HILTON COLLEGE

TRIAL EXAMINATION
AUGUST 2016

CORE MATHEMATICS PAPER 2

Time: 3 hours 150 marks

PLEASE READ THE FOLLOWING GENERAL INSTRUCTIONS CAREFULLY.

1. This question paper consists of 24 pages. There is also a **separate** yellow information sheet. Please check that your paper is complete.
2. Read the questions carefully.
3. This question paper consists of 13 questions. Answer all questions.
4. You may use an approved non-programmable and non-graphical calculator, unless a specific question prohibits the use of a calculator.
5. Round off your answers to one decimal digit where necessary, unless otherwise stated.
6. All necessary working details must be shown.
7. It is in your own interest to write legibly and to present your work neatly.
8. Please note that the diagrams are **NOT** necessarily drawn to scale.
9. Please ensure that your calculator is in DEGREE mode.

Please do not turn over this page until you are asked to do so

EXAMINATION NUMBER:

M	E	M	O

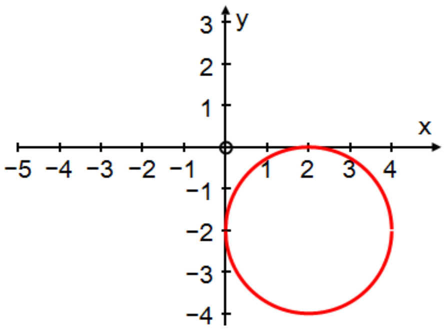
I pledge that I have neither given nor received help with this assessment.

Date: ______________________________ **Signed**: ______________________________

SECTION A

QUESTION 1

(a) Give the equation of the circle shown below: (2)

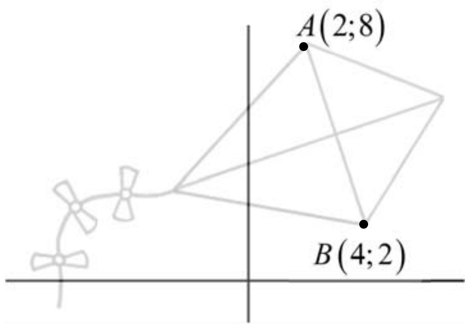


$(x-2)^2+(y+2)^2=4$ ✓a radius ✓a centre

(b) Give the equation of the line which has an angle of inclination of 45° and and passes through the point $(0;-3)$. (2)

$y=x-3$ ✓a slope ✓a intercept

(c) *"The longer diagonal of a kite bisects the shorter diagonal at 90°"*
Use the above statement to find the equation of the longer diagonal. (4)



$mid-pt \quad of \quad AB=(3;5)$ ✓a mid-pt

$gradient \quad of \quad AB=-3$ ✓a slope

$gradient \quad of \quad longer \quad diagonal=\frac{1}{3}$ ✓m -'ve reciprocal

$\therefore 5=3\left(\frac{1}{3}\right)+c$ ✓m substitution

$\therefore c=4$

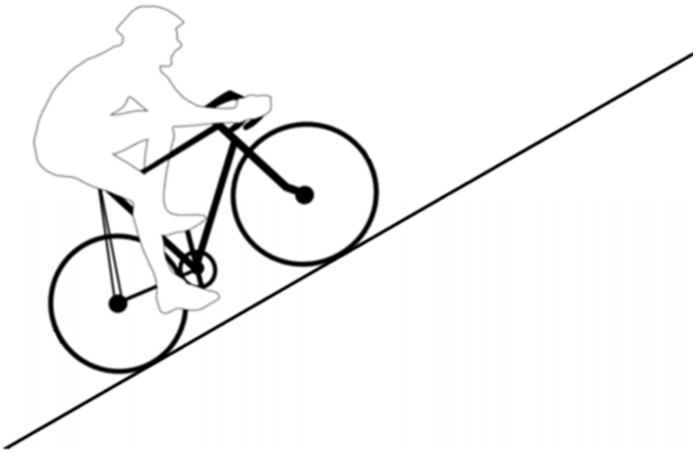
$\therefore y=\frac{1}{3}x+4$ ✓ca

8

http://www.clipartkid.com/kite-clipart-black-and-white-clipart-panda-free-clipart-images-wge8V1-clipart/

QUESTION 2

Consider the sketch of Guy cycling up a hill. The equation of the straight line representing the road is $y = \dfrac{x}{\sqrt{3}}$ and the equation of the circle representing the rear wheel is $x^2 - 8x + y^2 - 4y = 1004$



(a) Determine the angle the road makes with the horizontal. (2)

$m = \tan\theta$ ✓m link to gradient

$\therefore \theta = \tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right) = 30°$ ✓ca

http://cliparting.com/free-bike-clip-art-11887/

(b) Assuming that units are in cm, determine the **diameter** of the rear wheel, in cm. (4)

$x^2 - 8x + y^2 - 4y = 1004$

$\therefore (x-4)^2 + (y-2)^2 = 1004 + 16 + 4$ ✓m completing the square

$\therefore (x-4)^2 + (y-2)^2 = 1024$ ✓a

$\therefore radius \quad is \quad 32$ ✓ca

$diameter \quad is \quad 64 \quad cm$ ✓ca

6

QUESTION 3

Determine the possible value(s) of k if the point $D(3;k)$ is a distance of 10 units from the point $A(9;3)$.

