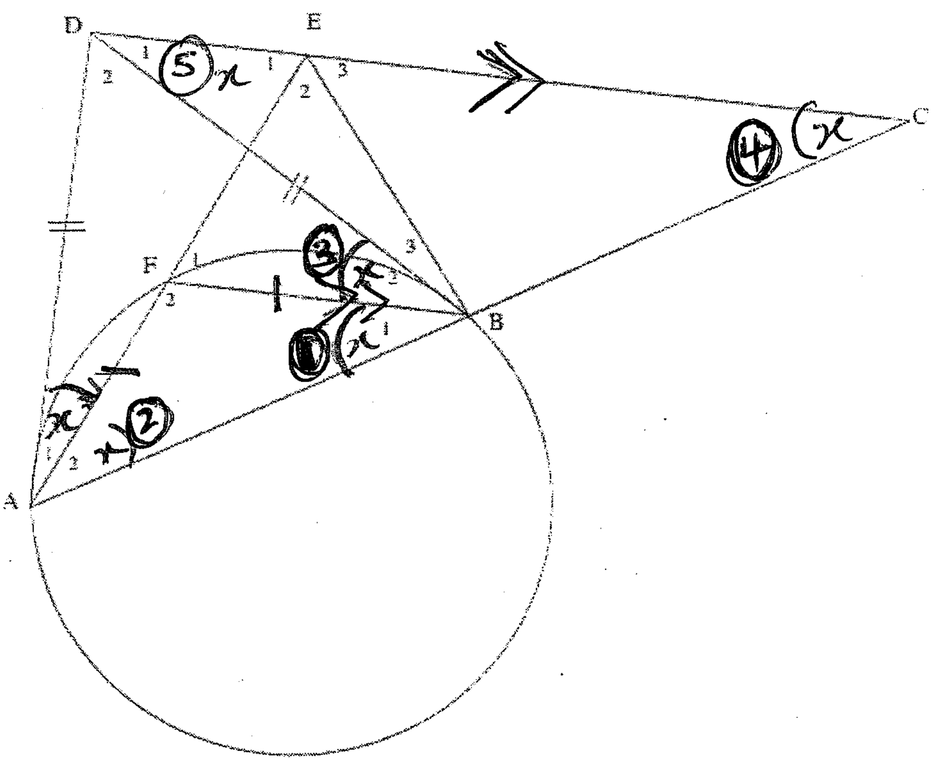
	Solution	Marks
8.1.1	$\hat{P} = 180° - 148°$ $= 32°$ ✓ (opp. $\angle^s$ of c.q) ✓	(2) (2)
8.1.2	$\hat{O}_1 = 2\hat{P}$ $= 64°$ ✓ (central $\angle$ TH.) ✓	(2) (2)
8.1.3	In $\triangle PMS$, $P\hat{M}S = 180° - (32° + 43°)$ $= 105°$ ✓ $\angle^s$ of $\triangle$ Thus $O\hat{M}S = 105° - 18° = 87°$ ✓	(2) (2)
8.1.4	$P\hat{M}T = (87° - 6°) + 18° = 99°$ ✓ $= \hat{R}_3$ (ext. $\angle$ of c.q.) ✓	(2) (2)

8.2



	Solution	Marks
8.2.1	$\hat{B}_1 = x$ (corresp $\angle^s =$; $AB \parallel DC$)	(1)
8.2.2	$\hat{C}_1 = B\hat{A}D = x$ (opp $\angle^s$ of parm) or $B\hat{A}D = \hat{B}_1$ (alt. $\angle^s$; $AD \parallel BC$) $\hat{B}_1 = \hat{E}_2$ (Tangent/chord TH) $= \hat{B}_2$ ($AE = AB$) $\hat{E}_3 = x$ (alt $\angle^s$, $AB \parallel DC$)	(6)
8.2.3	$A\hat{D}E = 180° - x$ (co-int $\angle^s$ ... $\parallel$ ..) Since opp $\angle^s = 180°$ Thus result.	(3)
		[18]

