MATHEMATICS TRIAL EXAMINATIONS MEMO

PAPER 1 AUGUST 2016

	ALGEBRA & EQUATIONS	PATTERNS& SEQUENCES	FINANCE	FUNCTIONS	CALCULUS	PROBABILITY
SAGS	25	25	15	35	35	15
THIS PAPER	**26**	**19**	**17**	**29**	**38**	**15**
	A] 1 (18)	A] 6 (5)	A] 4 (17)	A] 2 (10)	A] 5 (16)	B] 12 (15)
				A] 3 (9)	B] 9 (8)	
	B] 13 (8)	B] 7 (14)		B] 8 (9)	B] 10 (8)	
				B] 9b (1)	B] 11 (6)	
PROBLEM SOLVING Q14. [6 MARKS] – MENSURATION AND ALGEBRA						

	KNOWLEDGE	ROUTINE	COMPLEX	PROBLEM SOLVING
SAGS	20%	30%	35%	15%
THIS PAPER	22%	30%	35%	13%

SECTION A

QUESTION 1 [18 marks]

(a)

(1)

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$ √√ aa one for each x value

$x = 4 \text{ or } x = -2$

(2) $x = -3 \quad 0r \quad x = -6$ √√ aa one for each x value

(3) $x^2 - 28 > -3x$

$x^2 + 3x - 28 > 0$

$(x+7)(x-4) > 0$

$x < -7 \text{ or } x > 4$

√ m critical values √m number line

√√ aa correct answers

(b) From eqn 1: $x = 5 - 2y$ .....(3) √ a making x the subject of the formula

Subst (3) into (2) :

$(5-2y)^2 + y^2 = 340 - 10y$ √ m substitution

$25 - 20y + 4y^2 + y^2 - 340 + 10y = 0$

$5y^2 - 10y - 315 = 0$ √ m simplification of equation

$y \ - 2y - 63 = 0$

$(y-9)(y+7) = 0$ √ a correct y values

$y = 9 \ or \ y = -7$

$x = -13 \quad or \quad x = 19$ √ ca x values

$-2x^2 + 3x - 3 + p = 0$

(1) $a = -2 \ ; \ b = 3 \ ; \ c = -3 + p$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(-2)(-3+p)}}{2(-2)}$$ √ a correct subst into formula

$$= \frac{-3 \pm \sqrt{9 + 8(-3+p)}}{-4}$$ √ m simplification

$$= \frac{-3 \pm \sqrt{-15 + 8p}}{-4}$$ √ a answer

(2) $8p - 15 > 0$ √ ca delta>0

$8p > 15$

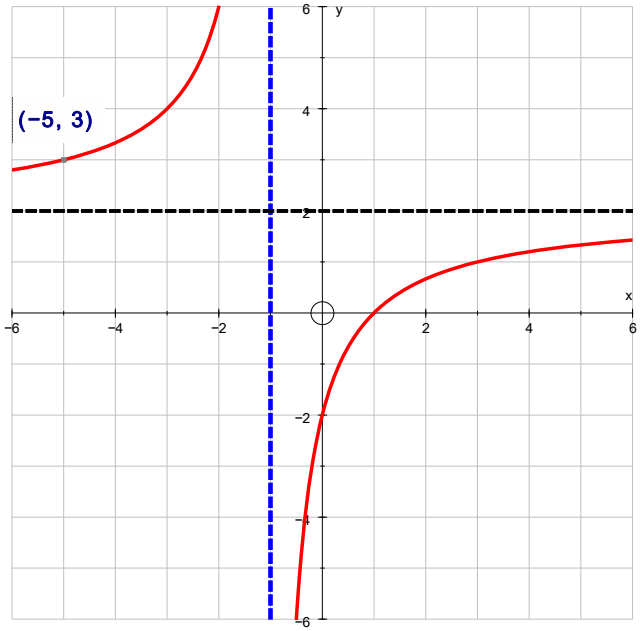
$p > \frac{15}{8}$ √ca answer

QUESTION 2 [10 MARKS]

a) $x = -1$ √ a
$y = 2$ √ a

b) $0 = \dfrac{-4}{x+1} + 2$ √ a y=o
$x = 1$ √ a

c) y int : $(0;-2)$ √ a
quadrants+shape √ a
asymptotes √ca



d) 1) $x < -1$ √ a

2) $\dfrac{-4}{x+1} + 2 > 0$

$x > 1$ *or* $x < -1$ √ a√ a

QUESTION 3 [9 marks]