$\sqrt{(3-9)^2+(k-3)^2}=10$ ✓m distance formula ✓a

$\therefore 36+(k-3)^2=100$

$\therefore (k-3)^2=64$ ✓m solving for k

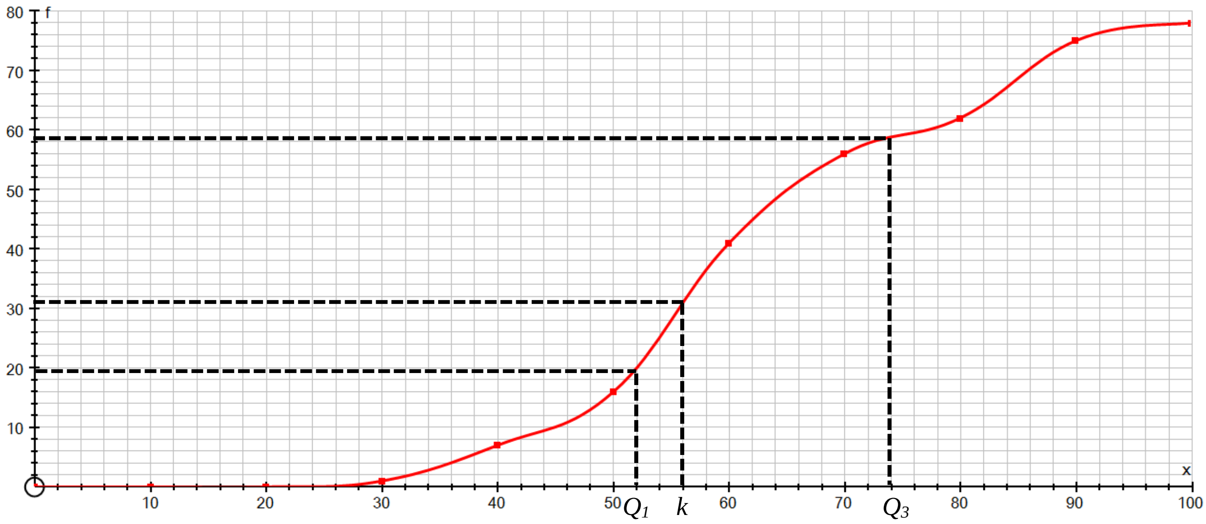
$\therefore k=3\pm 8$

$\therefore k=11 \quad or \quad -5$ ✓ca ✓ca

5

QUESTION 4

(a) The Grade 12 Core Maths Marks for Term 2 are depicted in the cumulative frequency curve shown below:



(i) How many matrics do Core Maths? (1)

78 ✓a

(ii) Determine the inter-quartile range of the marks. Show by means of dotted lines where you have read off any values you have used in your calculation. (3)

$Q_3 = 74$ ✓m for showing on graph

$Q_1 = 52$ ✓a for both correct

$\therefore IQR = 74 - 52 = 22$ ✓m for subtracting Q_1 from Q_3

(iii) What percentage of pupils achieved a distinction (80% or more)? (2)

$\frac{16}{78} \times 100 \approx 20.5\%$ ✓a
✓m for % calculation

(iv) Give a value for *k* if 40% of candidates achieved a mark of less than *k*? (2)

$40\% \equiv\sim 31 \quad candidates$ ✓a

$k = 56\%$ ✓ca

(b) Ten Grade 11 boys achieved the following marks in Science and Mathematics in the June 2016 Examinations.)

Science (x)	**Maths (y)**
90	93
74	61
49	60
87	88
77	82
72	62
77	90
62	43
64	77
89	80

(i) Determine the equation of the line of best fit, the least squares regression line. Give both parameters to 3 decimal places. (2)

$y = 0.886x + 7.988$ ✓a slope ✓a intercept

(ii) Use your answer to (b) (i) to predict the Maths mark for a boy who achieves a mark of 80% for Science. (2)

78.9% ✓a ✓m for substitution if shown

(iii) Calculate, to 2 decimal places, the correlation coefficient and comment on what it means for the relationship between the Science and Mathematics marks given. (3)

$r = 0.71$ ✓a

✓a ✓a

There is a reasonably strong, positive relationship between performance in Maths and performance in Science.

15

QUESTION 5

(a) In each case simply give the quadrant(s) in which θ must lie if:

(i) $\cos\theta > 0$ *and* $\tan\theta < 0$ (1)

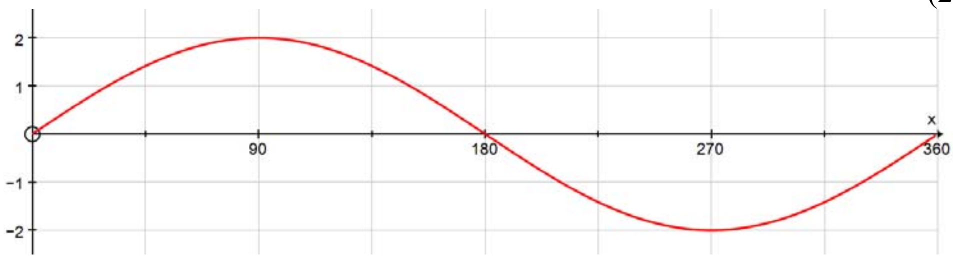
IV ✓a

(ii) $\sin\theta = -\frac{3}{5}$ *and* $\tan\theta = \frac{3}{4}$ (1)

III ✓a

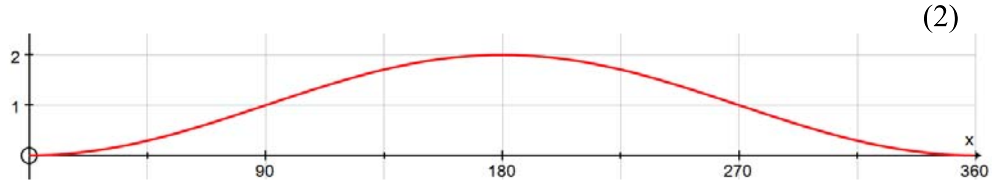
(b) Give the equations for each of the following graphs:

