



**HILTON COLLEGE
TRIAL EXAMINATION
AUGUST 2016**

CORE MATHEMATICS PAPER ONE

Time: 3 hours

Marks: 150

GENERAL INSTRUCTIONS

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This paper consists of 14 questions and is 14 pages. A separate YELLOW INFORMATION sheet is provided. Please check that your question paper is complete.
 2. Read the questions carefully.
 3. Answer all questions.
 4. Number your answers exactly as the questions are numbered.
 5. Please note that diagrams are not necessarily drawn to scale.
 6. All necessary working details must be shown.
 7. Approved non-programmable, non-graphing calculators may be used, unless otherwise stated.
 8. Round off your answers to one decimal digit where necessary.
 9. Ensure that your calculator is in DEGREE mode.
 10. It is in your own interest to write legibly and to present your work neatly.
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SECTION A**QUESTION 1**

(a) Solve for x :

(1) $x^2 - 2x = 8$ (2)

(2) $(x+3)\sqrt{x+6} = 0$ (2)

(3) $x^2 - 28 > -3x$ (4)

(b) Solve for x and y :

$x + 2y = 5$ and $x^2 + y^2 = 340 - 10y$ (5)

(c) Given: $-2x^2 + 3x - 3 = -p$

(1) Express the roots/solutions of the equation in terms of p . (3)

(2) Hence, find the value(s) of p for which the equation will have 2 unequal real roots? (2)

[18]

QUESTION 2

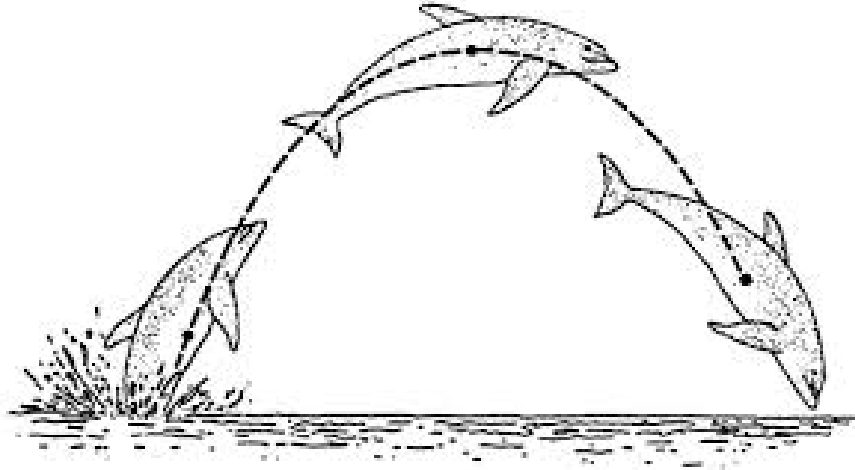
Given: $f(x) = \frac{-4}{x+1} + 2$

- (a) Write down the equations of the asymptotes. (2)
- (b) Find the value of x such that $f(x) = 0$ (2)
- (c) Sketch the graph of f , clearly indicating the coordinates of the intercepts and showing the asymptotes. (3)
- (d) Determine the value(s) of x for which:
- (1) $f(x) \geq 2$ (1)
- (2) $\frac{-4}{x+1} > -2$ (2)

[10]

QUESTION 3

Whilst on a sea cruise, Jenny noticed a dolphin jump out of the water. She observed the beautiful mammal and the path it took as shown below:



The path can be modelled by the function $h(t) = a(t + p)^2 + q$ where t is the time in minutes and $h(t)$ is the height in metres.

