



HERZLIA SENIOR HIGH SCHOOL

"If you will it, it is no legend"

GRADE 12

**MATHEMATICS PAPER 2
MONDAY 5TH SEPTEMBER 2016**

MARKS: 150

TIME: 3 HOURS

This question paper consists of 16 pages, 1 information sheet and a 20-page answer book.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams and graphs that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. An INFORMATION SHEET, with formulae, is included.
9. Write neatly and legibly.

QUESTION 1

On a certain day a tour operator sent 11 tour buses to 11 different destinations. The table below shows the number of passengers on each bus.

8	8	10	12	16	19	20	21	24	25	26
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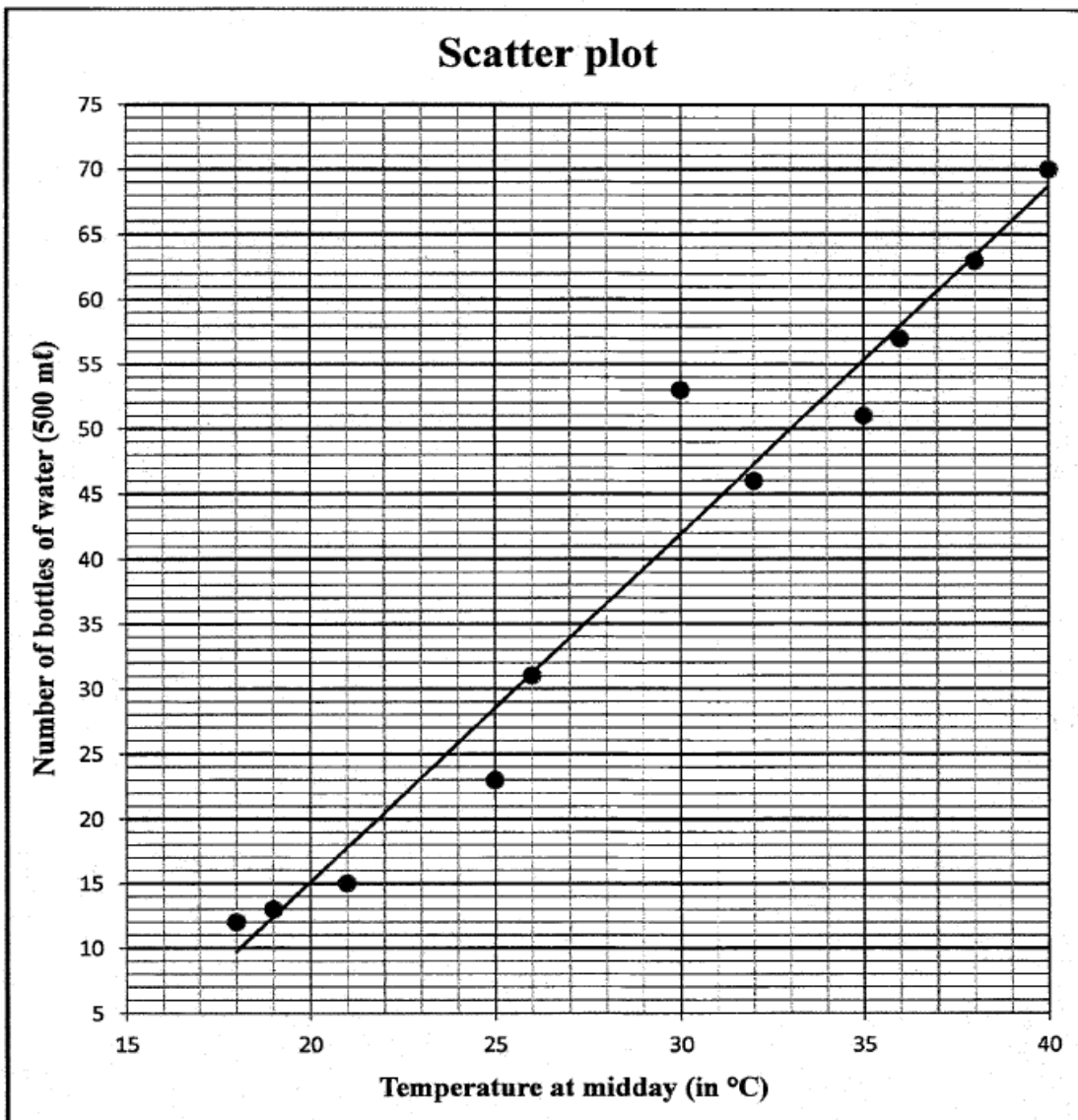
- 1.1 Calculate the mean number of passengers travelling in a tour bus. (2)
- 1.2 Write down the five-number summary of the data. (3)
- 1.3 Draw a box and whisker diagram for the data. (2)
- 1.4 Refer to the box and whisker diagram and comment on the skewness of the data set. (1)
- 1.5 Calculate the standard deviation for this data set. (2)
- 1.6 A tour is regarded as popular if the number of passengers on a tour bus is one standard deviation above the mean. How many destinations were popular on this particular day? (2)

[12]**PLEASE TURN OVER**

QUESTION 2

On the first school day of each month information is recorded about the temperature at midday (in $^{\circ}\text{C}$) and the number of 500 ml bottles of water that were sold at the tuck shop of a certain school during the lunch break. The data is shown in the table below and represented on the scatter plot. The least squares regression line for this data is drawn on the scatter plot.