(a) $y = a(t-1)^2 + 4$ — √a subst(1;4)

$0 = a(2-1)^2 + 4$ — √a subst(2;0)/(0;0)

$-4 = a$ — √ca neg value of a

$\therefore y = -4(t-1)^2 + 4$ — √ca equation

(b) TP $h(t-2) = (3;4)$ — √ca √ca co-ords

(c) *one* x *– many* y *values* — √a

(d) $x \geq 1$ *or* $x \leq 1$ — √a √a

QUESTION 4 [17 marks]

(a) $A = P(1-i)^n$

$$69\,750 = 275\,0009\left(1-\frac{7}{100}\right)^n$$ √m subst into of Pv formula

$$\frac{68\,750}{275\,000} = \left(1-\frac{7}{100}\right)^n$$

$$n = \log_{\left(1-\frac{7}{100}\right)}\left(\frac{68\,750}{275\,000}\right)$$ √a use of log

$n = 19{,}1\ldots$

$\therefore 19\ years$ √a answer

(b) (1) $$F = \frac{x[(1+i)^n - 1]}{i}$$

$$= \frac{1400\left[\left(1+\frac{18}{1200}\right)^{60} - 1\right]}{\frac{8}{1200}}$$ √a subst √m use of Fv formula

$= \text{R}134\ 700{,}51$ √a subst √m use of Pv formula

(2) $$A = 134\,700{,}51\left(1+\frac{5}{400}\right)^2$$ √m subst into of compound formula

$= R138\,089{,}07$ √ca answer

(c) (1) $Loan\ amt = 700\,000 - 70\,000 = R630\,000$ √a balance

$$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$630\,000 = \frac{x\left[1-\left(1+\frac{18}{1200}\right)^{-180}\right]}{\frac{18}{1200}}$$ √a subst √m use of Pv formula

$\therefore x = R10\,145{,}65$ √a answer

(2) $P = \dfrac{x\left[1-(1+i)^{-n}\right]}{i}$

$$P = \frac{10145,65\left[1-\left(1+\frac{18}{1200}\right)^{-130}\right]}{\frac{18}{1200}}$$

√a √m subst into Pv

√a n value

$\therefore P = R578\,742,21$

√a answer

QUESTION 5 [16 marks]

(a) $f(x)=1-3x^2$

$f(x+h)=1-3(x+h)^2$

$=1-3(x^2+2xh+h^2)$

$=1-3x^2-6xh-3h^2$ √a f(x+h)

$f'(x)=\lim_{h\to 0}\frac{1-3x^2-6xh-3h^2-1+3x^2}{h}$ √m subst into first principles formula

$=\lim_{h\to 0}\frac{h(-6x-3h)}{h}$ √m h- common factor

$=\lim_{h\to 0}-6x-3h$

$=-6x$ √a answer

(b)

(1) $y=x^2.x^{-\frac{1}{2}}-1.x^{-\frac{1}{2}}$ √a getting rid of roots and fractions

$y=x^{\frac{3}{2}}-x^{-\frac{1}{2}}$ √m simplification

$\therefore \frac{dy}{dx}=\frac{3}{2}x^{\frac{1}{2}}+x^{-\frac{3}{2}}$ √a √a differentiation