(i) (2)



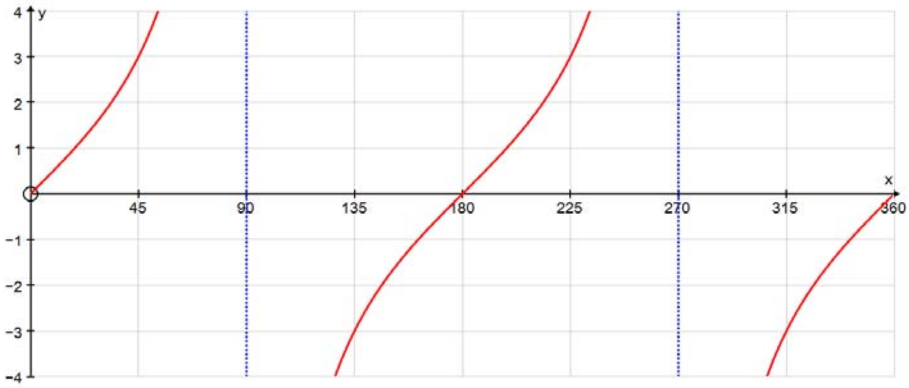
$y = 2\sin x$ ✓a sin ✓a amplitude of 2

(ii) (2)



$y = -\cos x + 1$ ✓a cos ✓a reflected and shifted

(iii) (2)



$y = 3\tan x$ ✓a tan ✓a amplitude

(c) If $\sin 20° = p$ then determine the following in terms of p:

(i) $\cos 20°$ (2)

$$\cos 20° = \sqrt{1-\sin^2 20} \quad \checkmark m \quad \text{identity / diagram}$$
$$= \sqrt{1-p^2} \quad \checkmark a$$

(ii) $\sin(-200°)$ (2)

$= \sin 20°$ ✓m reduction

$= p$ ✓a

(iii) $\cos 250°$ (2)

$= -\sin 20°$ ✓m reduction

$= -p$ ✓a

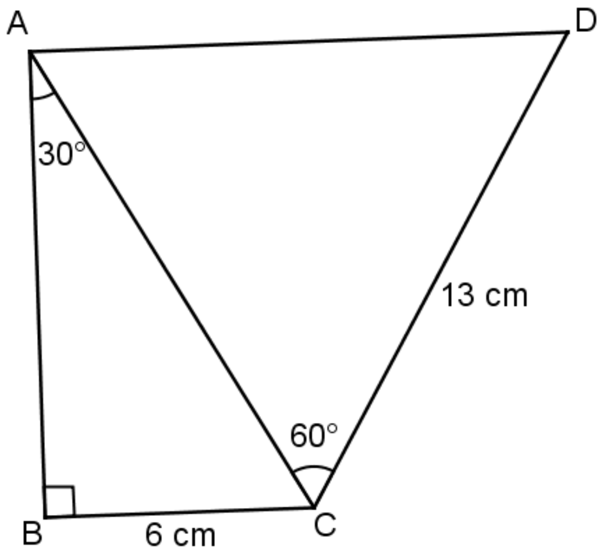
(iv) $\cos 140°$ (2)

$= -\cos 40°$

$= -(1-2\sin^2 20°)$ ✓m double angle formula

$= 2p^2 - 1$ ✓a

(d) Consider the diagram with lengths and angles as marked:



(i) Determine, to one decimal place if necessary, the length of AC. (2)

$\sin 30° = \dfrac{6}{AC}$ ✓m correct method

$\therefore AC = \dfrac{6}{\sin 30°} = 12 \quad cm$ ✓ca

(ii) Hence, or otherwise, determine, to one decimal place, the length of AD. (3)

$AD^2 = 12^2 + 13^2 - 2 \times 12 \times 13 \times \cos 60°$ ✓m cos rule ✓a

$= 157$

$\therefore AD = 12.5 \quad cm$ ✓ca

21

QUESTION 6

(a) Complete the following statements:

(i) If a line cuts two sides of a triangle in the same proportions then…. (1)

It is parallel to the third side ✓a

(ii) The exterior angle of a cyclic quadrilateral is …. (1)

Equal to the opposite interior angle ✓a

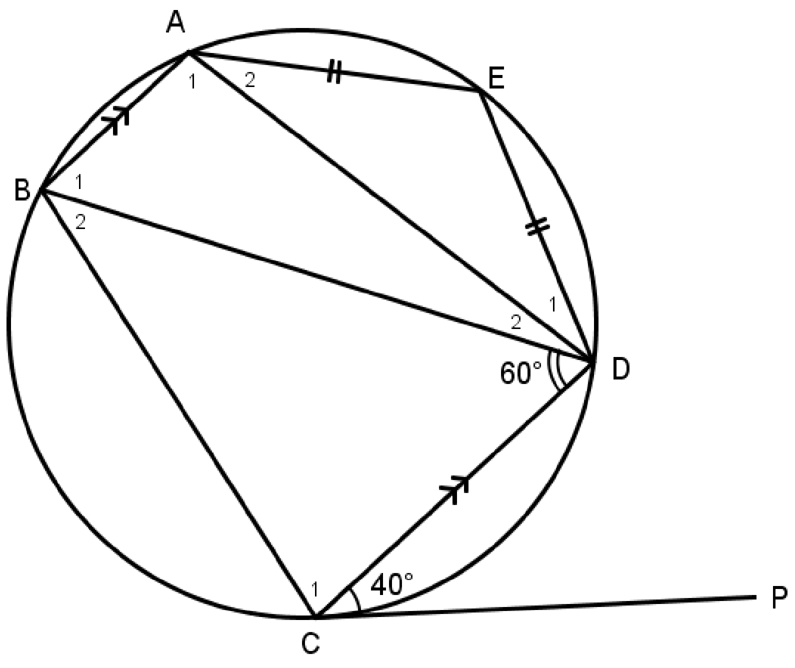
(iii) If two triangles have corresponding sides in the same proportion then …. (1)