Temperature at midday (in $^{\circ}\text{C}$)	18	21	19	26	32	35	36	40	38	30	25
Number of bottles of water (500 ml)	12	15	13	31	46	51	57	70	63	53	23

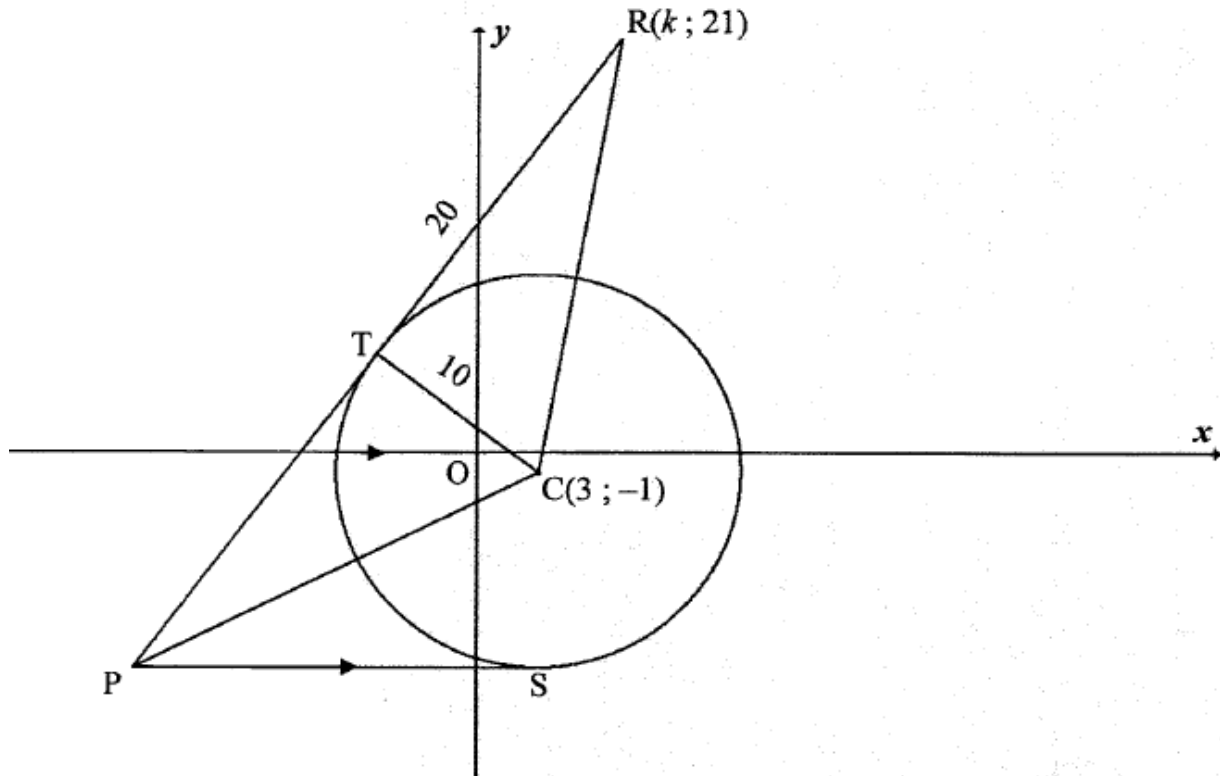


- 2.1 Identify an outlier in the data. (1)
- 2.2 Determine the equation of the least squares regression line. (3)
- 2.3 Estimate the number of 500 m bottles of water that will be sold if the temperature is 28 C at midday. (2)
- 2.4 Refer to the scatter plot. Would you say that the relation between the temperature at midday and the number of 500 m bottles of water sold is weak or strong? Motivate your answer. (2)
- 2.5 Give a reason why the observed trend for this data cannot continue indefinitely. (1)

[9]**PLEASE TURN OVER**

QUESTION 3

A circle having $C(3;-1)$ as centre and a radius of 10 units is drawn. PTR is a tangent to this circle at T. $R(k;21)$, C and P are the vertices of a triangle. $TR = 20$ units.