QUESTION 9

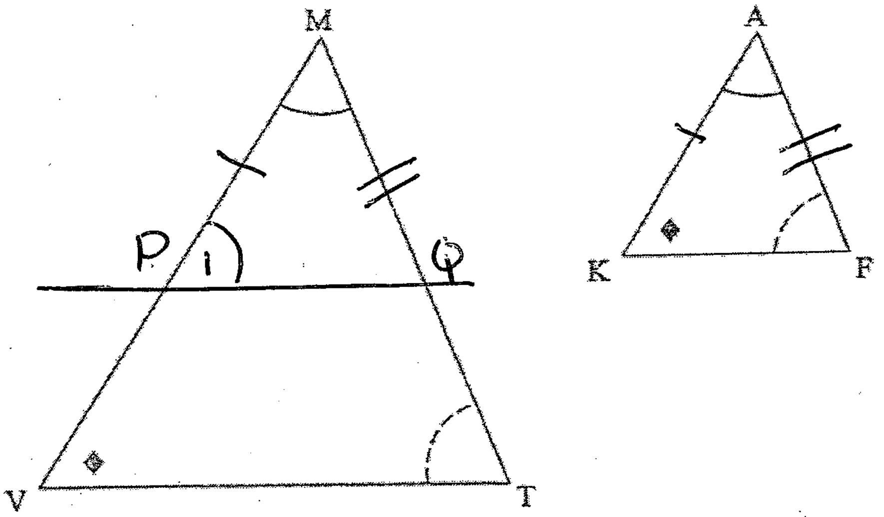


	Solution	Marks
9.1	$\hat{A}_1 = \hat{B}_1 = x$ (Tang/chord TH) $= \hat{A}_2$ (AF = FB) $= \hat{B}_2$ (AD = BD) OR (Tang/chord TH) $\hat{B}_1 =$ corresp. $\hat{C}$ (FB ∥ DC) $\hat{B}_2 =$ alt. $\hat{D}_1$ (" ")	(5)

9.2	Since BE subtends A_2 and D_1 $(\hat{A}_2 = \hat{D}_1)$, properties of c.q. Result	(2)
9.3	Since ABED is a c.q. (in 9.2) $\hat{B}_3 = \hat{A}_1 = x$ (subt. by DE) Thus $A\hat{B}E = x + x + x$ $= 3x$	(3)
9.4	Since $\hat{D}_1 = \hat{C} = x$ (proven) $\therefore$ BD = BC Already BD = AD (given) Thus result	(3)
		[13]