(2) $y=\frac{x(x^2-5x+6)}{(x-2)}$

$=\frac{x(x-3)(x-2)}{(x-2)}$ √m factorisation

$\therefore \frac{dy}{dx}=2x-3$ √a √a differentiation

(c) $\frac{d}{dx}(f(x))+\frac{d}{dx}(3.g(x))=3x^2+3x$ √a √a

(d) $f'(x)=3$ *and* $g(3)=11=f(3)$ √a √a

$\therefore f(3)+f'(3)$

$=11+3$ √ca answer

$=14$

QUESTION 6 [5 marks]

$$\begin{array}{ccccccc} x & & 17 & & y & & 57 & & 86 \\ & 17-x & & y-17 & & 57-y & & 29 & \\ & & y-34+x & & & 74-2y & & -28+y & \end{array}$$

√ a setting up differences

√m finding 2nd difference

$-28+y=74-2y$

$3y=102$

$y=34$

√m equating 2nd difference

√a answer

$y-34+x=74-2y$

$34-34+x=74-2(34)$

$x=74-68$

$x=6$

√a answer

SECTION A [75 MARKS]

SECTION B

QUESTION 7 [14 marks]

(a) (1) 600 ; 900 ; 1200 ; ….

300 300 √a finding d

$T_n = a + (n-1)d$

$T_{15} = 600 + (14)(300)$

$= 4800m$ √m substation into linear formula

$= 4,8km$

√a answer

(2)

$S_{15} = \frac{15}{2}[2(600) + (14)(300)]$

$= 40\,500m$ √a √m subst into sum formula

$= 40,5km$

√a answer

(3)

$42\,000 = 600 + (n-1)(300)$

$42\,000 - 600 = 300n - 300$ √m subst into Tn formula

$41\,700 = 300n$

$n = 139$

√ a answer

She would need just over 4 months to run 42km, therefore she will qualify and be eligible in 6 months. √ m conclusion

(b)

$r = m + 1$

1) $-1 < m + 1 < 1$ √ a value of r √ m (-1<r<1) √ a answer

$-2 < m < 0$

$S_\infty = \frac{m}{1-(m+1)}$

2) $= \frac{m}{-m}$ √ m Subst into correct formula√ a answer

$= -1$

QUESTION 8 [9 marks]

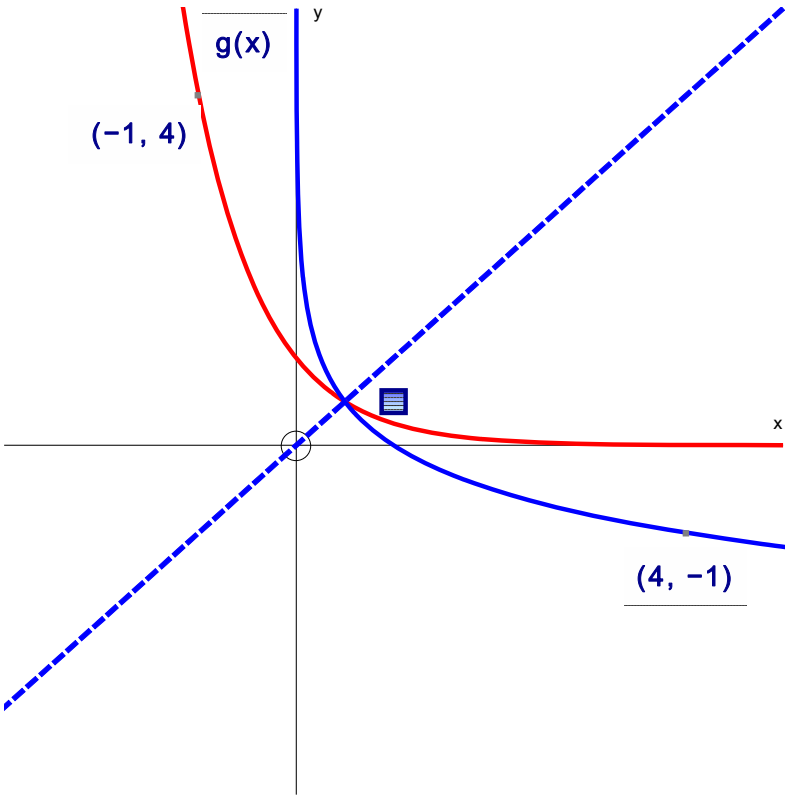
(a)