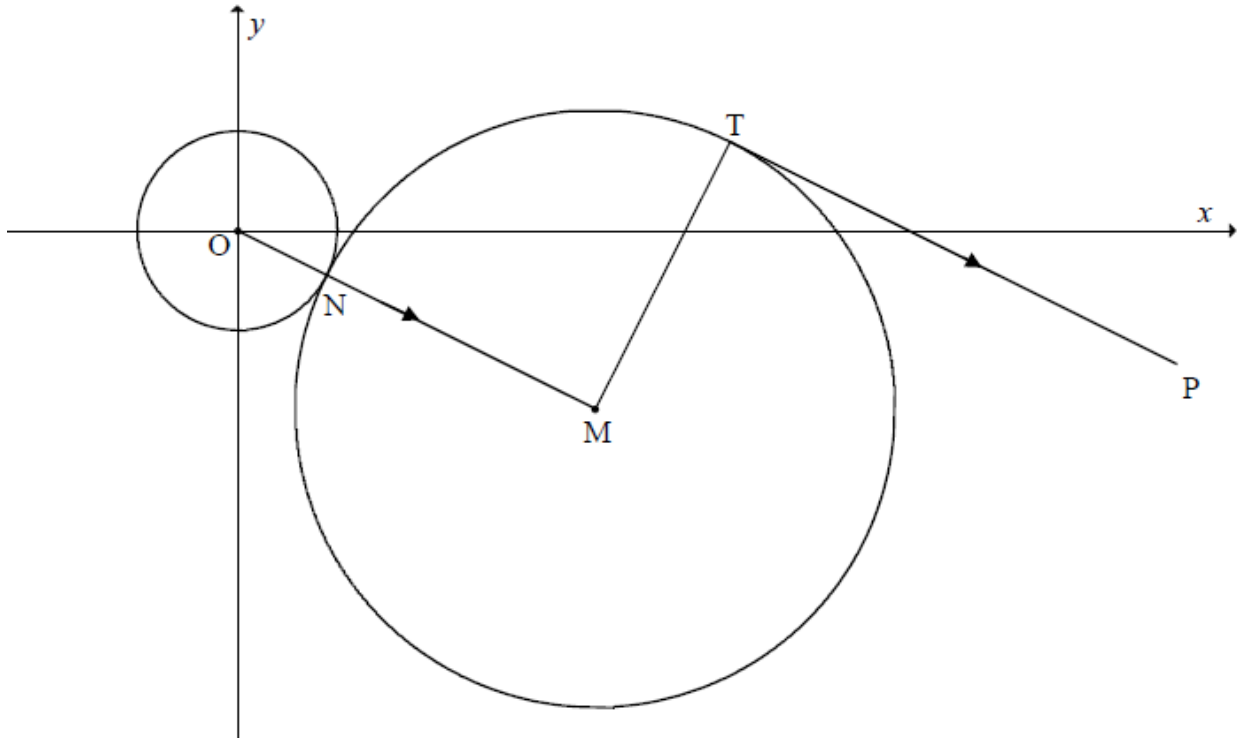
- 3.1 Give a reason why $TC \perp TR$. (1)
- 3.2 Calculate the length of RC. Leave your answer in surd form. (2)
- 3.3 Calculate the value of k if R lies in the first quadrant. (4)
- 3.4 Determine the equation of the circle having centre C and passing through T. Write your answer in the form $(x - a)^2 + (y - b)^2 = r^2$. (2)

- 3.5 PS, a tangent to the circle at S, is parallel to the x -axis. Determine the equation of PS. (2)
- 3.6 The equation of PTR is $3y - 4x = 35$.
- 3.6.1 Calculate the coordinates of P. (2)
- 3.6.2 Calculate the length of PT. Show the necessary working. (3)
- 3.7 Consider another circle with equation $(x - 3)^2 + (y + 16)^2 = 16$ and having centre M.
- 3.7.1 Write down the coordinates of centre M. (1)
- 3.7.2 Write down the length of the radius of this circle. (1)
- 3.7.3 Prove that the circle with centre C and the circle with centre M do not intersect or touch. (3)

[21]**PLEASE TURN OVER**

QUESTION 4

In the diagram below, the equation of the circle with centre M is $(x-8)^2 + (y+4)^2 = 45$. PT is a tangent to this circle at T and PT is parallel to OM. Another circle, having centre O, touches the circle having centre M at N.