They are equiangular ✓a

OR

They are similar ✓a

(b) In the diagram below CP is a tangent to the circle at C. AB || CD. AE = ED. $P\hat{C}D = 40°$ and $B\hat{D}C = 60°$



Calculate, giving reasons, the sizes of:

(i) $\hat{B}_2$ (2)

$40°$ ✓a $(tan\text{-}chord$ ✓a $theorem)$

(ii) $\hat{B}_1$ (1)

$60°$ ✓a with reason $(alt.\ \angle's\ on\ \|\ lines)$

(iii) $\hat{E}$ (2)

$120°$ ✓a $(opp.$ ✓a $\angle's\ of\ CQ)$

(iv) $\hat{D}_1$ (2)

$30°$ ✓a $(sum$ ✓a $of\ \angle's\ of\ isos\ \Delta)$

(v) $\hat{A}_1$ (2)

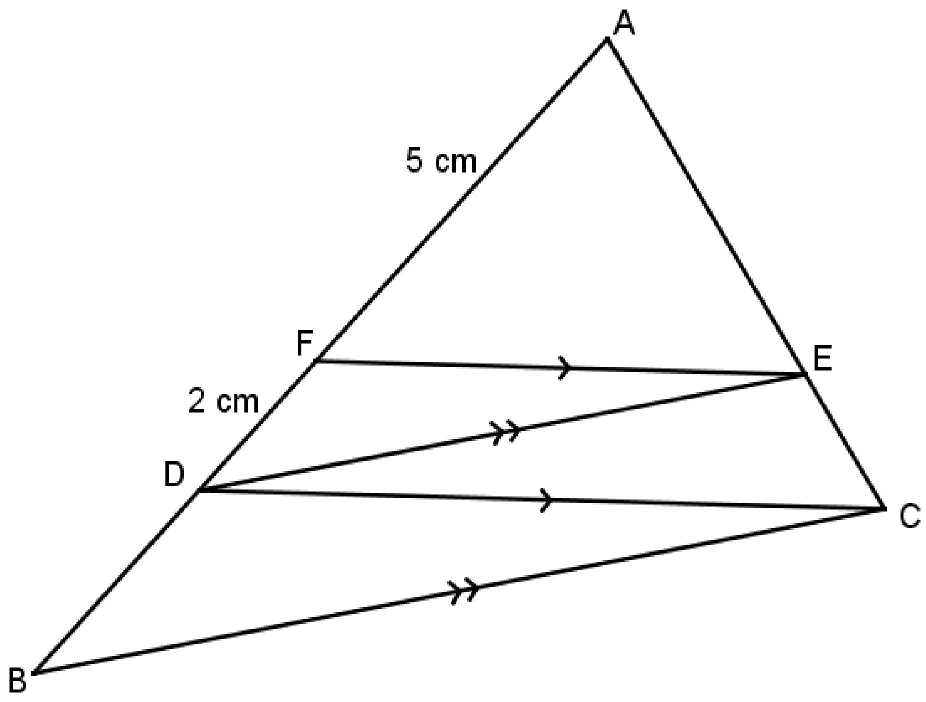
$\hat{D}_2 = 20°$ ✓a $(opp.$ ✓a $\angle s\ of\ CQ)$

$\therefore \hat{A}_1 = 100°\ (sum\ \angle s\ of\ \Delta)$

12

QUESTION 7

(a) In the diagram below, FE || DC and DE || BC. AF = 5 cm and FD = 2 cm.



Calculate, with reasons, the length of DB to one decimal place. (5)

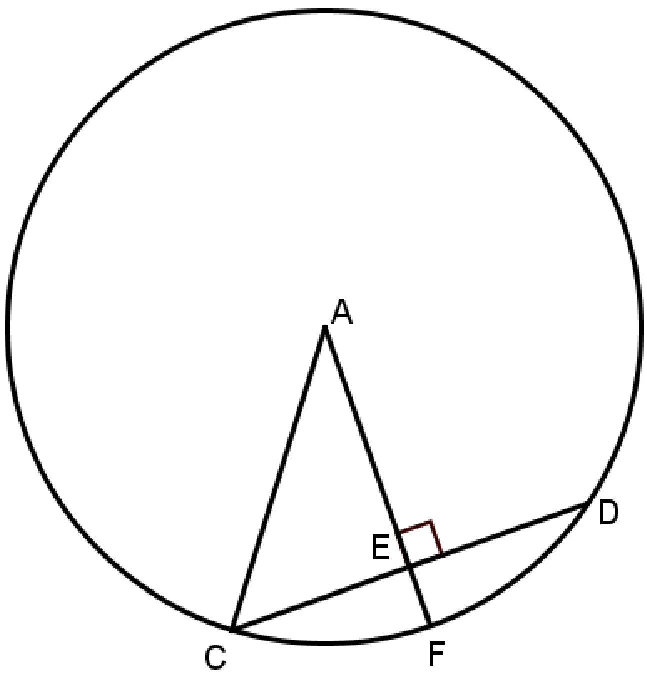
$\frac{AF}{FD} = \frac{AE}{EC} = \frac{5}{2}$ ✓a $(prop. \quad int. \quad theorem)$ ✓a