$y = 4^{-x}$

$x = \left(\frac{1}{4}\right)^{y}$ √m swop x and y

$\therefore y = \log_{\frac{1}{4}} x$ √a answer

(b)



√m √m shape of g and g^{-1}

√ a √a points on g and g^{-1}

(c)

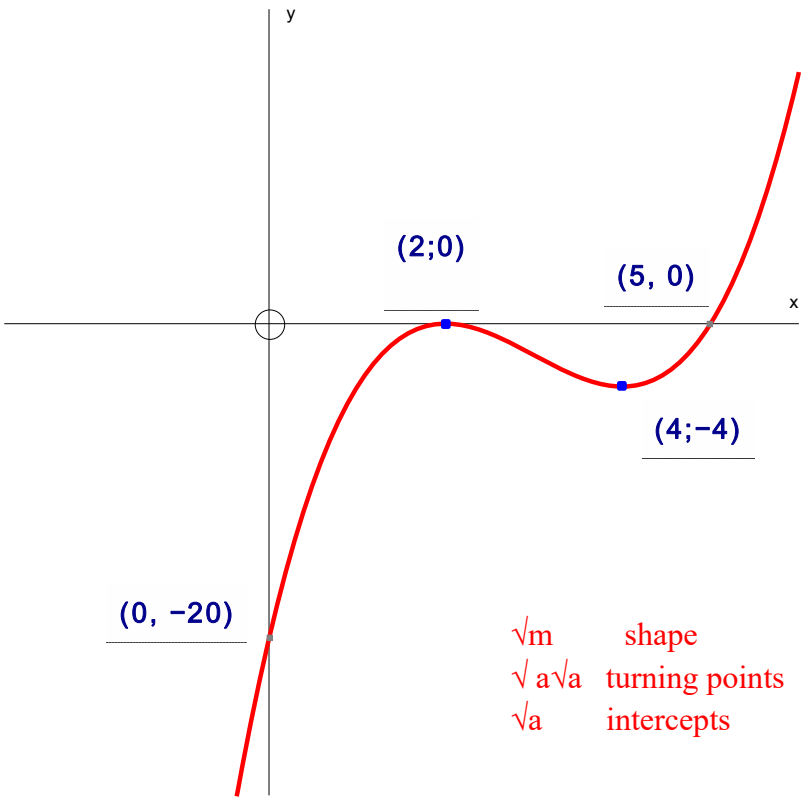
$\frac{\log x}{\log \frac{1}{4}} = \log_{\frac{1}{4}} x$ √a simplification of logs

$\therefore \log_{\frac{1}{4}} x \geq 0$

$\therefore 0 < x \leq 1$ √a √a answer, correct inequality signs

QUESTION 9 [8 marks]

(a)



√m shape
√ a√a turning points
√a intercepts

(b)

$f'(x) = 3x^2 - 18x + 24$

$f'(3) = -3$

$\therefore y = -3x + c$

$subst(3;-2)$

$-2 = -33) + c$

$7 = c$

$\therefore y = -3x + 7$

√a grad of tangent

√a equation

(c)