- (a) Determine the equation of the graph if at 1 minute, the maximum height the dolphin reached was 4 metres. (4)
- (b) Without any further calculations, determine the turning point of $h(t - 2)$. (2)
- (c) Explain why the inverse of h is not a function. (1)
- (d) Restrict the domain of h so that the inverse is a function. (2)

[9]

QUESTION 4

- (a) A motor car costing R275 000 depreciated at a rate of 7% per annum on the reducing balance method. Calculate how long it took for the car to depreciate to one quarter of its original value. Round off your answer to the nearest year. (3)
- (b) Peter started a five year savings plan and decided to make equal monthly deposits of R1400 into an account earning interest at a rate of 8% per annum compounded monthly.
- (1) What is the value of the investment at the end of the time period? (4)
- (2) Peter decides to leave the account open for 6 more months but makes no further deposits. If the rate changed to 5% per annum compounded quarterly, what was the final value of his investment? (2)
- (c) Amy wants to buy a flat for R700 000. She puts down a 10% deposit and takes out a loan for the balance at a rate of 18% per annum compounded monthly.
- (1) Calculate her monthly payment if she wishes to settle her loan in 15 years. (4)
- (2) Amy has just inherited some money and wishes to settle her loan earlier. What is her outstanding balance immediately after her 50th payment? (4)

[17]

QUESTION 5

Determine:

(a) $f'(x)$ from first principles if $f(x) = 1 - 3x^2$. (4)

(b) $\frac{dy}{dx}$ if

(1) $y = \frac{x^2 - 1}{\sqrt{x}}$ (4)

(2) $y = \frac{x^3 - 5x^2 + 6x}{x - 2}$ (3)

(c) Given that $f'(x) = 3x^2$ and $g'(x) = x$.

Determine $\frac{d}{dx} [f(x) + 3.g(x)]$. (2)

(d) The line $g(x) = 3x + 2$ is a tangent to the curve of a function f at the point where $x = 3$. Calculate the value of $f(3) + f'(3)$. (3)

[16]**QUESTION 6**

Consider the following quadratic pattern :

$$x \quad 17 \quad y \quad 57 \quad 86$$

Determine the values of x and y . **[5]**

TOTAL MARKS SECTION A : 75

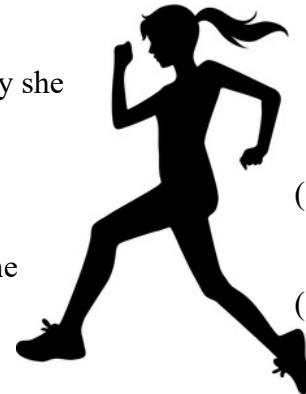
SECTION B**QUESTION 7**

- (a) An athlete trains by running 600m on the first day. Each day she increases her distance by 300m.

(1) Calculate the distance she runs on the 15th day. (3)

(2) What is the total distance, in km, that she has run at the end of the 15th day? (3)

(3) In order to participate in the Comrades marathon in 6 months, she must complete a qualifier race of 42km. Would she have sufficient time to run the 42km qualifier and be eligible for the Comrades? Show all calculations. (3)



- (b) Consider the infinite geometric series

$$m + m(m+1) + m(m+1)^2 + \dots$$

(1) For which values of m will the series converge?
(3)

(2) Determine S_{∞} . (2)

[14]

QUESTION 8

- (a) Consider the graph of $g(x) = 4^{-x}$.

Give the equation of g^{-1} in the form $y = \dots$ (2)

- (b) Sketch the graphs of g and g^{-1} on the same set of axes, showing the intercepts with the axes and clearly indicating g and g^{-1} . (4)

- (c) Use the graph to solve for x :

$$\frac{\log x}{\log \frac{1}{4}} \geq 0 \quad (3)$$

[9]

QUESTION 9

- (a) Sketch the graph of $f(x) = x^3 - 9x^2 + 24x - 20$, showing all intercepts and the turning points. (4)
- (b) Determine the equation of the tangent to $f(x)$ at $x = 3$. (2)
- (c) Give the values of x for which the graph is concave down. (2)

[8]

QUESTION 10

The graph of a cubic function f has turning points at $A(-1; p)$ and $B(2; q)$. The function f also has the following properties:

- $f'(x) > 0$ for $x < -1$ or $x > 2$
 - $f'(x) < 0$ for $-1 < x < 2$
 - $f(2) > 0$
- (a) Draw a neat sketch graph of f . Clearly label points A and B on the sketch (it is not necessary to show the x and y intercepts). (3)
- (b) Use the graph to deduce how many roots $f(x) = 0$ has. (1)
- (c) If $f(x) = x^3 + bx^2 + cx + d$, calculate the values of b and c . (5)

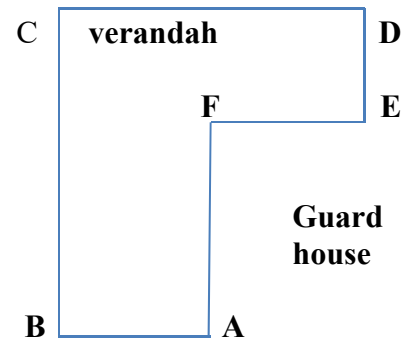
[9]

QUESTION 11

A verandah is to be built on the corner of a guard house as shown:



Plan:



A railing ABCDE is to be built around the outer edges of the verandah.

If $AB = DE = x$ and $BC = CD = y$ and the total length of the railing used is 30 metres, find the value of x and y for which the verandah will have a maximum area.

[6]

QUESTION 12

Idols season 12 – 2016 is upon us.

- (a) You have decided to audition for Season 12. The probability of getting a YES from the first judge in the first round of Idols is 0,6. If you get that YES, the probability of getting the next YES, from the next judge rises to 0,7. However, if the first judge said NO, the probability of getting a YES thereafter is only 0,4.



Determine, with the use of a tree diagram, the probability of getting a YES from the second judge.

(4)

- (b) Part of the planning for the competition is organising the tours for the top 5 contestants, in the final stages of the competition. They are to perform live across the country. The provinces and cities they are touring are :

Western Cape (Cape Town, Stellenbosch)
 KZN (Durban, Pietermaritzburg)
 Gauteng (Johannesburg, Soweto , Krugersdorp)
 Limpopo (Bela-Bela.

In how many ways can the tour be arranged if:

- (1) all cities must be visited and there are no restrictions? (2)
- (2) the first performance must be in Bela-Bela and the last performance in Durban? (2)
- (3) the performances in Gauteng and Limpopo must follow one another? (3)
- (c) The data below shows information obtained from the show:

	Qualifying	Not qualifying	TOTAL
Males	42	4983	5025
Females	37	5163	5200
TOTAL	79	10146	10225

- (1) Determine, the probability, that a person selected at random is a female who did not qualify. (1)
- (2) Is being female and qualifying independent? Use calculations to support your answer. (3)

[15]

QUESTION 13

The functions $f(x) = 2x^2 + 3px - 3$ and $g(x) = 2x^2 + (p - 2)x - 1$ have a common factor $(x - r)$.

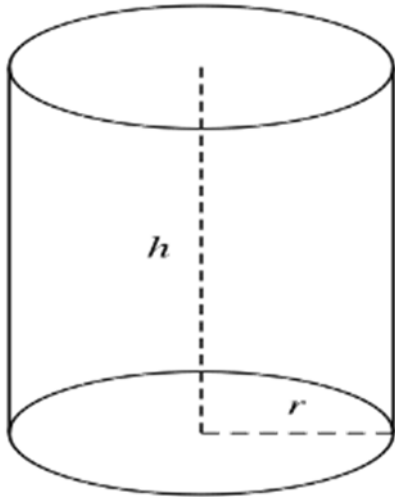
(a) Prove that $r = \frac{1}{p+1}$. (5)

(b) Hence, or otherwise determine the numerical value of r . (3)

[8]

QUESTION 14

A solid metal cylinder of 10 cm height and 14 cm diameter is melted and re-cast into two cones, whose volumes are in the proportion of 3 : 4, keeping the height 10 cm. What would be the percentage change in the flat surface area (top and bottom circles of cylinder) from the initial cylinder to the cones?

[6]

$$\text{Volume of a cylinder} = \pi r^2 h$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

TOTAL MARKS SECTION B : 75