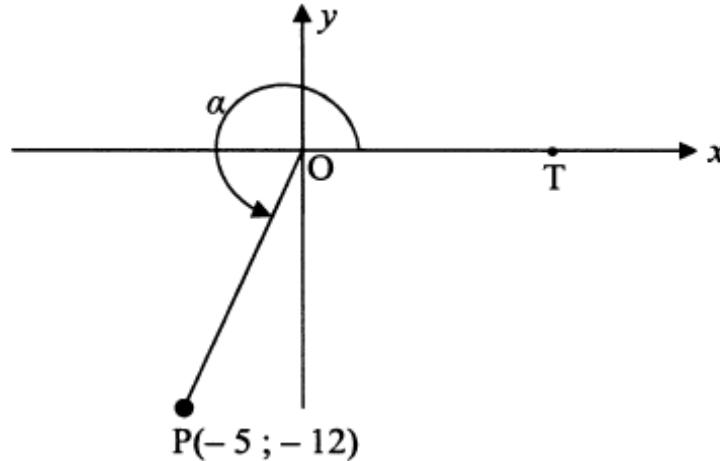


- 4.1 Write down the coordinates of M. (1)
- 4.2 Calculate the length of OM. Leave your answer in simplest surd form. (2)
- 4.3 Calculate the length of ON. Leave your answer in simplest surd form. (3)
- 4.4 Calculate the size of \widehat{OMT} , giving reasons. (2)
- 4.5 Determine the equation of MT in the form $y = mx + c$. (4)
- 4.6 Calculate the coordinates of T. (6)

[18]

QUESTION 5

- 5.1 In the diagram below, reflex $\widehat{TOP} = \alpha$ and P has coordinates $(-5; -12)$.



Determine the value of each of the following trigonometric ratios WITHOUT using a calculator:

5.1.1 $\cos \alpha$ (2)

5.1.2 $\sin(30^\circ - \alpha)$ (3)

- 5.2 Determine the value of:

$$\frac{1}{\cos(360^\circ - \theta) \cdot \sin(90^\circ - \theta)} - \tan^2(180^\circ + \theta)$$
 (5)

- 5.3 Determine the general solution of $4 \sin x + 2 \cos 2x = 2$. (6)

- 5.4 Prove the identity:

$$\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = 2 \tan 2x$$
 (5)

- 5.5 If $\sin x - \cos x = \frac{3}{4}$, calculate the value of $\sin 2x$ WITHOUT using a calculator. (4)

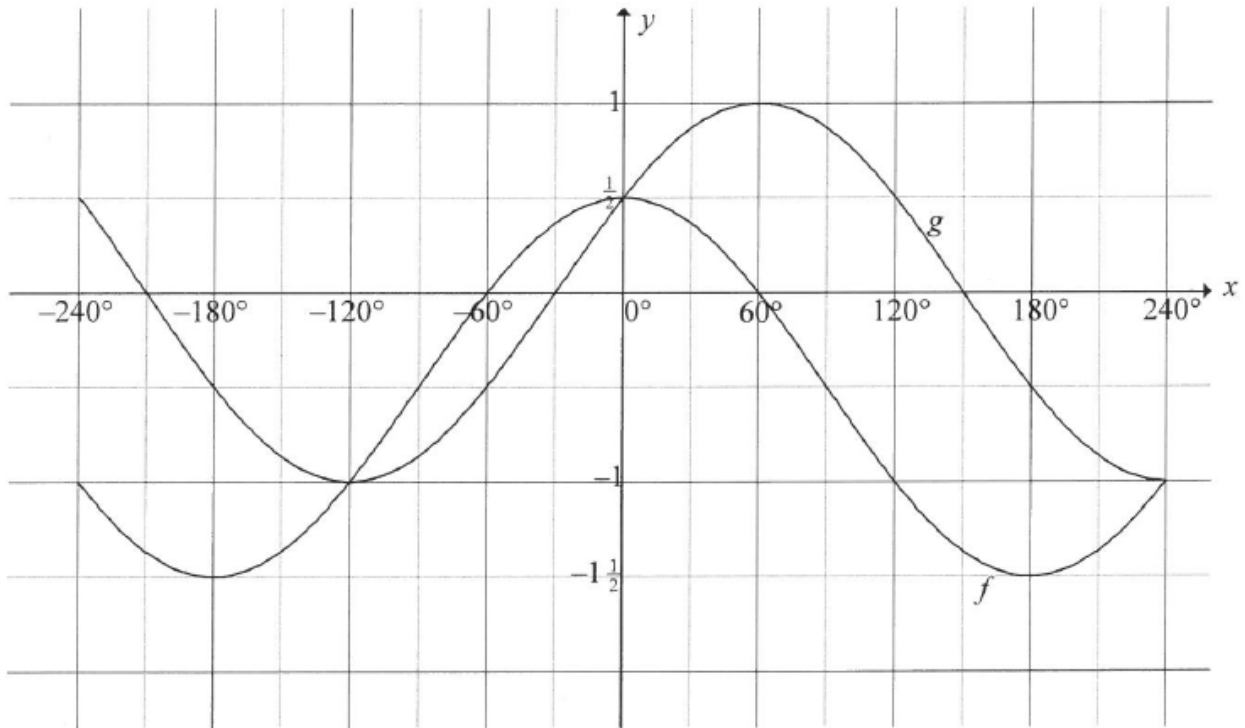
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QUESTION 6

In the diagram below, the graphs of $f(x) = \cos x + q$ and $g(x) = \sin(x + p)$ are drawn on the same system of axes for $-240 \leq x \leq 240$. The graphs intersect

at $(0; \frac{1}{2})$, $(-120; -1)$ and $(-240; -1)$.