$f''(x) < 0$

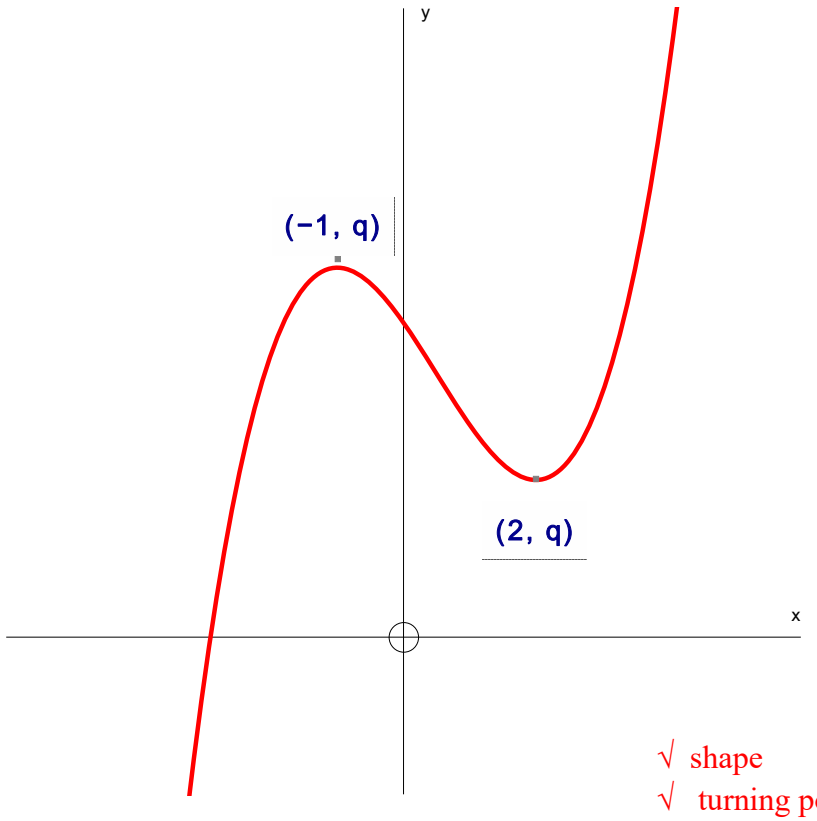
$6x - 18 < 0$

$x < 3$

√ca f "<0
√ca answer

QUESTION 10 [9 marks]

(a)



√ shape
√ turning points
√ one root

(b) One root

√a answer

(c) $y = x^3 + bx^2 + cx + d$

$\frac{dy}{dx} = 3x^2 + 2bx + c$

√a finding derivative and =0

$3x^2 + 2bx + c = 0$

$3(-1)^2 + 2(-1) + c = 0$

$3 - 2b + c = 0$

$-2b + c = 0$

$3(2)^2 + 2b(2) + c = 0$

$12 + 4b + c = 0$

$4b + c = -12$

$\therefore b = \frac{-3}{2}$ *and* $c = -6$

√m subst x values of TP

√a√a values for b and c

QUESTION 11 [6 marks]

$2x+2y=30$

$x+y=15$

$x=15-y$ √a x value

$A=l\times b$

$=x(y-x)+xy$

$=(15-y)(y-(15-y))+(15-y)(y)$

$=(15-y)(2y-15)+15y-y^2$ √m area

$=30y-225-2y^2+15y+15y-y^2$ √a answer

$=-3y^2+60y-225$

$\frac{dA}{dy}=-6y+60$

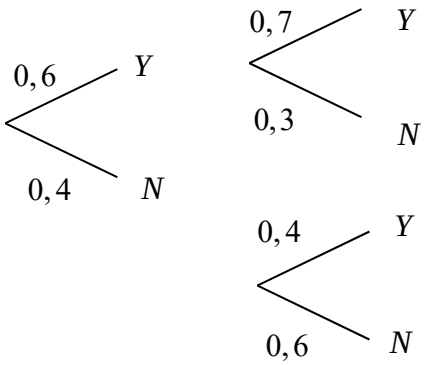
$0=-6y+60$ √ca derivative =0

$y=10$ √ca y value

$\therefore x=5$ √ca x value

QUESTION 12 **[15 marks]**

(a)



√m √m tree diagram

P(Y from 2nd judge)= $(0,6\times0,7)+(0,4\times0,4)$
$=0,58$

√ca from tree diagram

√a answer

(b) (1) $8!=40\,320$ √√a answer

(2) $6!=720$ √√a answer

(3) If we group Gauteng and Limpopo and consider them as "venues" then there are 5 "venues" which can be visited in 5! different ways but we need to multiply this by 2! for the different orders for Gauteng and Limpopo and then by 3! for the different orders possible within Gauteng.