$but \quad \frac{AD}{DB} = \frac{AE}{EC} = \frac{5}{2} \quad (prop. \quad int. \quad theorem)$ ✓m seocnd use of prop. int. theorem

$\therefore \frac{7}{DB} = \frac{5}{2}$ ✓a

$\therefore DB = 2.8\,cm$ ✓ca

(b) In the diagram below A is the centre of the circle. CD = 6 cm and EF = 1 cm. Calculate, giving reasons, the length of the radius.
Hint: let the radius be x. (3)



$CE = 3 \quad cm \quad (\perp \quad from \quad centre)$ ✓wj

$\therefore (x-1)^2 + 3^2 = x^2 \quad (Pythag.)$ ✓wj

$\therefore x = 5$ ✓ca

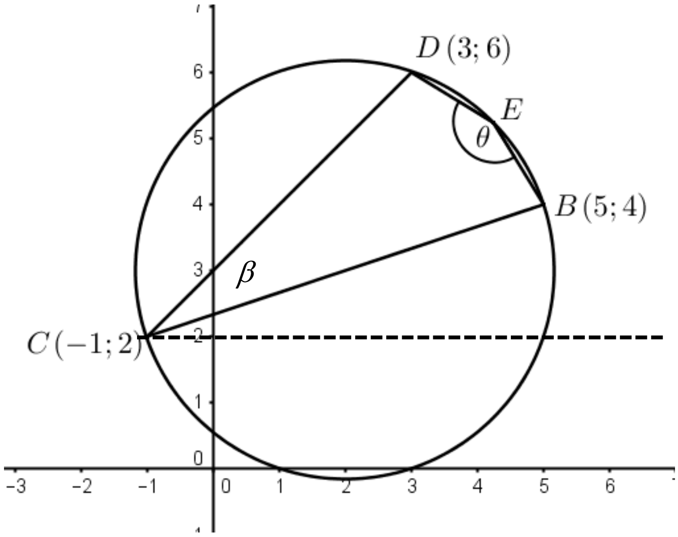
8

TOTAL FOR SECTION A: 75 MARKS

SECTION B

QUESTION 8

Consider the diagram:



(a) Determine θ to 1 decimal place, giving reasons where necessary. (5)

$\beta = \tan^{-1} 1 - \tan^{-1}\frac{1}{3}$ ✓m ✓a

$\therefore \beta = 45° - 18.4°$ ✓ca

$\therefore \beta = 26.6°$

$\therefore \theta = 153.4° \quad (opp \quad \angle s \quad of \quad CQ)$ ✓ca ✓reason

(b) If it is further given that BC is a diameter of the circle then find the equation of the circle. (4)

$centre \quad is \quad \left(\frac{-1+5}{2};\frac{2+4}{2}\right) = (2;3)$ ✓m ✓a

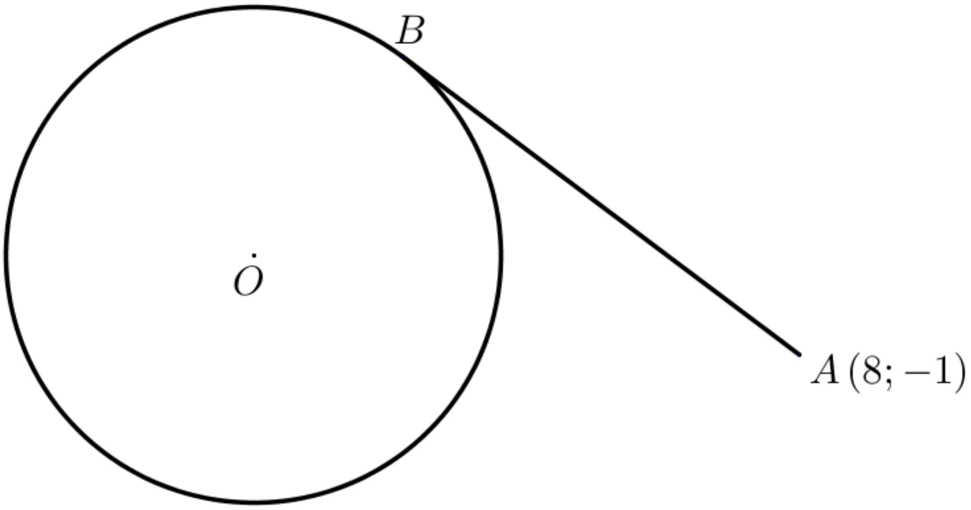
$r^2 = (4-3)^2 + (5-2)^2 = 10$ ✓a

$\therefore (x-2)^2 + (y-3)^2 = 10$ ✓ca

9

QUESTION 9

Consider the diagram below, showing the circle with equation $x^2 + y^2 + 6x - 2y - 15 = 0$



Determine, to 1 decimal place, the length of the tangent (*AB*) to the circle from the point $A(8;-1)$, to the point of contact *B,* giving reasons where necessary.