6.1 Determine the values of p and q . (2)

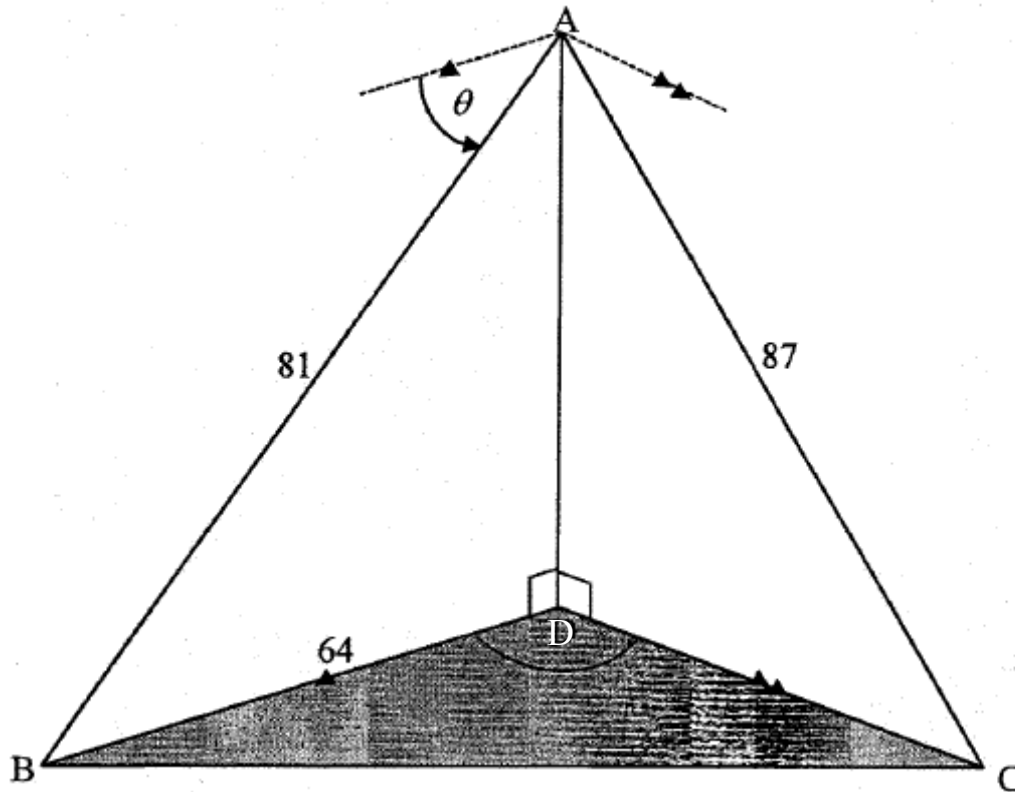
6.2 Determine the values of x in the interval $-240 \leq x \leq 240$ for which $f(x) > g(x)$. (2)

6.3 Describe the transformation that the graph of g has to undergo to form the graph of h , where $h(x) = -\cos x$. (2)

[6]

QUESTION 7

From point A an observer spots two boats, B and C, at anchor. The angle of depression of boat B from A is θ . D is a point directly below A and is on the same horizontal plane as B and C. $BD = 64$ m, $AB = 81$ m and $AC = 87$ m.



- 7.1 Calculate the size of θ to the nearest degree. (3)
- 7.2 If it is given that $\hat{BAC} = 82,6^\circ$, calculate BC, the distance between the two boats. (3)
- 7.3 If $\hat{BDC} = 110^\circ$, calculate the size of \hat{DCB} . (3)

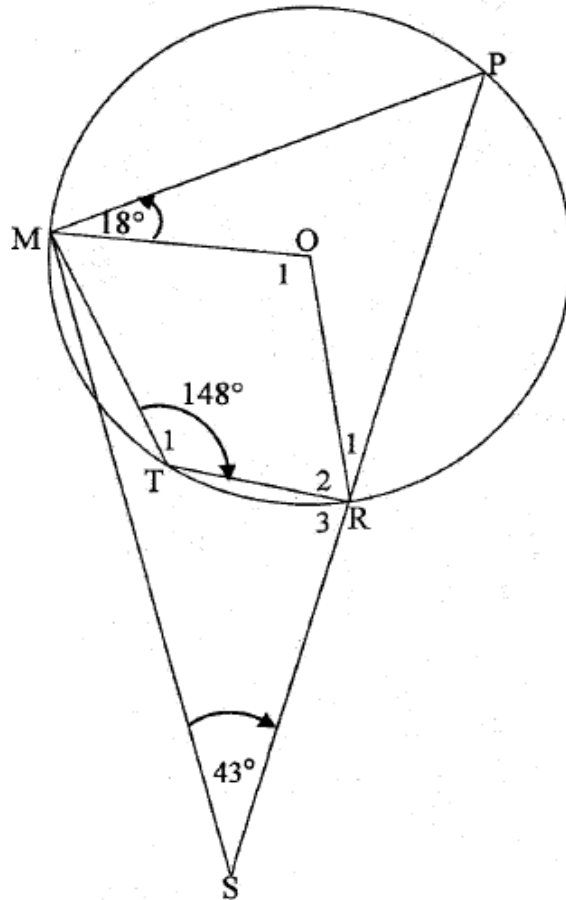
[9]