So, final answer is 5!3!2! = 1440.

√√√a answer

(c) (1) P(F, not qualifying)$=\frac{5163}{10225}$ √a answer

(2)

P(F) and P(qualifying)

$= \frac{5200}{10225} \times \frac{79}{10225}$

$= \frac{410800}{104550625}$

√m finding prob
√a correct prob

P(F and going through)

$= \frac{37}{10225}$

$\therefore \neq$

$\therefore$ *not independent*

√ca conclusion

QUESTION 13 [8 marks]

(a) $f(r)=0$ √ a

$2(r)^2+3p(r)-3=0$

$2r^2+3pr-3=0$

$2r^2=3-3pr$

$g(r)=0$

$2(r)^2+(p-2)(r)-1=0$ √ a

$2r^2+pr-2r-1=0$

$2r^2=2r+1-pr$

$3-3pr=2r+1-pr$

$-3pr+pr-2r=-2$

$-2pr-2r=-2$

$pr+r=1$ √ m equating √m simplification

$r(p+1)=1$

$\therefore r=\dfrac{1}{p+1}$

√ a answer

(b)

$2\left(\dfrac{1}{p+1}\right)^2+3p\left(\dfrac{1}{p+1}\right)-3=0$ √ m substitution

$2\left(\dfrac{1}{(p+1)^2}\right)+\dfrac{3p}{p+1}-3=0$ √ a LCD

$\dfrac{2}{(p+1)^2}+\dfrac{3p}{p+1}-3=0$ √ a answer

$2+3p(p+1)-3(p+1)^2=0$

$2+3p^2+3p-3(p^2+2p+1)=0$

$2+3p^2+3p-3p^2-6p-3=0$

$-3p-1=0$

$p=-\dfrac{1}{3}$

$\therefore r=\dfrac{1}{-\frac{1}{3}+1}$

$=\dfrac{3}{2}$

QUESTION 14 [6 marks]

Let radius of cylinder = R, cone 1 = r1, cone 2 = r2.

Let their volumes of cylinder = V, cone 1 = v1, cone 2 = v2.

Cylinder is melted and recast into cone 1 & cone 2, So V = v1 + v2.

Ratio of volumes of the two cones v1 : v2 is 3 : 4.

If v1 is 3x, v2 will be 4x, hence, volume of cylinder V= 7x √ m

V cylinder = $\pi R^2 h$ V cone 1= $1/3\ \pi r_1^2 h$ V cone 2= $1/3\ \pi r_2^2 h$

Ratio of the volumes V : v1: v2 = 7 : 3 : 4

So, $\pi R^2 h : 1/3\ \pi r_1^2 h : 1/3\ \pi r_2^2 h = 7 : 3 : 4$

Cancelling π and h, which are common to all terms, we get

$R^2 : 1/3\ r_1^2 : 1/3\ r_2^2 = 7 : 3 : 4$ √ a

So, if R^2 is 7k, $r1^2$ will be 9k and $r2^2$ will be 12k. √ a

Flat surface area of cylinder (sum of the areas of the two circles at the top and bottom of the cylinder) = $2 * \pi * R^2$

Flat surface area of cone 1 & 2 are: $\pi * r1^2$ & $\pi * r2^2$ respectively (areas of the circle at the bottom of each of the cones).

Ratio of the flat surface area of cylinder to that of the two cones is $2 * \pi * R^2 : (\pi * r1^2 + \pi * r2^2)$

Cancelling π on both sides of the ratio we get $2R^2 : (r_1^2 + r_2^2)$ Or 14k : 21k √ m

Change in flat surface area is 21k-14k=7k √ a

So percentage change in flat surface area = $\frac{7k}{14k} \times 100\% = 50\%$ √ a