$(x+3)^2 + (y-1)^2 = 15+1+9 = 25$ ✓m finding centre and radius ✓a

$\therefore r^2 = 25 \quad and \quad r = 5$

$AB^2 = AO^2 - r^2 \qquad (pythagoras, \quad radius \perp tangent)$ ✓m with reason

$but \quad AO^2 = 11^2 + 2^2 = 125$ ✓a

$\therefore AB^2 = 125 - 25$ ✓a

$\therefore AB = 10 \quad units$ ✓ca

6

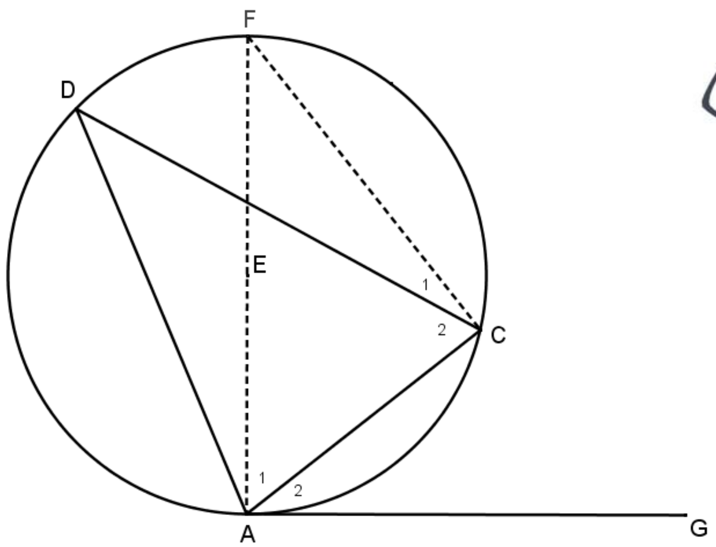
QUESTION 10

(a) John was asked to prove the ***tan-chord*** theorem, viz. that *"the angle between a tangent and a chord is equal to any angle subtended by that chord in the alternate segment"*.

He knows that he needs to prove that $\hat{A}_2 = \hat{D}$ and he has remembered that he needs to construct the diameter and to join FC.

He has made the constructions correctly with dotted lines but now he is stuck!

Complete the proof for John. (6)



$let \quad \hat{A}_2 = x$ ✓m

$\hat{A}_1 + \hat{A}_2 = 90° \quad (radius \perp tangent)$ ✓wj

$\therefore \hat{A}_1 = 90° - x$

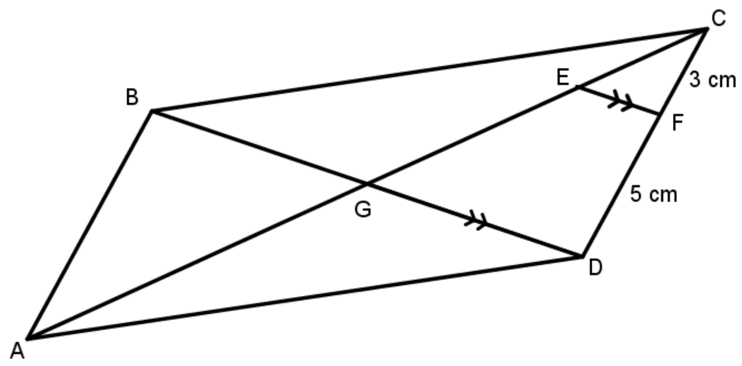
$but \quad \hat{C}_1 + \hat{C}_2 = 90° \quad (\angle \quad in \quad semi-circle)$ ✓wj

$\therefore \hat{F} = x \quad (\angle s \quad of \quad \Delta)$ ✓wj

$but \quad \hat{D} = x \quad (\angle s \quad in \quad same \quad segment)$ ✓wj

$\therefore \hat{A}_2 = \hat{D}$ ✓a

(b) Consider the diagram below showing parallelogram ABCD with diagonals AC and BD drawn, intersecting at G. EF||BD , CF = 3 cm and FD = 5 cm



(i) Determine the ratio $\frac{CE}{EA}$ giving reasons. (4)

$\frac{CE}{EG} = \frac{CF}{FD} = \frac{3}{5}$ $(prop.\ \ \text{int}.\ \ theorem)$ ✓wj

$let\ \ CE = 3p\ \ then\ \ EG = 5p$ ✓m

$\therefore GA = 8p\ \ (diagonals\ \ of\ \ parm.\ \ bisect\ \ one\ \ another)$ ✓a

$\therefore \frac{CE}{EA} = \frac{3p}{13p} = \frac{3}{13}$ ✓a

(ii) Determine $\frac{Area\ \ \Delta CEF}{Area\ \ ABCD}$ giving reasons. (4)

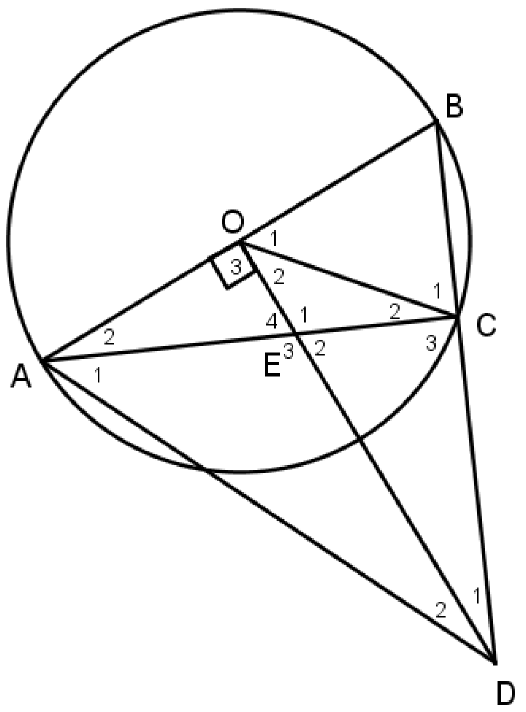
$\Delta CEF\ |||\ \Delta CGD\ \ (AAA)$ ✓a