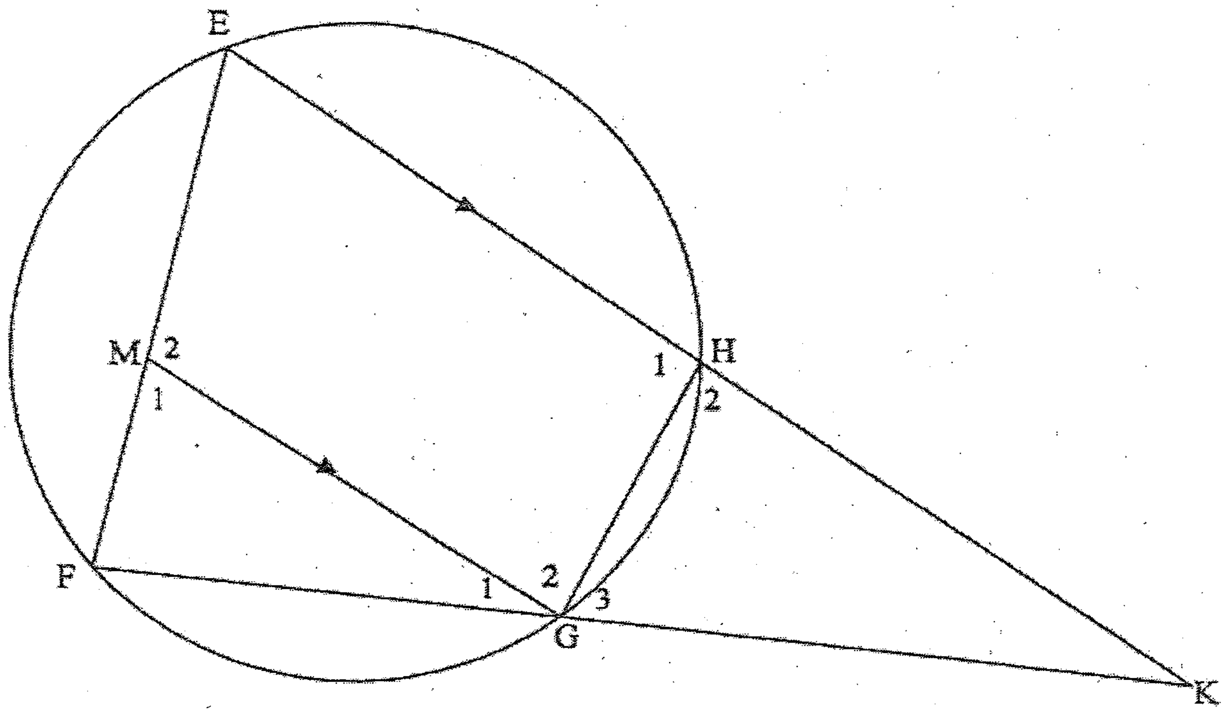
QUESTION 10

10.1



	Solution	Marks
	Constr: Make $MP = AK$ and $MQ = AF$ ✓	
	Proof: Thus $\triangle MPQ \equiv \triangle AKF$	Constr. ✓
	(SAS) ✓	Given ✓
	Hence $\hat{P}_1 = \hat{K}$ (from $\equiv$)	
	But $\hat{K} = \hat{V}$	
	$\therefore \hat{P}_1 = \hat{V}$ ✓	
	But they are corresp $\angle^s$	
	$\therefore PQ \parallel VT$ ✓	
	$\therefore \frac{MV}{MP} = \frac{MT}{MQ}$ ✓	
	from constr.	
	Result	
		(7)

10.2



	Solution	Marks
10.2.1 (a)	In Δ^s KGH and KEF ✓ 1) $\hat{K}$ is common ✓ 2) $\hat{G}_3 = \hat{E}$ (ext. ∠ of c.q) ✓ $\hat{H}_2$ or $\hat{F}$ " ∴ Result (AAA) ✓	(3)

10.2.1 (b)	From $\parallel$ in 10.2.1 (a) $\frac{KG}{GH} = \frac{KE}{EF}$ ✓ $\therefore KG.EF = KE.GH$ ✓ Since $KG = EF$ (given) ✓ $\therefore$ result.	(3)
10.2.1 (c)	Since $MG \parallel EK$ ✓ Prop. th. $\frac{EF}{EM} = \frac{KF}{KG}$ ✓ or $\therefore EF.KG = EM.KF$ Since $EF = KG$ (already) ✓ $\therefore$ Result	(3)
10.2.2	From (b) and (c) $EF^2 = KG^2$ ✓ Thus $KE.GH = EM.KF$ ✓ $\therefore EM = \frac{KE.GH}{KF}$ $= \frac{20.4}{16}$ $= 5$ units ✓	(3)
		[19]

	Additional space	Marks
	10.2.2.	
	$EF^2 = KE \times GH$	
	$= 20 \times 4$	
	$= 80$	
	$EF = 4\sqrt{5}$ ✓	
	$EF = KG$	
	$KG^2 = EM.KF$	
	$(4\sqrt{5})^2 = EM.16$ ✓	
	$\frac{80}{16} = EM$	
	$EM = 5$ ✓	