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Give reasons for ALL statements in QUESTIONS 8, 9 and 10.

QUESTION 8

8.1 In the diagram below, P, M, T and R are points on a circle having centre O. PR produced meets MS at S. Radii OM and OR and the chords MT and TR are drawn. $\hat{T}_1 = 148^\circ$, $\hat{PMO} = 18^\circ$ and $\hat{S} = 43^\circ$.



Calculate, with reasons, the size of:

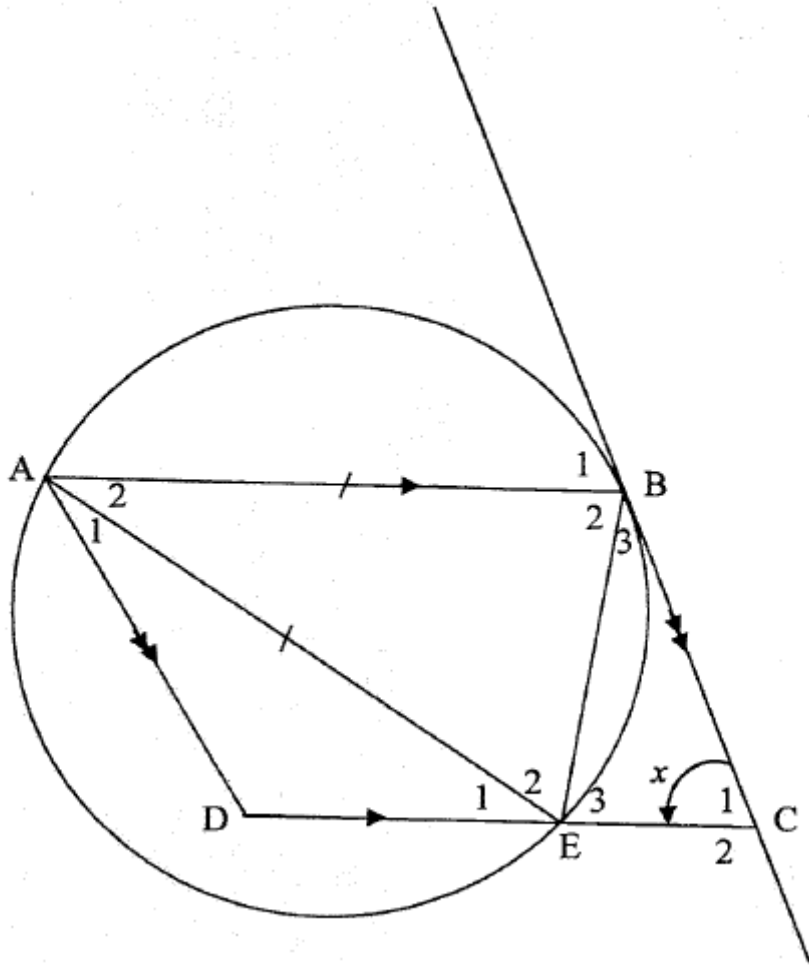
8.1.1 \hat{P} (2)

8.1.2 \hat{O}_1 (2)

8.1.3 \hat{OMS} (2)

8.1.4 \hat{R}_3 , if it is given that $\hat{TMS} = 6^\circ$ (2)

8.2 In the diagram below, the circle passes through A, B and E. ABCD is a parallelogram. BC is a tangent to the circle at B. $AE = AB$. Let $\hat{C}_1 = x$.