$\therefore \frac{\Delta CEF}{\Delta CGD} = \left(\frac{3}{8}\right)^2 = \frac{9}{64}$ $\left(\begin{matrix} areas\ \ of\ \ similar\ \ figures\ \ are\ \ in\ \ the\ \ ratio \\ equal\ \ to\ \ the\ \ square\ \ of\ \ the\ \ ratios\ \ of \\ their\ \ sides \end{matrix}\right)$ ✓wj

$but\ \ CGD = \frac{1}{4}\ \ parm.\ \ ABCD$ $\left(\begin{matrix} diagonals\ \ bisect\ \ area\ \ of\ \ parm\ \ and \\ \Delta CGD = \Delta DAG\ \ equal\ \ base\ \ and\ \ height \end{matrix}\right)$ ✓wj

$\therefore \frac{\Delta CEF}{ABCD} = \frac{9}{256}$ ✓a

(c) In the diagram below, AB is a diameter of circle ABC with centre O. Chord BC is produced to D. $OD \perp AB$ and OD cuts AC at E.



Prove, giving reasons:

(i) That AOCD is a cyclic quadrilateral. (4)

$\hat{C}_1 + \hat{C}_2 = 90°$ $(\angle$ *in semi – circle*$)$ ✓wj

$\therefore \hat{C}_3 = 90°$ $(\angle s$ *on straight line*$)$ ✓wj

$\therefore \hat{C}_3 = \hat{O}_3$ ✓a

$\therefore$ *AOCD is a cyclic quadrilateral* (*converse* $\angle s$ *in same segment*) ✓wj

(i) $\hat{C}_2 = \hat{D}_1$ (3)

$\hat{D}_1 = \hat{A}_2$ $(\angle s$ *in same segment*$)$ ✓wj

but $\hat{A}_2 = \hat{C}_2$ (*isos.* Δ *radii*) ✓wj

$\therefore \hat{C}_2 = \hat{D}_1$ ✓a

(iii) $\Delta OCE \parallel\!| \Delta ODC$ (4)

$\hat{C}_2 = \hat{D}_1 \quad (proved)$ ✓a

$\hat{O}_2 \quad is \quad common$ ✓a

$\therefore \hat{E}_1 = \hat{C}_2 + \hat{C}_3 \quad (third \quad \angle \quad of \quad \Delta)$ ✓wj

$\therefore \Delta OCE \parallel\!| \Delta ODC \quad (AAA)$ ✓wj

(iv) $OE.OD = OC^2$ (2)

$\frac{OC}{OD} = \frac{OE}{OC} \quad (\parallel\!| \quad \Delta s)$ ✓wj

$\therefore OC^2 = OE.OD$ ✓a

27

QUESTION 11

The marks of a class of 23 boys have a mean of 73% and a standard deviation of 10.

Angus and Gavin have marks of 79% and 67% respectively.

Calculate, to 2 decimal places, the standard deviation of the remaining boys if Angus and Gavin leave the class.

$Variance \quad initially = 100$ ✓a

$\therefore \sum_{i=1}^{23}\left(x_i - \overline{x}\right)^2 = 2300$ ✓a

Suppose Angus and Gavin are number 22 *and* 23 *then*

now, $\left(x_{22} - \overline{x}\right)^2 = 36 \quad and \quad \left(x_{23} - \overline{x}\right)^2 = 36$ ✓m

Since $\overline{x}$ *is unchanged by their leaving*:

$\sum_{i=1}^{21}\left(x_i - \overline{x}\right)^2 = 2300 - 36 - 36 = 2228$ ✓a

so the new variance $= \dfrac{2228}{21}$

and the new standard deviation $= \sqrt{\dfrac{2228}{21}} = 10.30$ ✓ca

5

QUESTION 12

(a) Determine the general solution of the following equations:

(i) $\sin 2\theta \cos 20° - \cos 2\theta \sin 20° = -\frac{1}{2}$ (4)

$\therefore \sin(2\theta - 20°) = -\frac{1}{2}$ ✓m

$\therefore key \quad angle = 30°$ ✓m

$\therefore 2\theta - 20° = 210° + 360k \quad or \quad 2\theta - 20° = 330° + 360k \quad with \quad k \in Z \quad for \quad both$

$\therefore \theta = 115° + 180k \quad or \quad \theta = 175° + 180k$ ✓a ✓a

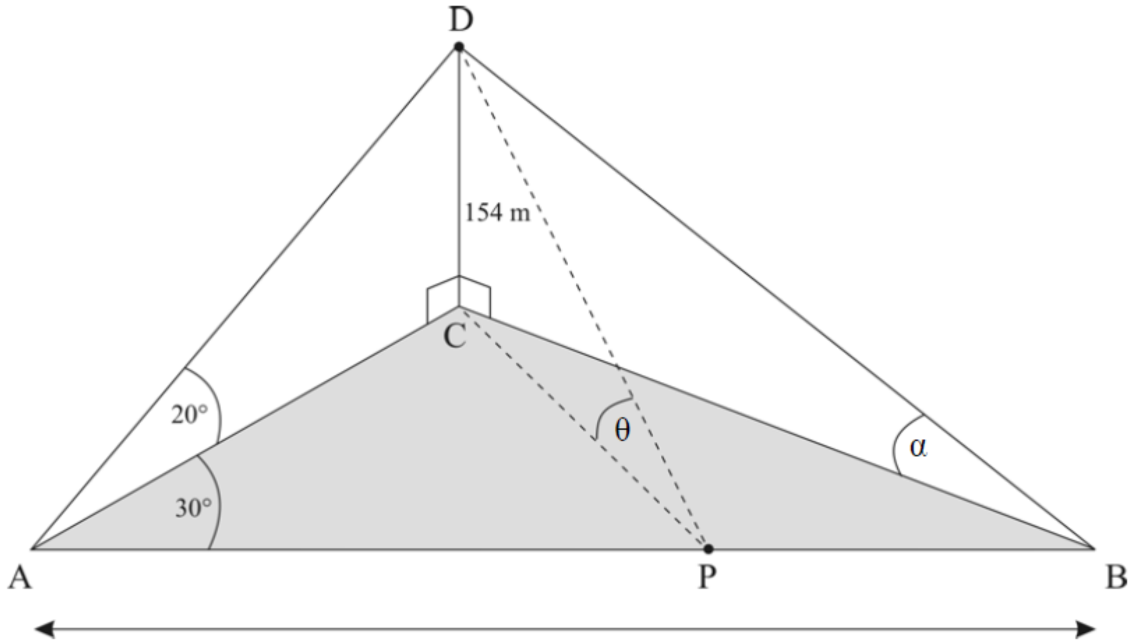
(ii) $\sin\theta = \cos 3\theta$ (5)