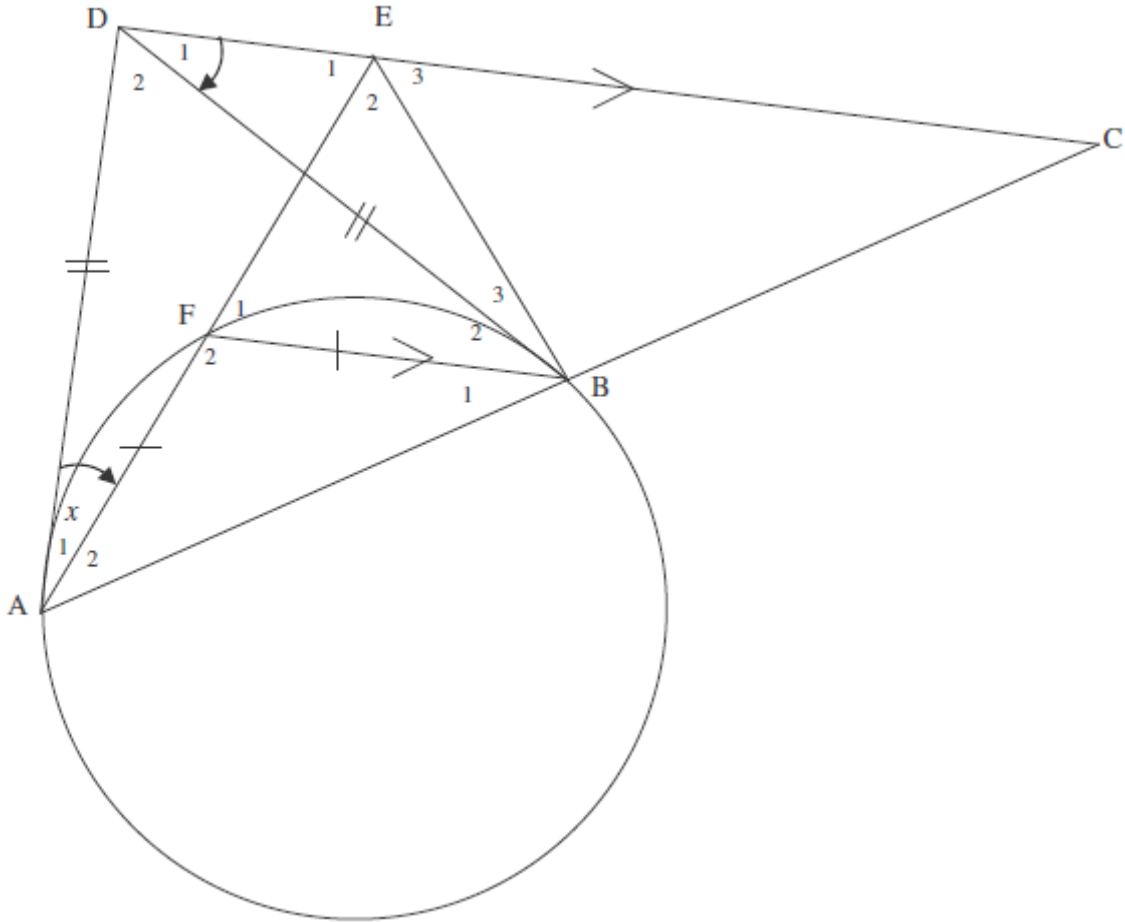
- 8.2.1 Give a reason why $\hat{B}_1 = x$. (1)
- 8.2.2 Name, with reasons, THREE other angles equal in size to x . (6)
- 8.2.3 Prove that ABED is a cyclic quadrilateral. (3)

[18]

PLEASE TURN OVER

QUESTION 9

In the diagram below, DA and DB are tangents to the circle at A and B. $AF = FB$. AB produced cuts the line through D, which is parallel to FB, at C. AF produced meets DC at E and $\hat{DAE} = x$.



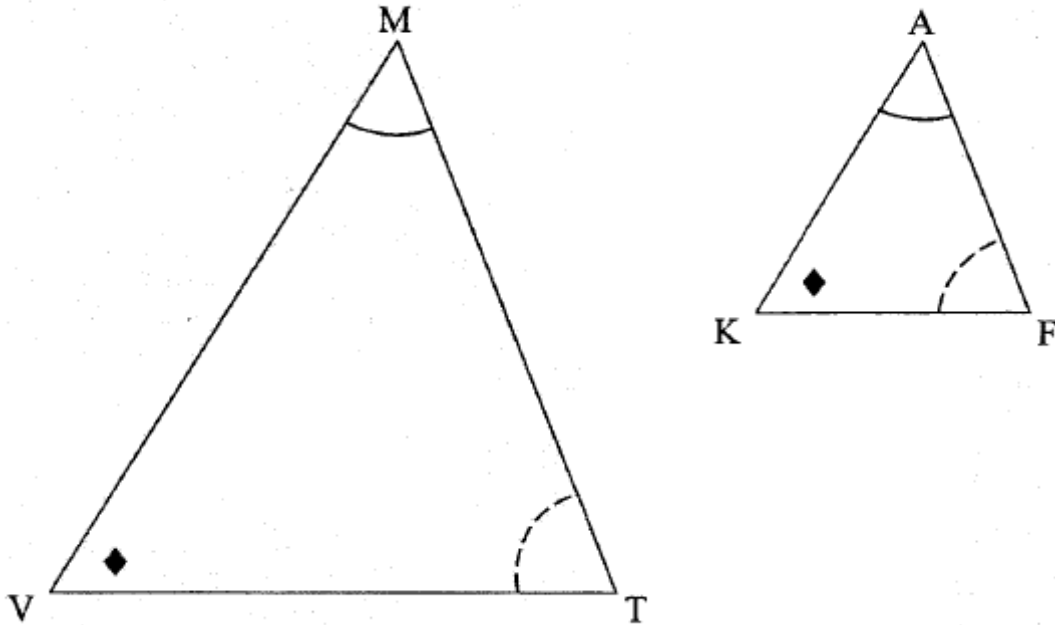
- 9.1 Find, with reasons, 5 angles each equal to x . (5)
- 9.2 Prove that ABED is a cyclic quadrilateral. (2)
- 9.3 Prove that $\hat{ABE} = 3\hat{DAE}$. (3)
- 9.4 Prove that $AD = BC$. (3)

[13]

QUESTION 10

10.1 In the diagram below, $\triangle MVT$ and $\triangle AKF$ are drawn such that

$$\hat{M} = \hat{A}, \hat{V} = \hat{K} \text{ and } \hat{T} = \hat{F}.$$

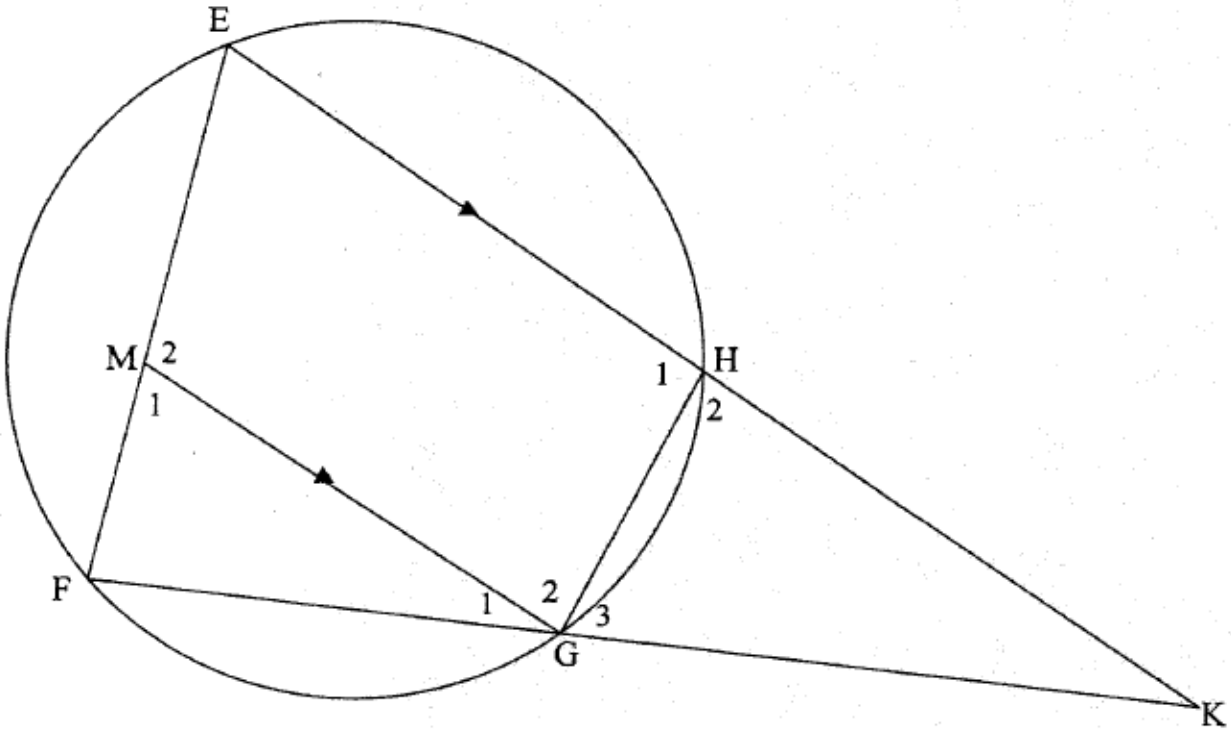


Use the diagram in the answer book to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are

in proportion, that is $\frac{MV}{AK} = \frac{MT}{AF}$. (7)

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10.2 In the diagram below, cyclic quadrilateral EFGH is drawn. Chord EH produced and chord FG produced meet at K. M is a point on EF such that $MG \parallel EK$. Also $KG = EF$.



10.2.1 Prove that:

(a) $\triangle KGH \parallel \triangle KEF$ (3)

(b) $EF^2 = KE \cdot GH$ (3)

(c) $KG^2 = EM \cdot KF$ (3)

10.2.2 If it is given that $KE = 20$ units, $KF = 16$ units and $GH = 4$ units, calculate the length of EM . (3)

[19]

TOTAL : 150