$\therefore \sin\theta = \sin(90° - 3\theta)$ ✓m

$\therefore \theta = 90° - 3\theta + 360k$ ✓a $\quad or \quad \theta = 180° - (90° - 3\theta) + 360k$ ✓m $\begin{pmatrix} with \quad k \in Z \\ for \quad both \end{pmatrix}$

$\therefore \theta = 22.5° + 90k$ ✓ca $\quad or \quad \theta = -45° + 180k$ ✓ca

(b) In the diagram below, AB is a straight line 1000m long. P represents an object moving along AB. DC is a vertical tower with C, A and B points in the same horizontal plane. The angles of elevation of D from A and B are $20°$ and α respectively.



(i) Find the length of AC rounded to 2 decimal places. (2)

$$\tan 20° = \frac{154}{AC}$$ ✓a

$$\therefore AC = \frac{154}{\tan 20°} = 423.11 \quad m$$ ✓ca

(ii) Find the value of α to the nearest degree. (6)

$$BC^2 = AC^2 + AB^2 - 2AC.AB\cos B\hat{A}C$$ ✓m cos rule

$$\therefore BC = \sqrt{423.11^2 + 1000^2 - 2\times 423.11\times 1000\times \cos 30°}$$ ✓a

$$\therefore BC = 667.96 \quad m$$ ✓ca

$$now \quad \tan\alpha = \frac{154}{667.96}$$ ✓m trig ratio ✓a

$$\therefore \alpha = \tan^{-1}\frac{154}{667.96} = 13.0°$$ ✓ca

(iii) Let θ be the angle of elevation of D from P. (5)
Determine the **maximum** value of θ to one decimal place.

The angle will be greatest when P is closest to C which will happen when CP is perpendicular to AB ✓m

When this happens:

$$\sin 30° = \frac{CP}{423.11}$$ ✓m using 90° triangle

$$\therefore CP = 423.11 \times \sin 30°$$

$$\therefore CP = 211.56 \quad m$$ ✓a

$$now \quad \tan\theta = \frac{154}{211.56}$$ ✓m

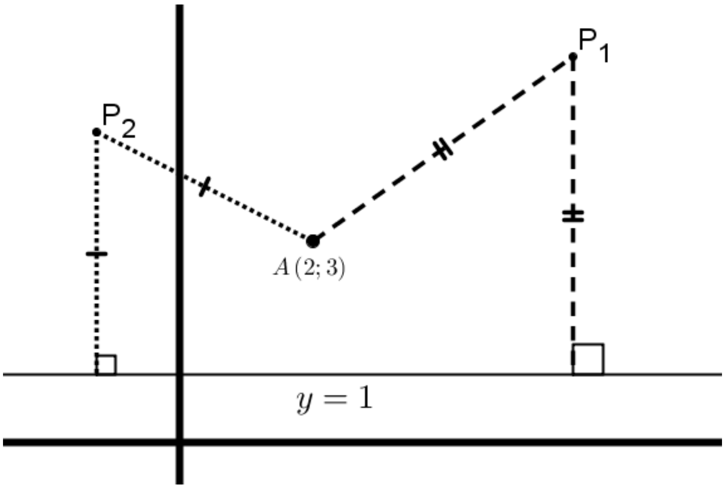
$$\therefore \theta = \tan^{-1}\frac{154}{211.56}$$

$$\therefore \theta = 36.1°$$ ✓ca

22

QUESTION 13

A point P moves in the plane in such a way that its distance from the point $A(2;3)$ is always equal to its distance from the line $y=1$. Two possible positions for P are shown and labelled as P_1 and P_2 to aid your understanding.



By letting the point P be the point $P(x;y)$, determine, in standard form, the equation of the function on which P moves while satisfying the condition of being equidistant from the point $A(2;3)$ and the line $y=1$.

$\sqrt{(x-2)^2+(y-3)^2}=\sqrt{(y-1)^2+0^2}$ ✓m distance formula ✓m equating distances

$\therefore (x-2)^2+(y-3)^2=(y-1)^2+0^2$ ✓m squaring

$\therefore x^2-4x+4+y^2-6y+9=y^2-2y+1$ ✓a

$\therefore x^2-4x+12=4y$ ✓ca

$\therefore y=\frac{x^2}{4}-x+3$ ✓ca

[6]

TOTAL FOR SECTION B: 75 MARKS

TOTAL FOR PAPER: 150 MARKS